ON CERTAIN MECHANISM OF AN EXCITEMENT OF ANGULAR MOMENTUM EQUATORIAL COMPONENTS.

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Abstract

Variations of non-axial components of angular momentum as a result of the interaction of Rossby waves with zonal flow at the presence of orography are investigated. Periods and amplitudes of such oscillations manifesting in the latitude of a station of observation are obtained.

Vector of angular momentum of atmosphere is deflected from the Earth's axis of rotation on small angle and it is desirable at least in qualitative manner to evaluate the influence of this deflection on general atmospheric circulation. More over, this problem has an applied interest. Last years it appeared the opportunity to conduct very accurate measurements as the latitude of the observation station relatively to the motionless star reference as the length-of-day. This chance was given by the development of a technique of astronomy measurements, especially by the elaboration of the method of the super-long base interferometry [1]. It has been found to be that variations of length-of-day and the latitude of observational station have the periods about 3-7 weeks [3,5]. On the other hand it is known that the period of exchange of angular momentum between solid mantle and the Earth's liquid core is about a years or larger therefore it is naturally to assume that the cause of these oscillations is the exchange of angular momentum between atmosphere and Earth. It is clear also that such exchange is possible only with availability of a friction on Earth. Here one of alternative mechanisms of exchange of the angular momentum is considered. This is the interaction of Rossby waves with large-scale topography. Comparing solution received from such model with a real variation of components of an angular momentum vector of atmosphere one can evaluate relative role, for various time scales, of orography and underlying friction in exchange of an angular momentum between atmosphere and Earth. We have investigated the simplest case allowing nontrivial variations of all three components of angular momentum. For this aim we have used low-order spectral model derived from barotropic vorticity equation for sphere including orography:

$$\frac{\partial}{\partial t}(\nabla^2\psi - \frac{1}{L_0^2}\psi) + (\psi, \nabla^2\psi + l + \eta - \frac{1}{L_0^2}\psi) = 0 \qquad (1)$$

where ψ - stream function; $\eta = l_0 h/H$ - orography function; H - the scale-height; h - height of mountains; $l = 2\Omega\cos\theta$ - the Coriolis parameter; l_0 - mean value of the Coriolis parameter; L_0 - Oboukhov's scale; θ - latitude; λ- longitude; r - Earth's radius. $(a,b)=(\partial a/\partial\theta\ \partial b/\partial\lambda - \partial a/\partial\lambda\ \partial b/\partial\theta)r^2\sin\theta$.

We have took as a finite-dimensional approximation of the solution of the barotropic vorticity equation next five modes

$$\psi = r^2\Omega(-\alpha(t)P_1^0(\theta)\sin\lambda + A_1^1(t)P_1^1(\theta)\sin\lambda + B_1^1(t)P_1^1(\theta)\cos\lambda + A_n^m(t)P_n^m(\theta)\sin m\lambda + B_n^m(t)P_n^m(\theta)\cos m\lambda$$

Orography was described by two spherical harmonics:

$$\xi = \Omega(C_1^1 P_1^1(\theta)\sin\lambda + D_1^1 P_1^1(\theta)\cos\lambda + C_n^m P_n^m(\theta)\sin\lambda)$$

The problem is reduced to solving of the 5-order ordinary differential equation system which can be written in the form (here we are using dimensionless time $t \Rightarrow t/\Omega^{-1}$)

$$\frac{dA}{dt} - qB\varepsilon_n = 0; \qquad \frac{dB}{dt} + \mathrm{q}\mathrm{A}\varepsilon_{\mathrm{n}} - \delta c = 0; \qquad \frac{d\varepsilon_n}{dt} + \prod r + \kappa cB = 0;$$

$$\frac{ds}{dt} + k_1 r = 0; \qquad \frac{dr}{dt} - k_1 s - 4\prod \varepsilon_n = 0 \qquad (2)$$

where

$$k_1 = \frac{2}{2 + a^2/L_0^2}; \qquad \mathrm{k_n} = \frac{2}{n(n+1) + a^2/L_0^2}$$

$$A = A_n^m - \frac{C_n^m}{n(+1) - 2}; \quad B = B_n^m; \quad \varepsilon_n = \alpha - \frac{2}{\sqrt{3}\,[n(n+1) - 2]};$$

$$s = \frac{A_1^1 C_1^1 + B_1^1 D_1^1}{\sqrt{(D_1^1)^2 + (C_1^1)^2}} + \frac{\sqrt{(D_1^1)^2 + (C_1^1)^2}}{n(n+1) - 2}; \quad r = \frac{B_1^1 C_1^1 - A_1^1 D_1^1}{\sqrt{(D_1^1)^2 + (C_1^1)^2}};$$

$$c = C_n^m m; \quad \kappa = \frac{\sqrt{3}\,k_1}{8}; \quad q = \frac{\sqrt{3}\,[n(n+1) - 2]\,k_n m}{2};$$

$$\delta = \frac{k_n}{n(n+1) - 2}; \quad \prod = \frac{\sqrt{3}\,k_1 \sqrt{(D_1^1)^2 + (C_1^1)^2}}{8}$$

This system has two independent integrals of motion

$$\frac{d}{dt}[(\frac{A^2 + \mathrm{B}^2}{2} + \frac{\delta}{\kappa}(\varepsilon_n - \frac{\prod}{k_1} s)] = \frac{dE}{dt} = 0 \qquad (3)$$

$$\frac{d}{dt}\left[\frac{s^2 + r^2}{2} + 2\varepsilon_n^2 + \frac{4\kappa c}{q} A\right] = \frac{dF}{dt} = 0, \qquad (4)$$

and its stationary solution is written as

$$\varepsilon_{n0} = const; \quad B_0 = 0; \quad r_0 = 0; \quad A_0 = \frac{\delta c}{q\varepsilon_{n0}}; \quad s = \frac{4\prod \varepsilon_{n0}}{k_1}. \qquad (5)$$

Linearization of the system (2) relative to stationary solution (5) leads to the system of linear differential equations. Roots of the characteristic equation of this system have approximately the following meanings(here we took into account that 2Π at least in 10 times less then k_1):

$$\sigma_1 = 0; \qquad \sigma_{2,3} \approx \pm\sqrt{k_1^2 - 4\prod^2/(1 + \delta\kappa c^2/k_1\varepsilon_{n0})};$$

$$\sigma_{4,5} \approx \pm\sqrt{-q^2\varepsilon_{n0} - 4\prod^2 + \frac{\delta\kappa c^2}{\varepsilon_{n0}} + 4\prod^2/(1 + \delta\kappa c^2/k_1\varepsilon_{n0})}.$$

It is easy to show, that if we take into consideration in the decomposition of stream function on spherical functions two additional terms $A_1^1(t)P_1^1(\theta)\sin\lambda + B_1^1(t)P_1^1(\theta)\cos\lambda$ which correspond to equatorial components of angular momentum, the region of orographic instability will narrow

$$\sqrt[3]{\frac{\delta\kappa c^2}{q^2}}(1 - \frac{4\Pi d}{3(\delta\kappa c^2/q^2)^{2/3} + k_1^2/q^2}) \geq \varepsilon_{n_0} \geq 0 \qquad (6)$$

comparing with triplet case [2,4], and the orographic instability will be observed under condition (here d is a number about 1). In comparison with the result [2] it has appeared additional term (second in the brackets describing non-axial symmetric components of orography), which became 0 when $\Pi\to 0$.

Under condition $\Pi\to 0$ system (2) is divided in two independent subsystems. One of them describes the evolution of harmonic Y_1^1 in free atmosphere, another subsystem describes the interaction of Rossby wave Y_n^m with zonal flow in the presence of orography represented by function $\xi=\Omega C_n^m P_n^m(\theta)\sin m\lambda$.

Let us consider directly the system of nonlinear equations (2) now. It is convenient to seek the solution of nonlinear system (2) by reducing it to one differential equation with one sought function ε_n. For this aim we will consider the last two equations of (2) as linear non-homogeneous subsystem with permanent coefficients. Solution of it can be written as

$$s = c_1 \sin k_1 t + c_2 \cos k_1 t - 4\Pi \sin k_1 t \int \varepsilon_n \cos k_1 t\, dt + 4\Pi \cos k_1 t \int \varepsilon_n \sin k_1 t\, dt$$

$$r = -c_1 \cos k_1 t + c_2 \sin k_1 t - 4\Pi \cos k_1 t \int \varepsilon_n \cos k_1 t\, dt + 4\Pi \sin k_1 t \int \varepsilon_n \sin k_1 t\, dt \qquad (7)$$

Coefficients c_1 and c_2 are found from the initial conditions. Assuming that at an initial moment s and r are satisfied to stationary solution (5) then c_1 and c_2 equal zero, and solution (7) using formula for integration by parts can be rewritten as

$$s = -4\Pi / k_1 (\varepsilon_n - \varepsilon_n^{II} / k_1^2 + \varepsilon_n^{IV} / k_n^4 - ....)$$

$$r = 4\Pi / k_1 (\varepsilon_n^{I} - \varepsilon_n^{III} / k_1^3 + \varepsilon_n^{V} / k_1^5 - ....) \qquad (8)$$

At present the low-frequency oscillations with characteristic period of 3 weeks and greater is interesting to us. Let us make a substitution of variables: introducing a new time $\bar{t}=t\,\omega_0$ ($\omega_0=2\Pi/T_0$; T_0 - corresponds to 3 weeks) and function $\bar{\varepsilon}_n = \varepsilon_n/\varepsilon_{no}$ (ε_{no} is characteristic value such that $q\varepsilon_{no}\sim\delta\kappa c^2/\varepsilon_{no}\sim\omega_0$). Substituting now (8) in (3), (4), and excluding from (3) A and B with the help of (4) and third equation of system (2) we will yield

$$\left[F - \frac{8\Pi^2}{k_1^2}\bar{\varepsilon}_n^2 - 2\bar{\varepsilon}_n^2\right]^2 + \frac{16k_1^2\tau^2}{q^2\varepsilon_{n_0}^2}\left[\frac{d\bar{\varepsilon}_n}{d\bar{t}}\left(1+\frac{4\Pi^2}{k_1^2}\right)\right]^2 + \frac{32\delta k c^2}{q\varepsilon_{n_0}^3}\left[\bar{\varepsilon}_n\left(1+\frac{4\Pi}{k_1}\right)\right] +$$

$$+\tau^2\left[2\left(F - 2\bar{\varepsilon}_n^2\left(1+\frac{4\Pi^2}{k_1^2}\right)\right)(\bar{\varepsilon}_n'^2 - 2\bar{\varepsilon}_n'\bar{\varepsilon}_n'') + \frac{32k_1^2\tau^2}{q^2\varepsilon_{n_0}^2}\left(1+\frac{4\Pi^2}{k_1^2}\right)\frac{d\bar{\varepsilon}_n}{d\bar{t}}\bar{\varepsilon}_n''' - \frac{128\delta k c^2\Pi^2\bar{\varepsilon}_n''}{q^2k_1^2\varepsilon_{n_0}^3}\right] +$$

$$+\ldots\tau^n(\ldots) = \frac{32\kappa c^2 E}{q^2\varepsilon_{n_0}^4}, \qquad (9)$$

here $\tau = \omega_0/k_1$.

Looking for the solution of this equation in the form

$$\bar{\varepsilon}_n = \bar{\varepsilon}_n^{(0)} + \tau\bar{\varepsilon}_n^{(1)} + \tau^2\bar{\varepsilon}_n^{(2)} + \ldots \qquad (10)$$

Substituting (10) into (9) and equalizing the coefficients under the same power of τ, in zero approximation we yield (we return here to old variables t and ε_n)

$$\frac{d\varepsilon_n}{dt} = \left\{\frac{32E\kappa^2c^2 - q^2F^2}{16\left(1+\frac{4\Pi^2}{k_1^2}\right)} - \frac{\varepsilon_n^4 q^2}{4} + \frac{q^2F\varepsilon_n^2}{4\left(1+\frac{4\Pi^2}{k_1^2}\right)} - \frac{2\kappa c^2\delta\varepsilon_n}{\left(1+\frac{4\Pi^2}{k_1^2}\right)}\right\}^{1/2} \qquad (11)$$

This equation is integrated in elliptic functions. When $\Pi\to 0$, the equation (12) turns into equation that was investigated in [2] and which describes zonal velocity oscillations appearing as result of interactions of Rossby wave with zonal flow in the presence of orography represented by unique spherical harmonic $C_n^m P_n^m(\theta)\sin m\lambda$. It is follow from our evaluations that $\Pi^2/k_1^2\leq 0.1$ and corrections to periods of oscillations obtained earlier for the case Π=0 are very insignificant. At the same time, taking into consideration the non-axial components ξ_1^1 in the orography decomposition results to deflection the vector of angular momentum of atmosphere from Earth's rotation axis on some angle. Interaction of zonal flow with wave ψ_1^1 causes changes of this angle which are displayed as variations of geographic latitude of observational station (as a consequence of conservation law of the whole angular momentum of Earth and atmosphere) and which are registered during precious astronomical measurings.

The angle between Earth's rotation axis and vector of angular momentum of atmosphere approximately equals

$$\theta \approx \left[\frac{3}{2}({A_1^1}^2 + {B_1^1}^2)\right]^{1/2} = \left\{s^2 + r^2 - \frac{2s\sqrt{{D_1^1}^2 + {C_1^1}^2}}{[n(n+1)-2]} + \frac{{D_1^1}^2 + {C_1^1}^2}{[n(n+1)-2]}\right\}^{1/2} \quad (12)$$

Substituting in (12) formulae (8) we will obtain for θ next expression

$$\theta \approx \frac{4\Pi}{k_1}\left\{(\varepsilon_n + \frac{2}{\sqrt{3}[n(n+1)-2]})^2 + (\frac{1}{k_1^2}\frac{d\varepsilon_n}{dt})^2\right\}^{1/2}, \quad (13)$$

where ε_n is a solution of (11), expressed by elliptic functions. The main term in formula (13) under radical is first. Therefore oscillations of θ (which is appearing in oscillations of the latitude of observational station θ_{st}) will take place with the same frequency as the oscillations of ε_n, which are connected with variations of length of the day ΔT_e. Previous analysis of records of oscillations of θ_{st} and ΔT_e proposed on figures in [3,5] don't contradict this conclusion.

On the fig. 1 it is proposed the graphics of spectrum of ΔT_e oscillation which were obtained from BIH (Bureau International de L'heure) data proposed by themselves the measurings of the deviations of length of day from mean value. Spectrum on the fig. 1a is calculated using records from 4 January 1967 to 31 December 1980. On the fig. 1b the spectra obtained from the records for time interval equals one year consequently for 1972, 1973 and 1974. The pikes corresponding to the oscillations with periods 4 and 6 weeks are clearly seen. As it was shown in [2] these periods are reproduced by model, which takes into account the interaction Rossby wave with zonal flow in the presence of orography. This model, connecting variations of angular momentum components of Earth with the exchange of angular momentum between atmosphere and rigid core of Earth at the expense of interaction of Rossby waves with orography, can become the basis for the construction more precise theory, ascertaining the correlation between the oscillations of angular velocity of Earth’s rotation and the changes of latitude of observation station. It gives the possibility to use long astronomical series of measurings for diagnostic investigations of atmospheric circulation.

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Oscillation spectrum of variations of the length-of-day

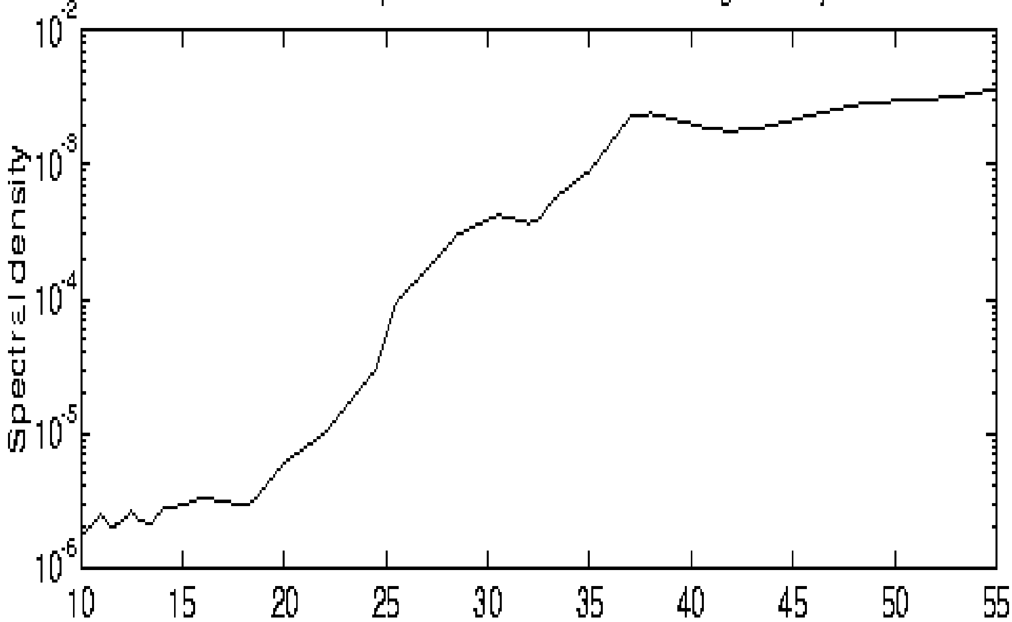

Fig.1a Spectrum obtained from BIH data for period from 4.1.1967 to 31.12.1980

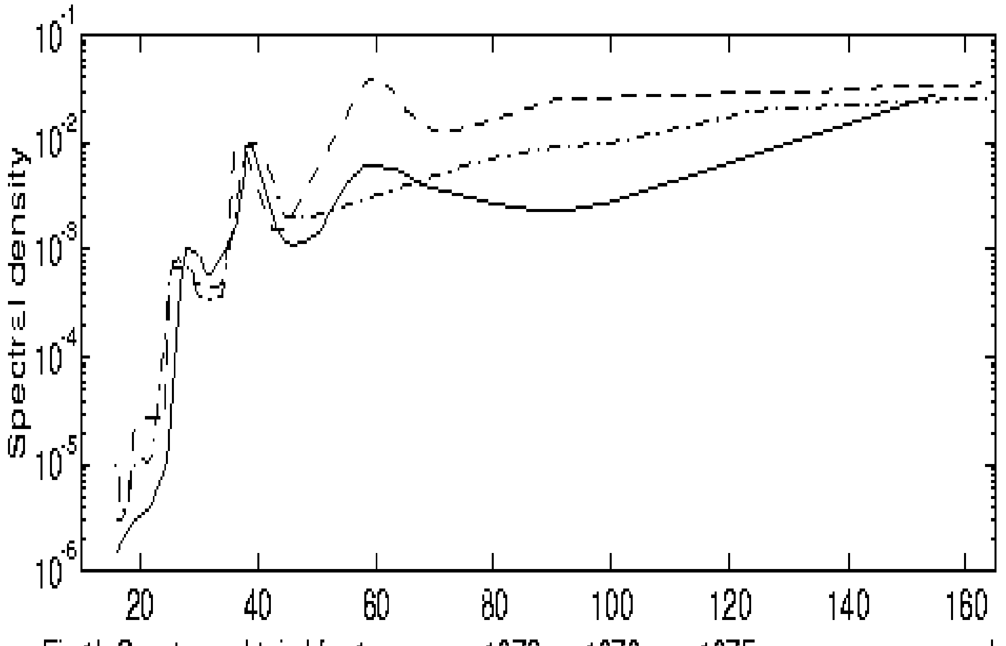

Fig.1b Spectrum obtaind for 1 year. ___ 1972, --- 1973 -.-.-. 1975