Leveling Cartoons, Texture Energy Markers and Image Decomposition

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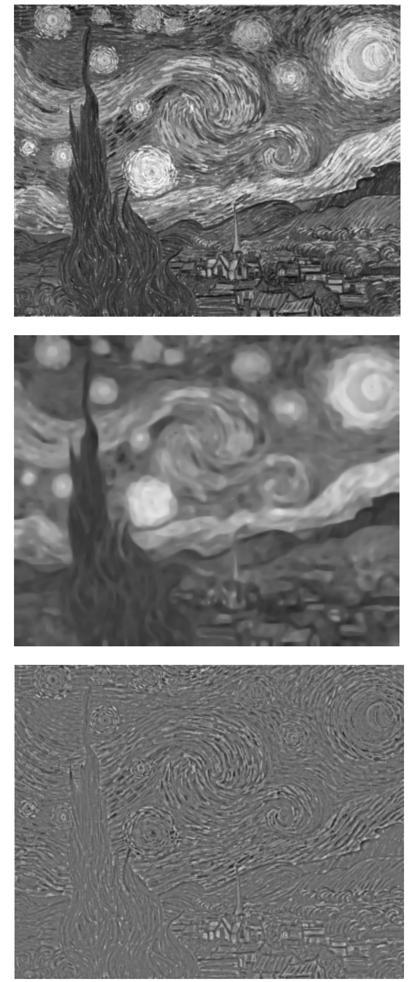
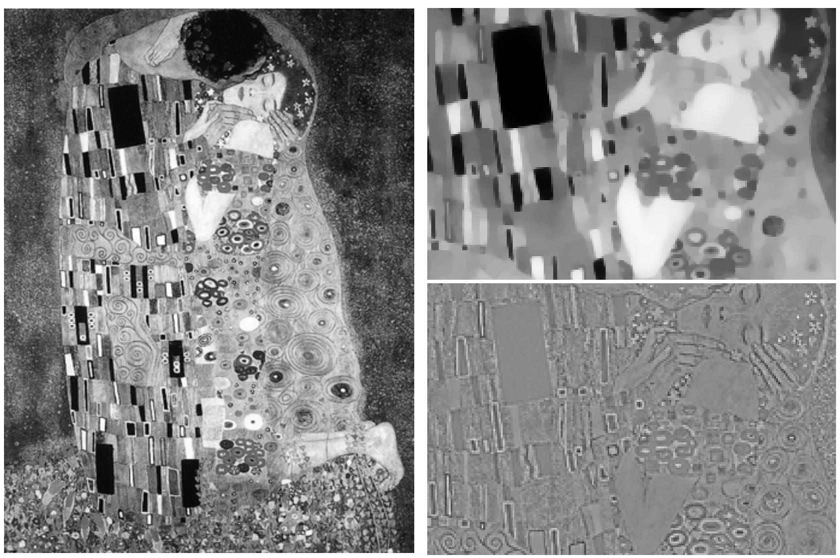
Image decomposition

- Image (u+v) model $f = u + v, \quad f, u, v : \Omega \subset \Re^2 \to \Re$
  - «cartoon» u (edges, contours, objects, shapes)
  - texture v (oscillations, details, noise)

- Inverse problem: image decomposition
  - Energy minimization
    - Total variation, convex optimization, PDE's
    - Wavelets and projections in function bases, dictionaries
  - Applications: image restoration, inpainting, analysis





Variational schemes

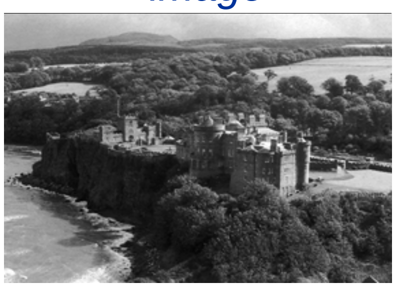
- *Mumford-Shah* image simplification
- Total Variation minimization (*Rudin, Osher & Fatemi):* $\|u\|_{\mathrm{TV}} = \iint_{\Omega} \|\nabla u\| dxdy$

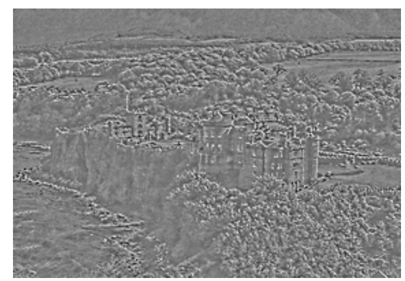
$$E_{ROF}(u) = \|u\|_{\mathrm{TV}} + \lambda \|u - f\|_2^2$$

- Texture = Oscillatory functions *(Y. Meyer):* $v = \mathrm{div}\vec{g} = \partial_x g_1 + \partial_y g_2$
- u+v *(Vese & Osher):* $E_{VO}(u,\vec{g}) = \|u\|_{\mathrm{TV}} + \lambda \| f - (u + \mathrm{div}\vec{g})\|_2^2 + \mu \|\vec{g}\|_p$

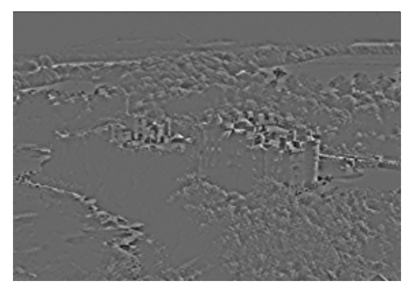
cartoon *texture*

image





ROF



VO

A VARIATIONAL FORMULATION OF PDEs FOR DILATION AND LEVELINGS

Petros Maragos

ISMM 2005 – Paris

Volume Extremization with Sup-Inf Constraints

Theorem : Maximizing the volume functional by keeping invariant the global supremum

$$\max \int\int u\, dxdy \quad \text{s.t.} \quad \bigvee u = \bigvee u_0$$

has a gradient flow governed by the **PDE generating flat dilation by disks**

$$u_t = \|\nabla u\|, \qquad u(x, y, 0) = u_0(x, y)$$

The **dual problem** of minimizing the volume functional by keeping invariant the global infimum

$$\min \iint u\, dxdy \qquad \text{s.t.} \quad \bigwedge u = \bigwedge u_0$$

has a gradient vector flow governed by the **isotropic flat erosion PDE**:

$$u_t = -\|\nabla u\|, \qquad u(x, y, 0) = u_0(x, y)$$

Create a **Cartoon Simplification** of a **reference** image $f(x,y)$ consisting of several parts by using a **marker** $u_0(x,y)$ that intersects some of these parts and evolves towards f in a monotone way such that all evolutions $u(x,y,t)$ satisfy:

$$t_1 < t_2 \Rightarrow f(x,y) \preceq_f u(x,y,t_2) \preceq_f u(x,y,t_1) \preceq_f u_0(x,y)$$

$\preceq_f$ is a **inf-semilattice order** w.r.t. a reference f

$$\Rightarrow \left| f(x,y) - u(x,y,t) \right| \leq \left| f(x,y) - u_0(x,y) \right| \quad \forall t, x, y$$

Partition the regions R^- and R^+ formed by zero-crossings of $f - u_0$:

$$R^- = \{(x,y) : f(x,y) \geq u_0(x,y)\} = \sqcup_i R_i^-$$

$$R^+ = \{(x,y) : f(x,y) < u_0(x,y)\} = \sqcup_i R_i^+$$

Evolution of u is done by maintaining all local min/max of u_0 inside subregions R_i^- / R_i^+ :

$$\bigvee_{R_i^-} u = \bigvee_{R_i^-} u_0 \quad \text{and} \quad \bigwedge_{R_i^+} u = \bigwedge_{R_i^+} u_0 ,$$

Variational Formulation of Levelings

Theorem: The gradient flow for the optimization problem

$$\min \iint |u-f|\,dxdy \quad \text{s.t.} \quad \bigvee_{R_i^-} u = \bigvee_{R_i^-} u_0, \quad \bigwedge_{R_i^+} u = \bigwedge_{R_i^+} u_0$$

is given by:

$$\partial u(x,y,t)/\partial t = -\operatorname{sgn}(u-f)\,\|\nabla u\|$$

$$u(x,y,0) = u_0(x,y)$$

$$R^- = \{(x,y) : u_0(x,y) \geq f(x,y)\} = \bigsqcup_i R_i^-$$

$$R^+ = \{(x,y) : u_0(x,y) < f(x,y)\} = \bigsqcup_i R_i^+$$

Leveling Cartoons, Texture Energy Markers and Image Decomposition …

Petros Maragos and Georgios Evangelopoulos

ISMM 2007

Leveling-based cartoons

- Leveling cartoon approximations $u = \Lambda\left(M \mid f\right)$
  - u: leveling of image f
  - M: marker (e.g. Gaussian, anisotropic)

- Residual $r = f - u$
  - finer scales information
  - *contains* texture v

- Multi-scale levelings
  - hierarchy of cartoons/residuals

$$\boxed{u_i = \Lambda\left(M_i \mid u_{i-1}\right), i = 1, 2, \ldots, n}$$

$$u_0 = f \qquad \boxed{r_i = f - u_i}$$

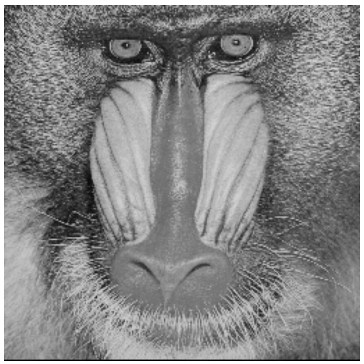
  - causality property: *u_j is a leveling of u_i for j>i*
  - markers are samples of a scale-space $M_i = f * G_{\sigma_i}$

Multiscale leveling decomposition (example)

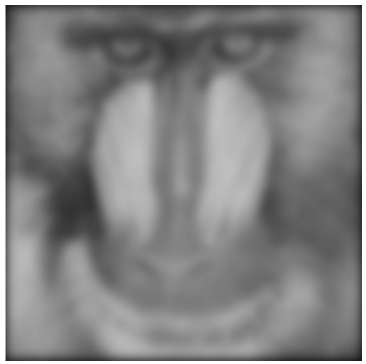
- 2-level cartoons and residuals via levelings with markers-samples of an isotropic Gaussian scale-space

image | *marker* | *leveling* | *residual*

level 1, (σ1=4)

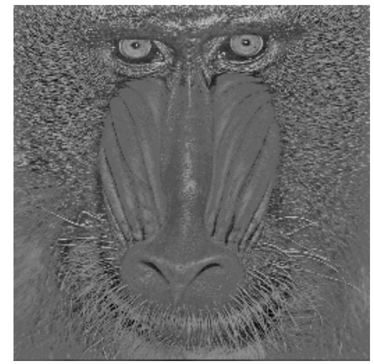


f



$M_1 = f * G_{\sigma 1}$

$u_1 = \Lambda\left(M_1 \mid f\right)$



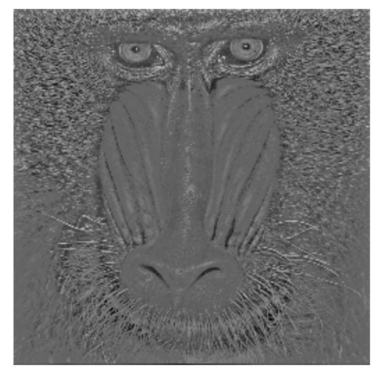
$r_1 = f - u_1$

level 2, (σ2=8)

$u_1 - u_2$

$M_2 = f * G_{\sigma 2}$

$u_2 = \Lambda\left(M_2 \mid u_1\right)$



$r_2 = f - u_2$

Comparisons with TV cartoons

- Levelings decrease the Total Variation norm
  - Creation of flat plateaus on which the gradient becomes zero

$$\iint \|\nabla u_{i+1}\| \leq \iint \|\nabla u_i\| \leq \iint \|\nabla f\|$$

Leveling cartoons

a. preserve regional maxima & minima and do not create new

b. preserve the sense of variation between neighbour pixels

c. TV norm decreases monotonically

d. scale controlled by (the scale) of the marker image

TV cartoons

a. preserve the global mean

b. preserve the global variance

c. scale controlled by the regularizing constant

AM-FM Texture Model

- Locally narrowband image texture *(Bovik, Havlicek)*

$$f(x,y) = a(x,y)\cdot\cos\left[\phi(x,y)\right], \qquad \nabla\phi(x,y) = \vec{\omega}(x,y)$$

  - analogies between AM-FM and Y.Meyer's oscillating functions for texture

- Amplitude and frequency estimation
  - Multiband Gabor filtering
  - 2D Energy Operator $\Psi(f) = \|\nabla f\|^2 - f\nabla^2 f$
  - Demodulation via ESA *(Maragos & Bovik 1995)*

$$\frac{\Psi(f)}{\sqrt{\Psi(\partial f/\partial x) + \Psi(\partial f/\partial y)}} \approx |a(x,y)|$$

$$\sqrt{\Psi(\partial f/\partial x)/\Psi(f)} \approx |\omega_1(x,y)|, \qquad \sqrt{\Psi(\partial f/\partial y)/\Psi(f)} \approx |\omega_2(x,y)|$$

Multiband Texture Energy Tracking (I)

- *Texture modulation energy* of a locally narrowband component

$$f(x,y) = a(x,y)\cdot\cos\left[\phi(x,y)\right], \qquad \Psi\left[a\cos(\phi)\right] \approx a^2 \left\|\nabla\phi\right\|^2$$

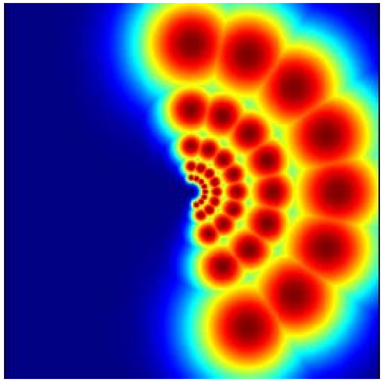
- Bandpass filter the “texture image part” to isolate components

$$f_k(x,y) = (v * g_k)(x,y) \approx (v_k * g_k)(x,y),$$

- Impulse Responses of a 2D Gabor filterbank

$$h_k(x,y) = \exp\left\{-a_k^2 x^2 - b_k^2 y^2\right\}\exp\left\{j\vec{\Omega}_k\cdot(x,y)\right\}$$

- k-filter:
  - Bandwidth parameters (a_k, b_k),
  - central frequency vector $\vec{\Omega}_k$



- Filterbank design
  - polar arrangement in spectral domain
  - octave bandwidth, equal bandwidth params
  - typical design (40 filters, 5 scales, 8 orientations)

Multiband Texture Energy Tracking (II)

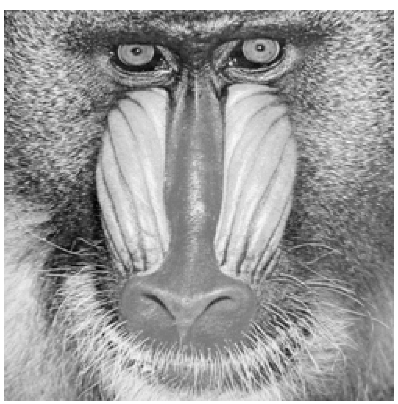
- Maximum Average Teager *(MAT)* energy
  - Energy tracking from the set of filtered, narrowband texture components

$$\Psi_{\mathrm{mat}}\left(v(x,y)\right) = \arg\max_{k}\left\{\left(\Psi\left(v * h_k\right) * h_a\right)\left(x,y\right)\right\}$$

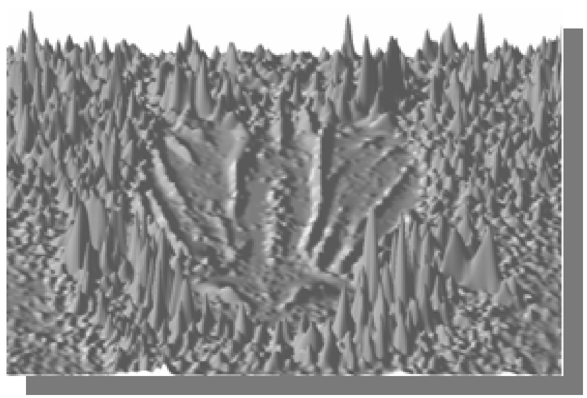
  - h_a: local averaging filter, h_k: the k-th Gabor filter-channel

- Indicates texture structure (analysis, detection, classification)
  - Criterion for the extraction of the texture dominant component
  - Dominant modulation features (amplitude, frequency, energy)

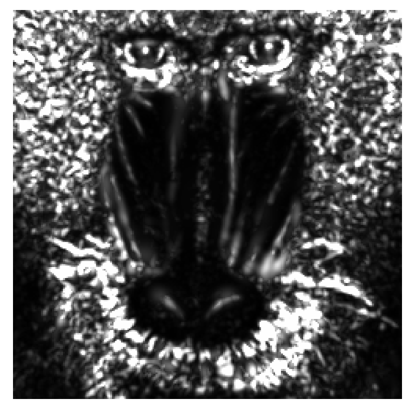
image



MAT energy

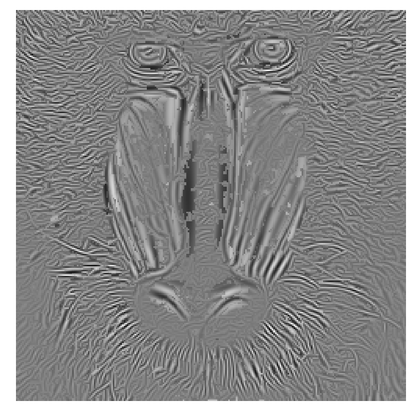


3D



grayscale

dominant component



MAT energy for Texture detection

- Texture energy measurements for texture markers
  - indicate texture areas
  - quantify region ‘texturdeness’
  - roots of the MAT energy
- Markers extracted from the texture image part (e.g. leveling residual)
  - absence of large scale, geometric structures and features (edges, contours, blobs, contrast)
  - $f=u+v$: texture + objects, v: texture, details, oscillations

$\Psi_{\text{mat}}(\cdot)$ | $\sqrt{\Psi_{\text{mat}}(\cdot)}$ | $\left(\Psi_{\text{mat}}(\cdot)\right)^{1/4}$

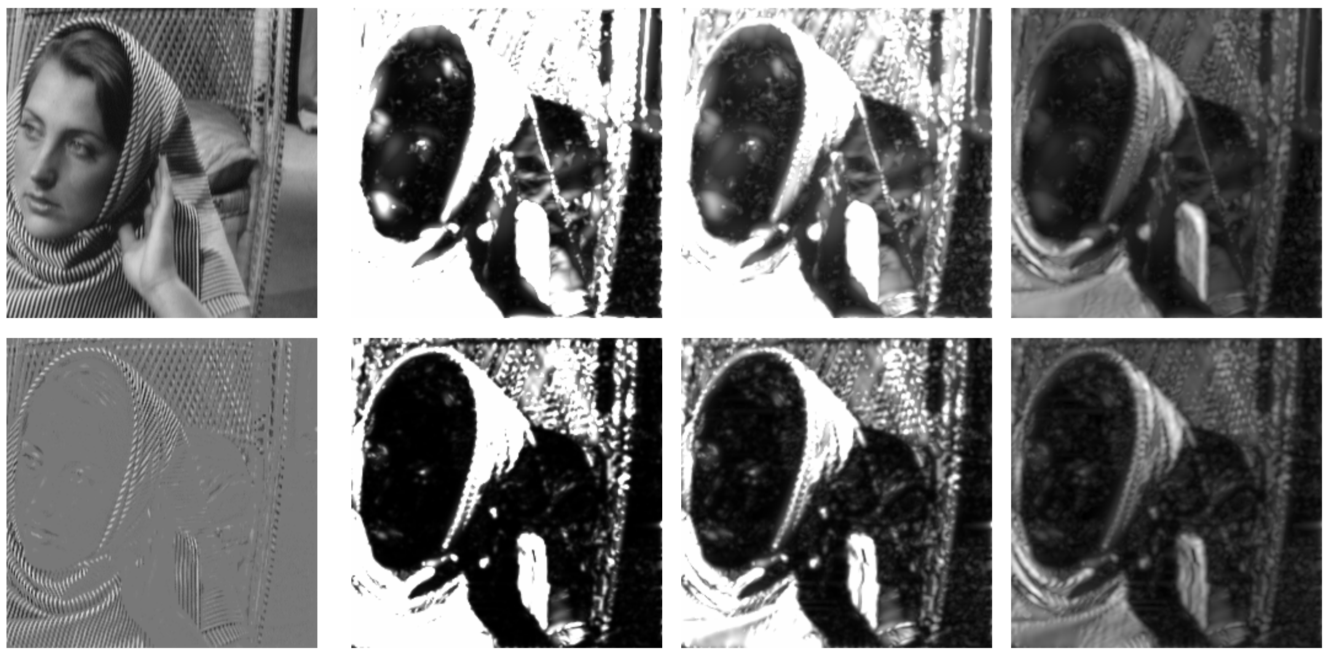
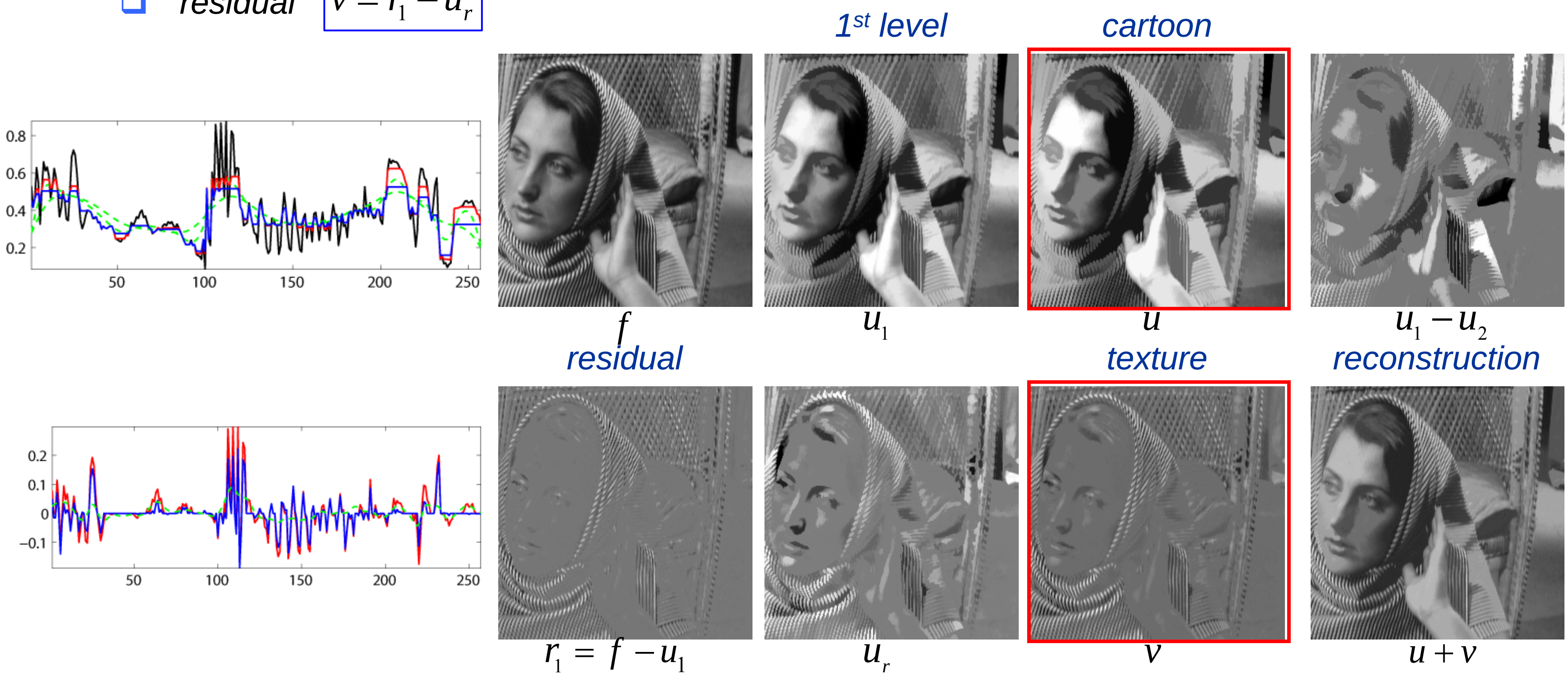


image f

leveling (σ=2) residual $r_1 = f - u_1$

Leveling-based decomposition (Gaussian Markers)

- Cartoon, second-order leveling by Gaussian markers
  - *1st level* $u_1 = \Lambda\left(M_1 \mid f\right),\quad M_1 = f * G_{\sigma 1}\quad r_1 = f - u_1$
  - *2nd level* $\boxed{u = u_2 = \Lambda\left(M_2 \mid u_1\right),}\quad M_2 = f * G_{\sigma 2}$
- Texture, residual from 'leveling on residual'
  - *1st level* $u_r = \Lambda\left(M_3 \mid r_1\right),\quad M_3 = r_1 * G_{\sigma 3},\quad \sigma 3 = \sigma 1/2$
  - *residual* $\boxed{v = r_1 - u_r}$



Comparisons of u+v models

- Comparisons of leveling (u_Λ, v_Λ) decomposition with Vese, Osher (u_{VO}, v_{VO}) model
  - *decomposition* (u, v) : Sharper object contours and small-scale features in u_Λ, texture components seem similar
  - *noise residuals* (w=f-u-v): Leveling preserves structure while VO preserves texture. Does w_Λ, model image 'noise' ?
  - reconstruction from the model (u+v): Leveling quantizes intensity values.
- Parameters are chosen to enforce $\|v_{VO}\|_2 = \|v_\Lambda\|_2$

cartoon *texture* *noise residual* *reconstruction*

Vese-Osher, u+v
(λ,μ)=(5,0.1)

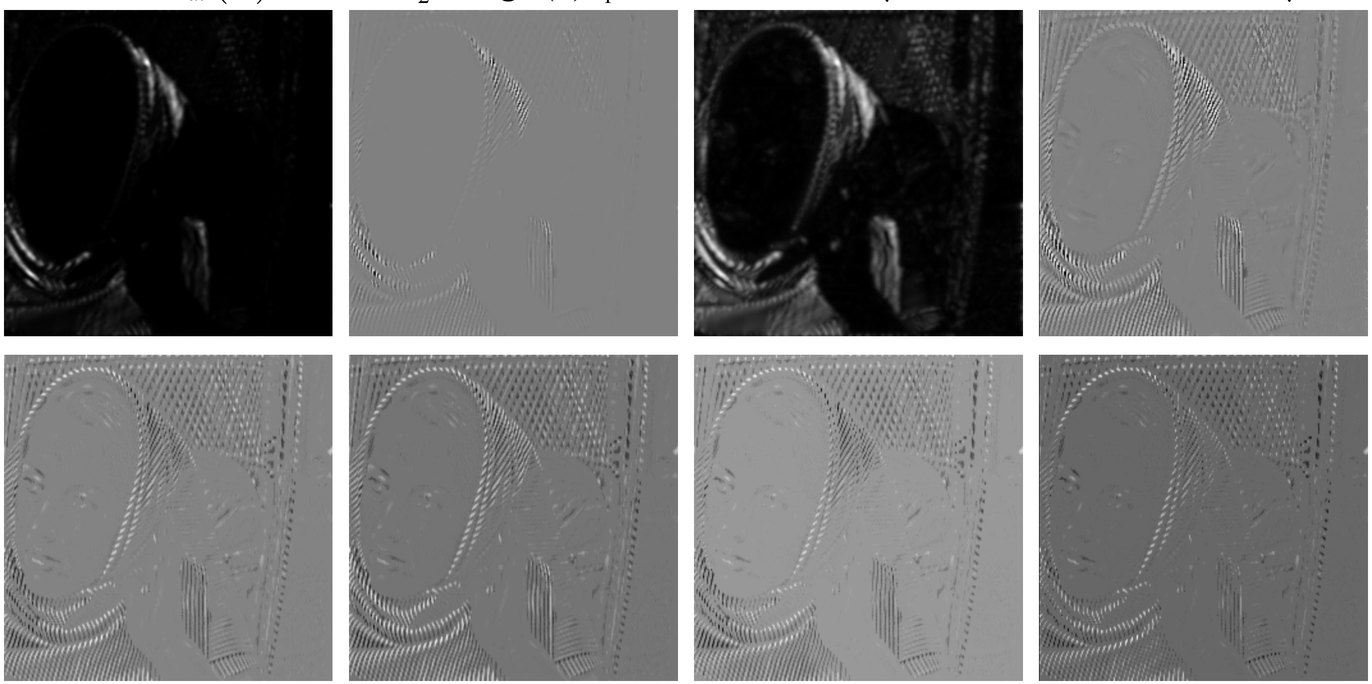
Leveling,
Gaussian markers
(σ1,σ2)=(10,16)

Leveling-based decomposition (Energy markers)

- Texture component v is retrieved by
  a. leveling the 'cartoon' residual $r_1 = f - \Lambda\left(M_1 \mid f\right)$ using texture-based markers
  b. keeping the 'new' residual

- Energy-based texture markers $\boxed{v = r_1 - \Lambda\left(\operatorname{sign}(r_1)\left[\left(\Psi_{\text{mat}}\left(r_1\right)\right)^{1/k}\right] \mid r_1\right), \quad k = 1, 2 \ldots}$
  - mappings/transforms of the texture MAT operator (e.g. signed roots)

$T_1 = \Psi_{\text{mat}}\left(r_1\right)$ $\quad T_2 = \operatorname{sign}(r) T_1$ $\quad T_3 = \sqrt{T_1}$ $\quad T_4 = \operatorname{sign}(r)\sqrt{T_1}$

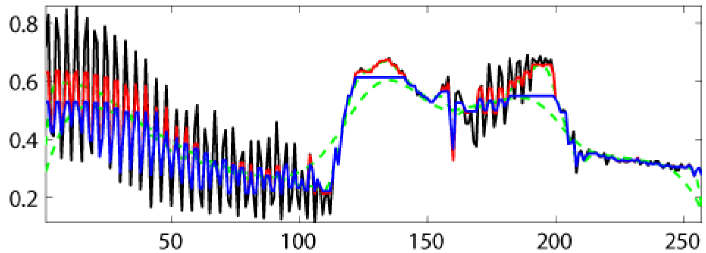


Markers
T

Texture components
$v = r_1 \text{-} \Lambda\left(T \mid r_1\right)$

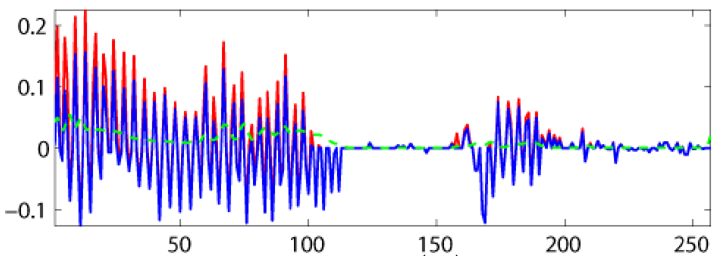
Decomposition process in 1D (profiles)

line 240

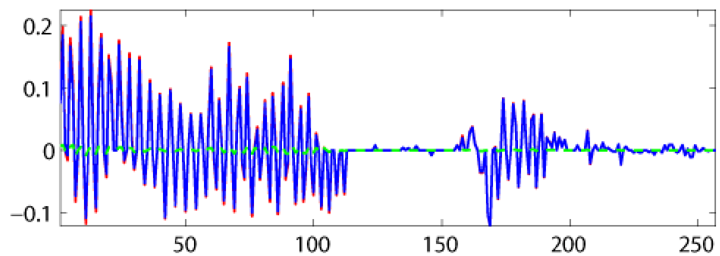


cartoon

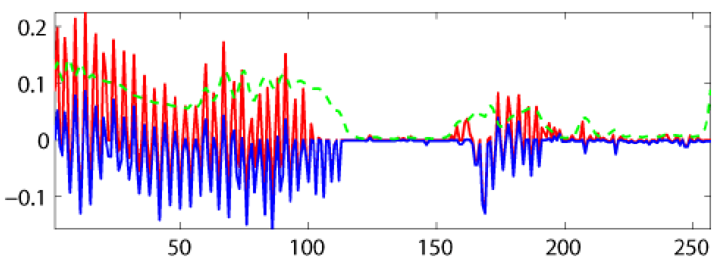
Black: Image
Red: 1st level
Blue: 2nd level (cartoon)
Green: Marker (Gaussian)



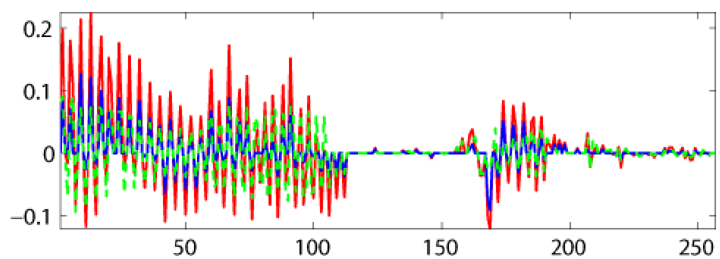
$T_1 = \Psi_{\text{mat}}(r_1)$



$T_2 = \text{sign}(r)T_1$



$T_3 = \sqrt{T_1}$

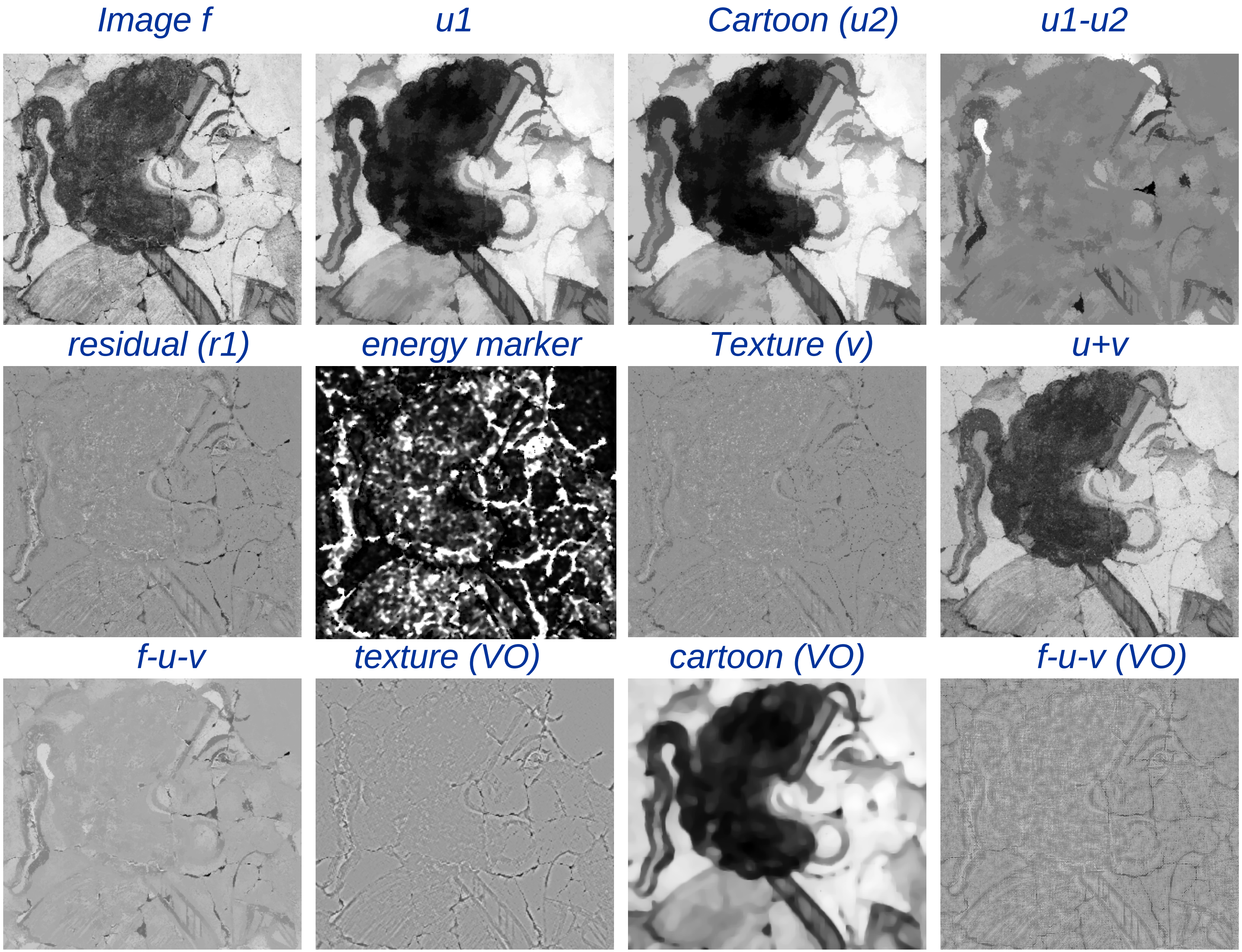


$T_4 = \text{sign}(r)\sqrt{T_1}$

texture

Red: Residual (r1)
Blue: Residual (texture)
Green: Marker (energy)

Application (Prehistoric Wall-Painting Restoration)



Section of ‘Potnia’ prehistoric wall painting in Thira, Acrotiri