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Coulomb screening in the simultaneous presence of a radiation field and a strong DC magnetic field[†]

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Abstract. The effective static dielectric constant of an electron plasma in the simultaneous presence of an electromagnetic wave and a uniform DC magnetic field is discussed. It is found that as the radiation field frequency approaches the plasma frequency a breakdown in screening occurs. As the radiation frequency approaches the electron cyclotron frequency, however, the potential of a static charge felt by the electrons becomes vanishingly small.

1. Introduction

The advent of high-intensity radiation sources in the microwave, infrared and optical spectra has stimulated considerable interest in the study of the interaction of electromagnetic radiation with semiconductors (Ephstein 1970, Bass and Granovskii 1971, Puchkov and Ephstein 1974, Luzzi and Miranda 1978) and plasmas (Cohn *et al* 1972, Seely and Harris 1973, Seely 1974, Amato and Miranda 1976). In a recent paper (Lima and Miranda 1978) the effect of an electromagnetic wave on the screening of a static charge in an electron plasma has been investigated. It was demonstrated that as the radiation field frequency approaches the plasma frequency ω_p , breakdown in screening occurs which becomes apparent as an enhancement of the electron–nuclei Coulomb interaction. In a later paper (Lima *et al* 1978), it has been predicted that such an effect will strongly enhance the contribution of the inverse Bremsstrahlung which is the chief mechanism for heating the plasma. This result is supported by recently reported experimental work (Offenberger *et al* 1978, Meyer *et al* 1978).

In this paper we shall extend our previous work on the modification of the Coulomb screening in the presence of an electromagnetic wave (Lima and Miranda 1978) by including the additional effects of a strong DC magnetic field, encouraged by the possibility of exploring the effects of laser-cyclotron resonance (Seely 1974). In fact, the resonance condition, where the laser frequency equals the electron cyclotron frequency, may be approached either by increasing the magnetic field strength or by using intense longer-wavelength lasers. Intense submillimetre lasers are becoming available (Lax and Cohn 1973) and it is therefore important to consider the effects of the cyclotron resonance on

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the various properties of the electron plasma. Our system, then, consists of an electron gas in a uniform DC magnetic field and a radiation beam, perturbed by the presence of a static charge. Our aim is to calculate the effective potential of this static charge, taking into account the plasma effects. This is performed within the usual RPA by treating the Coulomb interaction between the electrons as a self-consistent field. The radiation beam is treated as a classical plane electromagnetic wave in the dipole approximation, whereas the electron states are described by the solution to the Schrödinger equation for an electron in the radiation field and a uniform static magnetic field.

2. Static charge potential

The solution to the time-dependent Schrödinger equation for an electron in the presence of a right-handed circularly polarised plane wave propagating parallel to a uniform DC magnetic field (z axis) can be written as (Seely 1974; Miranda 1976, 1977)

$$\Psi_{a}(\mathbf{x},t) = U\phi_{a}(\mathbf{x},t) \tag{1}$$

where

$$U = \exp(i\delta(t) \cdot \hat{p}/\hbar) \exp(i\zeta(t) \cdot x/\hbar) \exp(-i\eta(t)/\hbar)$$
(2)

with

$$\begin{split} \delta(t) &= -r(t)e_x + s(t)e_y & \zeta(t) = Q(t)e_y \\ r(t) &= \frac{1}{m} \int^t dt' \big[G(t') - (e/c)A_x(t') \big] & s(t) = \frac{G(t)}{m\omega_c} \\ \eta(t) &= \frac{1}{m} \int^t dt' \Big(\frac{e^2A^2}{c^2}(t') + Q^2(t') - \frac{2e}{c} Q(t')A_y(t') - G^2(t') \Big). \end{split}$$

Here, $\phi_{\alpha}(x, t)$ is the solution to the Schrödinger equation for an electron in a uniform magnetic field (no radiation field present) (Landau and Lifshitz 1958), $\alpha = (n, p_x, p_z)$ are the Landau quantum numbers, $\omega_c = eH/mc$ is the electron cyclotron frequency, and $A_x(t)$ and $A_y(t)$ are the components of the vector potential $A(t) = (cE_0/\omega_0)(e_x \cos \omega_0 t + e_y \sin \omega_0 t)$, describing the radiation field. The real functions of time G(t) and Q(t) are determined by the equation (Seely 1974)

$$G(t) + iQ(t) = \frac{e\omega_{\rm c}}{c} \int^t dt' [A_y(t') - iA_x(t')] \exp[i\omega_{\rm c}(t - t')].$$
(3)

Physically, equation (1) means that by performing spatial and momentum translations we move from an EM field-dependent (ψ) representation into an EM field-independent (ϕ) representation. Hence, if we now consider the one-electron problem in the simultaneous presence of the radiation and magnetic fields interacting with a static charge described by $V(\mathbf{x})$, i.e.,

$$\mathscr{H} = (1/2m) [\hat{p} + (eH/c)y e_x - (e/c)A(t)]^2 + V(x)$$
(4)

and go from the ψ representation to the ϕ representation by means of a canonical transformation based upon U, we transfer the EM field dependence from the first term of (4) to the term describing the static charge distribution, namely,

$$\mathscr{H} \to \widetilde{\mathscr{H}} = U^{t} \big[-i\hbar(\partial/\partial t) + \mathscr{H} \big] U = \mathscr{H}_{0} + V(\mathbf{x} + \delta(t))$$
(5)

where

$$\mathscr{H}_{0} = (1/2m) [\hat{p} + (eH/c)ye_{x}]^{2}$$

is the Landau Hamiltonian (Landau and Lifschitz 1958). Then, in the ϕ representation, our Hamiltonian is that of an electron in a uniform magnetic field moving in the $\delta(t)$ displaced potential, $V(\mathbf{x} + \delta(t))$. Hence, using equation (5) and allowing for the intrinsic self-consistent field, the Hamiltonian in the second quantisation formalism (in the ϕ representation) for the electron gas interacting with a static charge is then written as

$$H = \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{+} c_{\alpha}^{-} - e \sum_{\beta \alpha k} \tilde{\varphi}(k, t) \langle \beta | \exp(i k \cdot x) | \alpha \rangle c_{\beta}^{+} c_{\alpha}^{-}.$$
(6)

Here $\epsilon_x = (n + \frac{1}{2})\hbar\omega_c + P_z^2/2m$ is the electron energy in the Landau state $\alpha = (n, p, p_z)$, $\beta = (n', p_x + \hbar k_x, p_z + \hbar k_z), \langle \beta | \exp(i \mathbf{k} \cdot \mathbf{x}) | \alpha \rangle$ is the overlap between Landau wavefunctions (Gomes and Miranda 1975) and $\tilde{\phi}(\mathbf{k}, t)$ describes the Fourier components of the displaced static charge and self-consistent fields, which are given by Poisson's equation, neglecting transverse currents (Mermin and Canel 1964)

$$k^{2}\tilde{\varphi}(\boldsymbol{k},t) = 4\pi\tilde{\rho}(\boldsymbol{k},t) - 4\pi e \sum_{\beta,\alpha} \langle \beta | \exp(-i\boldsymbol{k}.\boldsymbol{x}) | \alpha \rangle \langle c_{\beta}^{+}c_{\alpha} \rangle_{t}.$$
(7)

Here, $\tilde{\rho}(\mathbf{k}, t)$ is the Fourier component of the displaced static charge distribution and $\langle \ldots \rangle_t$ denotes averaging with the complete Hamiltonian. We note that if $\tilde{\varphi}(\mathbf{k}, t)$ and $\rho(\mathbf{k})$ denote the Fourier components of the scalar potential and the static charge distribution, respectively, in the ψ representation; they are related to $\tilde{\varphi}(\mathbf{k}, t)$ and $\tilde{\rho}(\mathbf{k}, t)$ by

$$\tilde{\varphi}(\boldsymbol{k},t) = \varphi(\boldsymbol{k},t) \exp(i\boldsymbol{k},\boldsymbol{\delta}(t)) \qquad \tilde{\rho}(\boldsymbol{k},t) = \rho(\boldsymbol{k}) \exp(i\boldsymbol{k},\boldsymbol{\delta}(t)). \tag{8}$$

We make a few remarks on the approximations underlying equation (7). Firstly, the well-known long-wavelength limit (dipole approximation) was used for the external EM field, a long-standing practice (Seely 1974) when dealing with problems involving the interaction of a quantum plasma and a radiation pump field. Replacing the EM field vector potential A(x, t) by its spatially uniform counterpart A(t) is, of course, a valid approximation in those applications where the dimensions of the interaction region are much smaller than the EM wavelength. Secondly, we used the so-called electrostatic approximation (EA) in a magnetised plasma. It is true that in the presence of a magnetic field, the general normal modes in the plasma involve contributions from both longitudinal and transverse currents and, except under certain conditions, the latter need not be negligible. The EA has been carefully considered (Mermin and Canel 1964, Celli and Mermin 1964) and the conditions for its validity established. It describes a normal mode well if the frequency (ω) and the wavelength (k^{-1}) of the mode satisfy $\omega \ll kc$, and the longitudinal currents in the mode are not negligible compared with the transverse currents. It is under these assumptions that equation (7) is reached. This means that in handling the self-consistent spatially non-uniform fields in the RPA the assumption has been made that only Coulomb interactions are important (the electrostatic approximation). In

short, the condition $\omega_{\text{mode}} \ll ck_{\text{mode}}$ underlies the validity of equation (7). Denoting $\alpha = (n, p_x, p_z)$ and $\alpha' = (n', p_x - \hbar k_x, p_z - \hbar k_z)$, constructing the equation of motion for $\langle c_x^+ c_x \rangle_t$ within the usual RPA (Pines 1961, Zyryanov 1961, Mermin and Canel 1964), and solving it with the initial condition $\langle c_x^+ c_x \rangle_{t=-\infty} = 0$, we obtain

$$\langle c_{\alpha'}^{+} c_{\alpha} \rangle_{t} = \exp[-\mathrm{i}(\epsilon_{\alpha} - \epsilon_{\alpha'})t/\hbar] \int_{-\infty}^{t} \mathrm{d}t' \, \mathrm{i}e\tilde{\varphi}(\boldsymbol{k}, t') \langle \alpha | \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}) | \alpha' \rangle (f_{\alpha'} - f_{\alpha}) \\ \times \exp[\mathrm{i}(\epsilon_{\alpha} - \epsilon_{\alpha'})t/\hbar]$$
(9)

where f_{α} is the electron occupation number for the Landau state α . Substituting equation (9) into (7) we have

$$\tilde{\varphi}(\boldsymbol{k},t) = \frac{4\pi\tilde{\rho}(\boldsymbol{k},t)}{k^2} - \frac{4\pi ie^2}{k^2} \int_{-\infty}^t dt' \tilde{\varphi}(\boldsymbol{k},t) \sum_{\boldsymbol{x},\boldsymbol{\alpha}'} |\langle \boldsymbol{\alpha}'| \exp(-i\boldsymbol{k}.\boldsymbol{x}) |\boldsymbol{\alpha}\rangle|^2 (f_{\boldsymbol{\alpha}'} - f_{\boldsymbol{\alpha}}) \\ \times \exp[-i(\epsilon_{\boldsymbol{\alpha}} - \epsilon_{\boldsymbol{\alpha}'})(t - t')/\hbar].$$
(10)

It follows from (10) that the temporal Fourier components of $\tilde{\varphi}$ and $\tilde{\rho}$ are related by

$$\tilde{\varphi}(\boldsymbol{k},\omega) = \left[4\pi\tilde{\rho}(\boldsymbol{k},\omega)\right] / \left[k^2 \boldsymbol{\epsilon}(\boldsymbol{k},\omega)\right]$$
(11)

where

$$\epsilon(\mathbf{k},\omega) = 1 - \frac{4\pi e^2}{k^2} \sum_{\alpha,\alpha'} |\langle \alpha'| \exp(-i\mathbf{k} \cdot \mathbf{x}) |\alpha \rangle|^2 \frac{(f_{\alpha'} - f_{\alpha})}{(\epsilon_{\alpha'} - \epsilon_{\alpha} - \hbar\omega)}$$
(12)

is the usual linear response function (i.e., the longitudinal part of the dielectric tensor) of an electron gas in a DC magnetic field (Mermin and Canel 1964, Celli and Mermin 1964). Using equations (8) and (12) we have

$$\varphi(\boldsymbol{k},t) = \exp(-i\boldsymbol{k}\cdot\boldsymbol{\delta}(t)) \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{4\pi\rho(\boldsymbol{k})}{k^2\epsilon(\boldsymbol{k},\omega)} \exp(-i\omega t) \int_{-\infty}^{+\infty} dt' \exp(i\boldsymbol{k}\cdot\boldsymbol{\delta}(t')) \exp(-i\omega t').$$
(13)

Expanding the periodic factors $\exp[i\mathbf{k} \cdot \delta(t)]$ in Fourier series

$$\exp(\mathbf{i}\boldsymbol{k}\cdot\boldsymbol{\delta}(t)) = \sum_{\nu=-\infty}^{+\infty} F_{\nu}(\boldsymbol{k}) \exp(-\mathbf{i}\nu\omega_{0}t)$$
(14)

and substituting them in (13) finally gives

$$\varphi(\mathbf{k},t) = \sum_{\mu,\nu=-\infty}^{\infty} \frac{4\pi\rho(\mathbf{k})}{k^2 \epsilon(\mathbf{k},\nu\omega_0)} F_{\mu}^*(\mathbf{k}) F_{\nu}(\mathbf{k}) \exp[i(\mu-\nu)\omega_0 t].$$
(15)

As before (Lima and Miranda 1978), in the presence of a radiation field, the potential of a static carge has components at the radiation field frequency and its harmonics.

In the remaining discussion we shall consider only the static component $\varphi_0(r)$ of the potential ($\mu = \nu$). We have from equation (15)

$$\varphi_0(\boldsymbol{x}) = \frac{1}{(2\pi)^3} \int d^3k \frac{4\pi\rho(\boldsymbol{k})}{k^2\epsilon_{\text{eff}}} \exp(-i\boldsymbol{k}\cdot\boldsymbol{x})$$
(16)

where

$$\frac{1}{\epsilon_{\rm eff}} = \sum_{\nu = -\infty}^{+\infty} \frac{J_{\nu}^2 \left[ek_{\perp} E_0 / m\omega_0 (\omega_0 - \omega_c) \right]}{\epsilon(\mathbf{k}, \nu\omega_0)}.$$
(17)

In arriving at (17) we have used the fact that $|F(k)|^2 = J_v^2 (ek_\perp E_0/m\omega_0 |\omega_o - \omega_c|)$ (Miranda 1976), where J_v is the Bessel function of order v. Equation (16) implies that the effect of a radiation field on the static potential of a charge distribution can be taken into account by introducing an effective dielectric constant, ϵ_{eff} , dependent on both the frequency and the strength of the electromagnetic field. Only in the limit of zero EM field does the ϵ_{eff} reduce to the usual static dielectric constant.

3. Effective dielectric constant

Equation (17) is the expression for the effective dielectric constant to be investigated. However, this requires an expression for the dielectric constant, $\epsilon(\mathbf{k}, \omega)$, in the presence only of a uniform magnetic field. The general expression for $\epsilon(\mathbf{k}, \omega)$ is quite complicated but it becomes substantially simplified in the long-wavelength limit for collective oscillation. The original work of Mermin and Canel (1964) gives a more detailed discussion of this point; here we shall only use their main results.

In the above long-wavelength limit, $\epsilon(\mathbf{k}, \omega)$ is given by

$$\epsilon(\mathbf{k},\omega) = 1 - \left[\omega_{\rm p}^2 \sin^2 \theta / (\omega^2 - \omega_{\rm c}^2)\right] - (\omega_{\rm p}^2 \cos^2 \theta / \omega^2) \tag{18}$$

where θ is the angle that k makes with the magnetic field (z axis). Two resonances dominate expression (18) in this limit. They are the roots ω_+ and ω_- of $\epsilon(\mathbf{k}, \omega) = 0$, namely,

$$\omega_{\pm}^{2} = \frac{1}{2} \{ (\omega_{p}^{2} + \omega_{c}^{2}) \pm [(\omega_{p}^{2} + \omega_{c}^{2})^{2} - 4\omega_{p}^{2}\omega_{c}^{2}\cos^{2}\theta]^{1/2} \}$$
(19)

and two limiting regimes of high- and low-density plasmas are of interest. In the high-density regime ($\omega_p^2 \gg \omega_c^2$) the roots (19) become

$$\omega_{+}^{2} = \omega_{p}^{2} + \omega_{c}^{2} \sin^{2} \theta \qquad \qquad \omega_{-}^{2} = \omega_{c}^{2} \cos^{2} \theta.$$
⁽²⁰⁾

The mode ω_+ is, in this case, the ordinary longitudinal plasmon; the mode ω_- is associated with a motion which is very close to a circular motion about the direction of propagation with frequency ω_c .

In the opposite regime of a low-density plasma ($\omega_c^2 \gg \omega_p^2$) the roots (19) reduce to

$$\omega_{+}^{2} = \omega_{c}^{2} + \omega_{p}^{2} \sin^{2} \theta \qquad \qquad \omega_{-}^{2} = \omega_{p}^{2} \cos^{2} \theta.$$
(21)

The mode ω_{-} corresponds to an oscillation that is very close to being linear and parallel to H; it corresponds to a plasmon in which, because of the strong magnetic field, the particles move parallel to H instead of k. The mode ω_{+} is the usual cyclotron mode.

With these results in mind we now turn back to the discussion of the effective dielectric constant. Rewriting equation (18) in terms of the roots ω_+ and ω_- , equation (17) becomes

$$\frac{1}{\epsilon_{\rm eff}} = \sum_{\nu=-\infty}^{+\infty} J_{\nu}^{2}(Z) \frac{(\nu\omega_{0})^{2} [(\nu\omega_{0})^{2} - \omega_{\rm c}^{2}]}{(\nu\omega_{0})^{4} - (\omega_{+}^{2} + \omega_{-}^{2})(\nu\omega_{0})^{2} + \omega_{+}^{2} \omega_{-}^{2}}$$
(22)

with

$$Z = ek_{\perp}E_0/(m\omega_0|\omega_0 - \omega_c|).$$
⁽²³⁾

In the case of a high-density plasma, using equation (20) gives

$$\frac{1}{\epsilon_{\rm eff}} = \sum_{\nu = -\infty}^{+\infty} \frac{J_{\nu}^{2}(Z)(\nu\omega_{0})^{2} [(\nu\omega_{0})^{2} - \omega_{\rm c}^{2}]}{(\nu\omega_{0})^{4} - \omega_{\rm p}^{2}(\nu\omega_{0})^{2} + \omega_{\rm p}^{2}\omega_{\rm c}^{2}\cos^{2}\theta}.$$
(24)

Hence, for $\omega_0 \simeq \omega_p$ the $v = \pm 1$ term is dominant, with the result that ϵ_{eff}^{-1} becomes very large; i.e., a breakdown in screening similarly to that previously reported (Lima and Miranda 1978) occurs. On the other hand, for $\omega_0 \simeq \omega_c$ equation (24) vanishes since the argument of the Bessel functions for $\omega_0 = \omega_c$ becomes infinite. In this case there is a breakdown of the electron interactions ($\varphi(\mathbf{r}) \rightarrow 0$). The same conclusions can easily be verified for the case of a low-density plasma.

In summary, we note that the behaviour of $\epsilon_{\text{eff}}^{-1}$ for ω_0 near the plasma frequency is completely opposite to the case where the radiation frequency is near the cyclotron frequency. In the former case, a breakdown in screening occurs, whereas at the lasercyclotron resonance condition there is a breakdown of the electron interaction. Physically, the fact that $\epsilon_{\text{eff}}^{-1}$ vanishes for $\omega_0 \simeq \omega$ may be understood as follows: consider the classical problem of one electron in the presence of both an electromagnetic field (described in the dipole approximation by A(t)) and a uniform magnetic field and acted upon by a potential V (i.e., the potential of the other electrons, the ions or impurities). We have

$$H = \frac{1}{2}mv_{\parallel}^{2} + \frac{1}{2}mv_{\perp}^{2} + V$$

where v_{\parallel}^2 and $v_{\perp}^2 = [eE_0/m(\omega_0 - \omega_c)]^2$ are the time-averaged squares of the longitudinal and the transverse electron velocities, respectively. Then, when $\omega_0 = \omega_c (v_{\perp} \to \infty)$ the transverse kinetic energy becomes much larger than V so that H is dominated by the kinetic energy term. This is equivalent to saying that at the cyclotron frequency, energy from the radiation field is resonantly pumped into the transverse (cyclotron) motion of the electron. This increases the radius of the cyclotron orbit to very large values, so that the electron no longer sees the potential V; it becomes a free particle. This implies that $\varphi_0(\mathbf{x})$ in the left-hand side of equation (16) vanishes at $\omega_0 = \omega_c$ which, in turn, implies that $\epsilon_{\rm eff}^{-1}$ is vanishingly small; this is consistent with our previous deduction of electron interaction breakdown. In other words, in contrast to the case where ω_0 is near ω_p , at the cyclotron resonance condition, instead of a breakdown in screening, the electron interactions freeze out.

These two contrasting behaviours of $\epsilon_{\rm eff}^{-1}$ can also be appreciated from yet another point of view. In the case of screening breakdown, the screening cloud (the plasma polarisation medium) undergoes increasingly larger amplitude oscillations as its frequency (ω_p) approaches the radiation frequency. The net result is therefore an enhancement of the electron-nucleus interaction. Conversely, if the cyclotron resonance condition ($\omega_0 \simeq \omega_c$) is reached, it is the indivual rather than the collective character of the electron motion (the circular motion of the screening cloud) that becomes important. Here the single electron cyclotron motion exhibits a very large amplitude typical of resonant behaviour, thereby entailing vanishingly small electron interactions.

4. Conclusions

In this paper we have investigated the effects of an additional uniform magnetic field on the modification of the Coulomb screening in the presence of an electromagnetic wave. We have shown that as the frequency of the electromagnetic wave approaches the plasma frequency the effective interaction potential due to static charged scattering centres in the plasma is enhanced as a consequency of the breakdown in screening. This behaviour has also been found (Lima and Miranda 1978) in the absence of a DC magnetic field.

On the other hand, as the radiation frequency approaches the cyclotron frequency the opposite effect appears, namely, the interaction potential becomes vanishingly small, i.e. at the cyclotron resonance condition the electron interactions are essentially frozen out. A similar conclusion has been reported by Seely (1974) although he did not consider the effect of Coulomb screening in the presence of the radiation field. On that account his results differ from ours in that his approach produced an effective electron-nuclei collision frequency which vanishes as $|\omega - \omega_c|^3$ at the cyclotron resonance. In our case, the inclusion of that effect brought out a rather differnt dependence on $|\omega - \omega_c|$ for the collision rate. This point, as well as its consequences towards the plasma kinetics will be fully discussed in a forthcoming paper (Lima *et al* 1979).

In conclusion, we have obtained in this paper a seemingly more complete description of the laser assisted electron Coulomb interactions in a strongly magnetised plasma. In particular, we have accounted for changes in the effective interaction potential induced by the laser and DC magnetic fields for such processes. Our main results are quite insensitive to a few simplifying assumptions introduced to facilitate calculation.

Finally, we would like to point out briefly some situations where these results could be tested. For instance, our predicted enhancement $(\omega_0 \simeq \omega_p)$ or weakening $(\omega_0 \approx \omega_c)$ of the potential of a static charge could be observed experimentally as an increase or reduction, respectively, in the electrical resistance of a semiconductor caused by the scattering of condution electrons by impurity ions. They may also prove to be of relevance to the laser fusion problem. In fact, an increase (decrease) in the electrical resistance of the plasma should be accompanied by a noticeable increase in the plasma heating (cooling), under the conditions described in this paper.

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