News From ViscousLand

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CONTENT

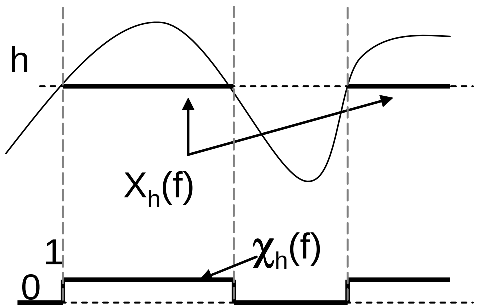
- Standard Morphological Operators process all image points identically.
- In some cases however, the regularization must be locally adapted to the local information in order to
  - preserve accuracy at certain points
  - strongly regularize the data at other points
- Hypothesis : high luminance values correspond to points of high precision while low luminance values correspond to uncertainties.

Ideas :

1. The filtering activity at a point p is defined as a function of the local luminance $f(p)$.
2. Instead of updating the filtering activity at each point of the space, families of filters of increasing activities are combined in order to produce the desired effect at each point of the space.
3. Two models are proposed. The first one considers the luminance ; in the second one, the filtering activity is function of the contrast.

- Let $(t_n)_{0\leq n\leq N}$ denote a family of increasing operators of increasing activity.
  1. $f < g \Rightarrow t_n(f) \leq t_n(g)$
  2. $n > p \rightarrow |t_p(f) - f| \leq |t_n(f) - f|$
     if t_n is extensive : $f \leq t_p(f) \leq t_n(f)$
  3. $t_0 = Id$
- The goal is to combine the operators $(t_n)_{0\leq n\leq N}$ in order to realize a compromise between the identity and the coarsest filter t_N : $Id \leq T^v \leq t_N$.
- Points having the same luminance in the image are processed identically, so the function can be examined level set by level set.

  Let $(X_h(f))_{0\leq h\leq N}$ denote the level sets of f : $X_h(f) = \{p \in E,\ f(p) \geq h\}$ and $(\chi_h(f))$ the associated indicatrix functions.



$$\chi_N \leq ... \leq \chi_h \leq ... \leq \chi_0$$

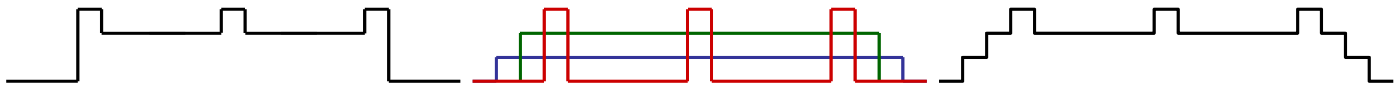
- A viscous transform will be specified by defining the filtering to be applied to each level h.

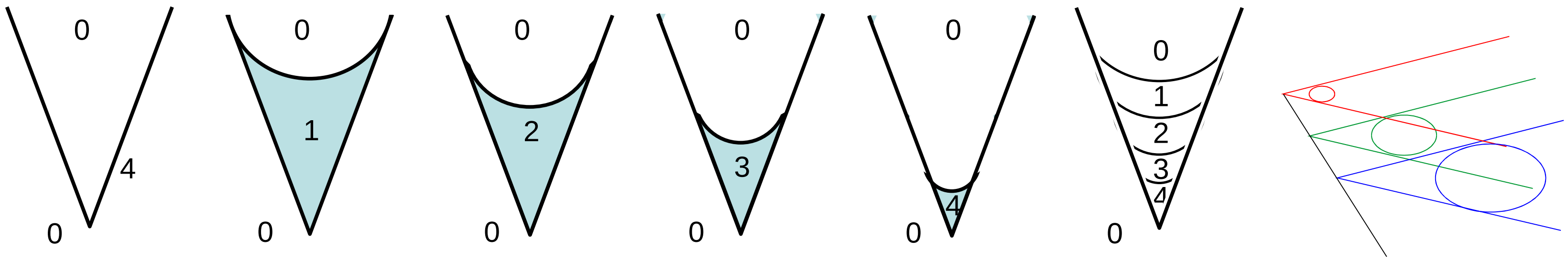
THE TWO VISCOUS TRANSFOMATIONS MODELS : Oil and Mercury

- **Oil type**
  - Points of high luminance are less filtered than points of low luminance : $t_{N-h}(\chi_h)$

$$
\begin{array}{ccc}
f & \longrightarrow & \{\chi_h(f)\}_{h \geq 0} \\
T^v \downarrow & & \downarrow t_{N-h} \\
\bigvee_{h \geq 0} h.t_{N-h}[\chi_h(f)] & \longleftarrow & \{t_{N-h}[\chi_h(f)]\}_{h \geq 0}
\end{array}
$$

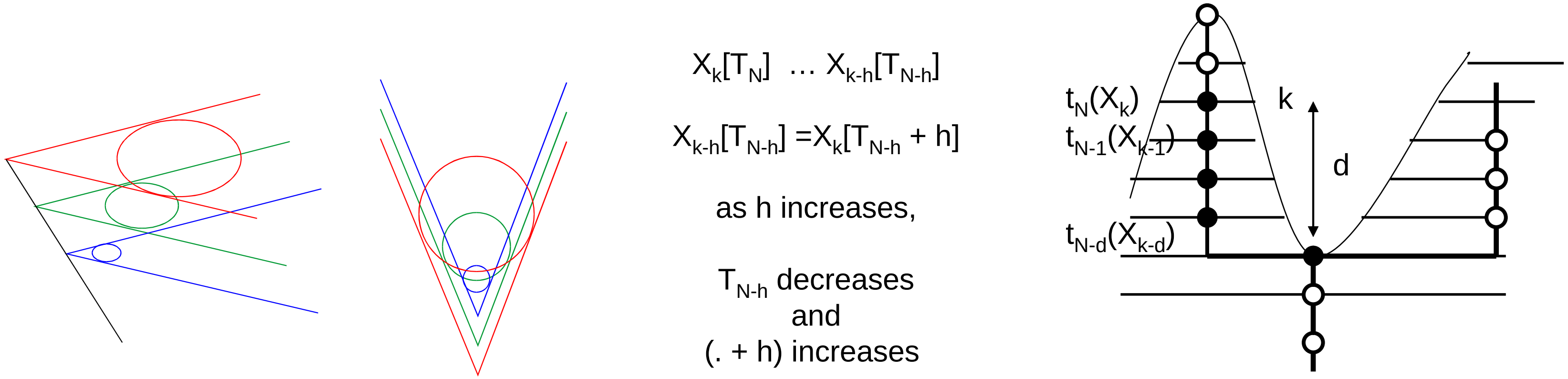
  - The viscous transform T^v results from the superposition of the whole family of sets.





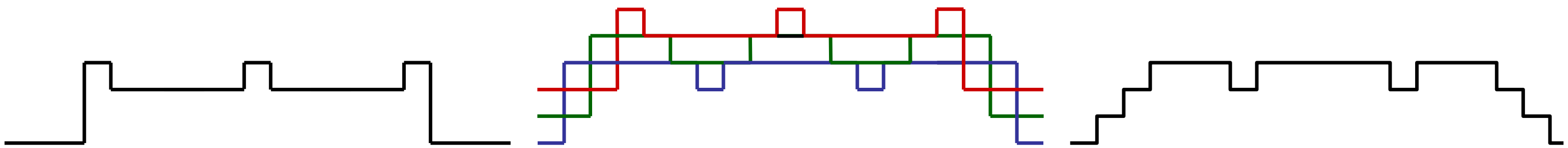
THE TWO VISCOUS TRANSFOMATIONS MODELS : Oil and Mercury

- Mercury type
  - The filtering activity is indexed on the contrast rather than on the luminance. At altitude k, points processing results from the combination of transformation of sets located at lower altitude :



$$\bigwedge_{h=0:d} X_k[t_{N-h}(.)+h] = X_k[\bigwedge_{h\geq 0} t_{N-h}(.)+h] \quad \text{and} \quad \widetilde{T}^v(.) = \bigwedge_{t=0:N} t_{N-k}(.+k)$$

  - The transformation can no more be processed level set by level set.



$$T^v(.) = \bigvee_{h \geq 0} h.t_{N-h}[\chi_h(.)] \qquad \widetilde{T}^v(.) = \bigwedge_{h \geq 0} t_{N-h}(.) + h$$

- t_n being increasing, T^v and $\widetilde{T}^v$ are increasing too.
- Comparison (case of extensive operators)

$$Id \leq T^v \leq \widetilde{T}^v \leq t_N$$

- As consequence : $T^v \leq T^v\widetilde{T}^v \leq t_N$ and $\widetilde{T}^v \leq \widetilde{T}^v T^v \leq t_N$

  and it can be proved that : $T^v\widetilde{T}^v = \widetilde{T}^v$

- Viscous closings and quasi-closings

$$\phi^v(.) = \bigvee_{h\geq 0} h.\varphi_{N-h}[\chi_h(.)] \quad \widetilde{\phi}^v(.) = \bigwedge_{h\geq 0} \varphi_{N-h}(.) + h$$

  – ϕ^v is a closing but $\widetilde{\phi}^v$ is not (not idempotent).

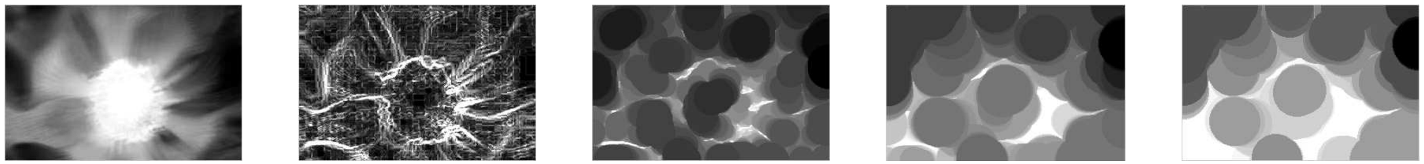
- Viscous (quasi-)openings are defined by duality :

$$\Gamma^v(f) = -\phi^v(-f) = \bigvee_{h\geq 0} h.\gamma_h[\chi_h(f)]$$

$$\widetilde{\Gamma}^v(f) = -\widetilde{\phi}^v(-f) = \bigvee_{h\geq 0} \gamma_{N-h}(f) - h$$

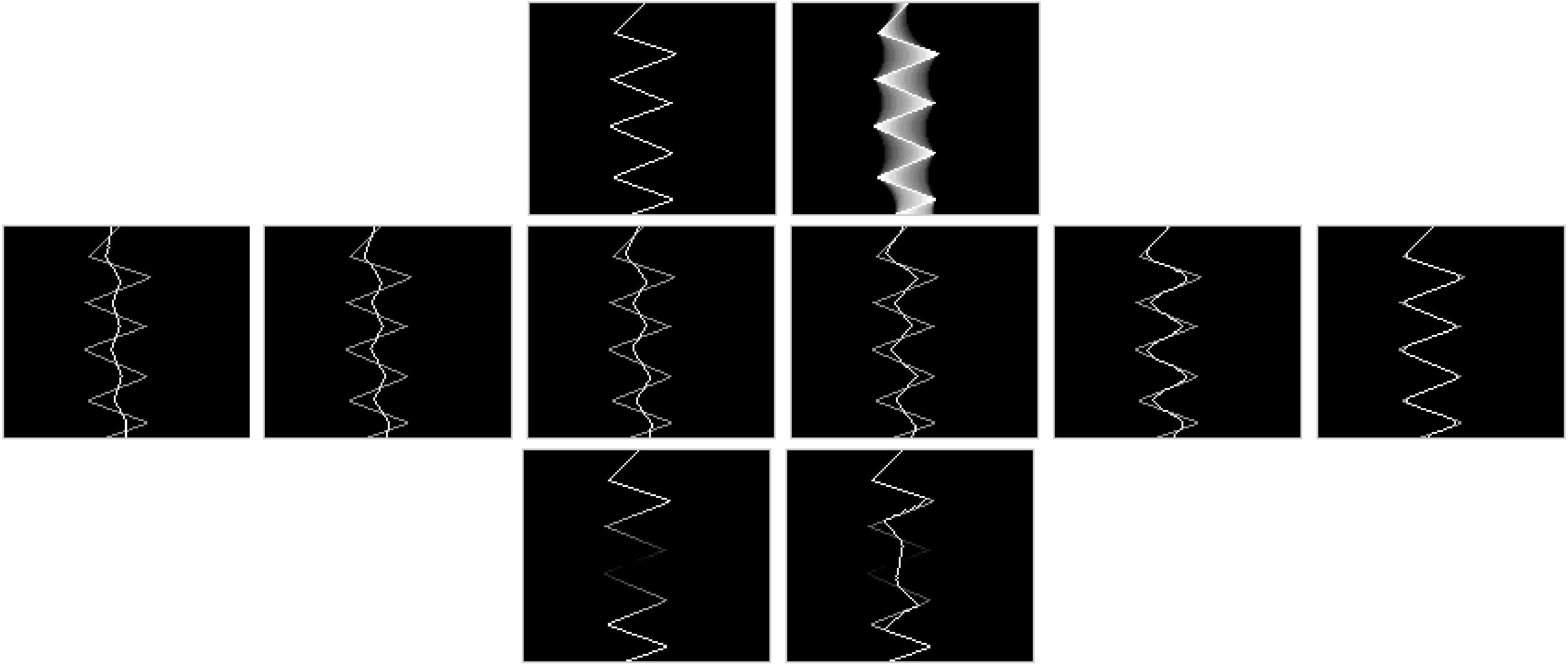
VISCOUS OPENINGS AND CLOSINGS

(1)Original image (2)result of a morphological opening (3)result of the viscous opening (of mercury type)

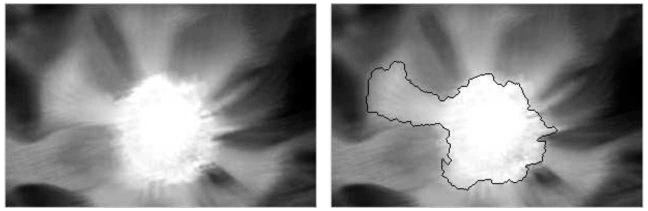


(1)Original image (2)gradient (3)viscous closing (oil) (4)viscous closing (mercury) (5) morphological closing

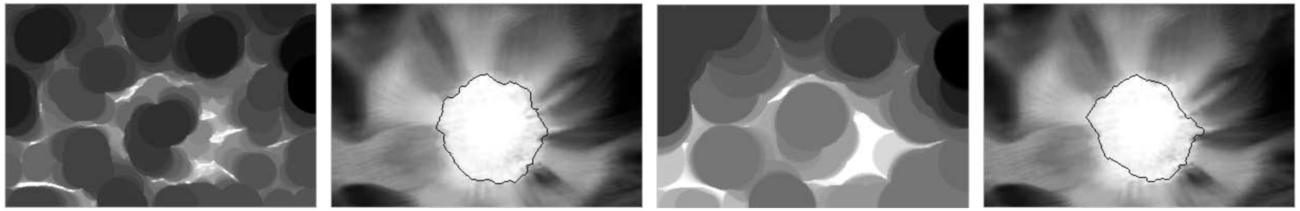
- Extraction of the watershed line after a viscous closing of the relief : **The Viscous Watershed Transform**



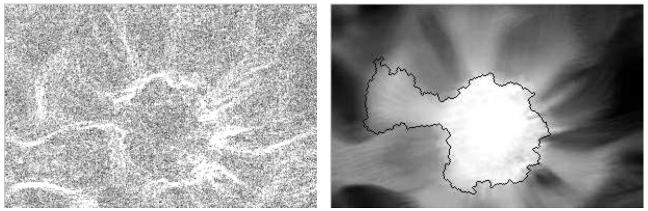
VISCOUS OPENINGS AND CLOSINGS



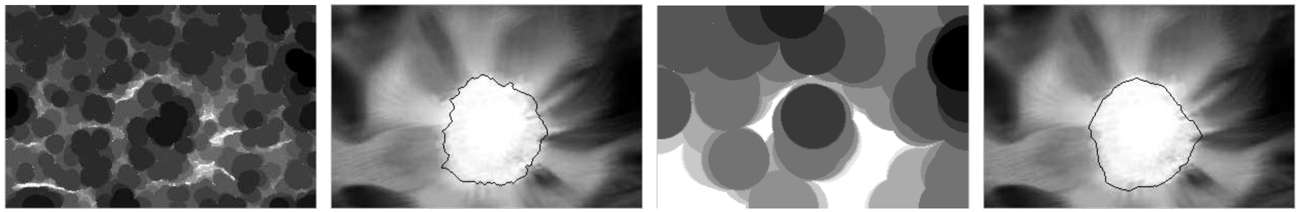
Standard Watershed Transform



Viscous Watershed Transforms (oil-mercury)



WT after addition of a gaussian noise



Viscous closings and VWT computed on the noisy image (oil-mercury)

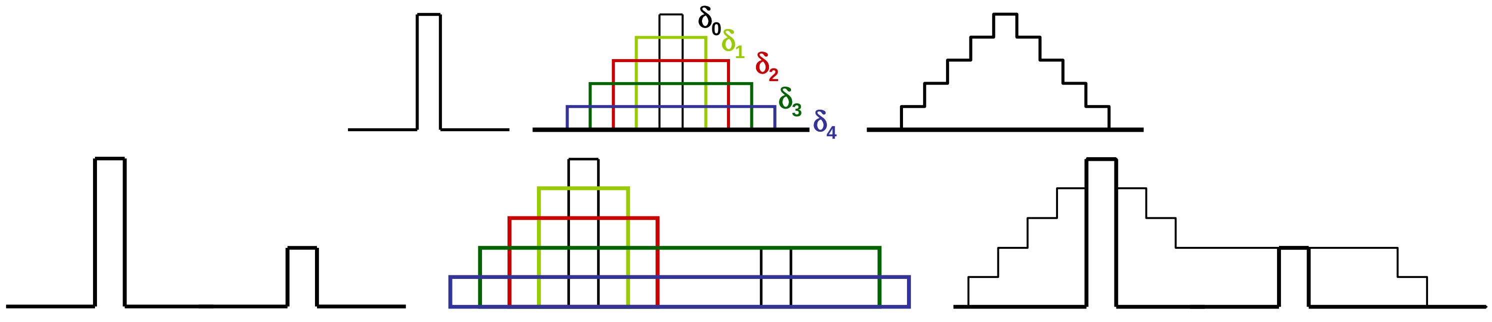
VISCOUS DILATIONS AND EROSIONS

- Viscous pseudo-dilation (mercury type) and viscous dilation (oil type)

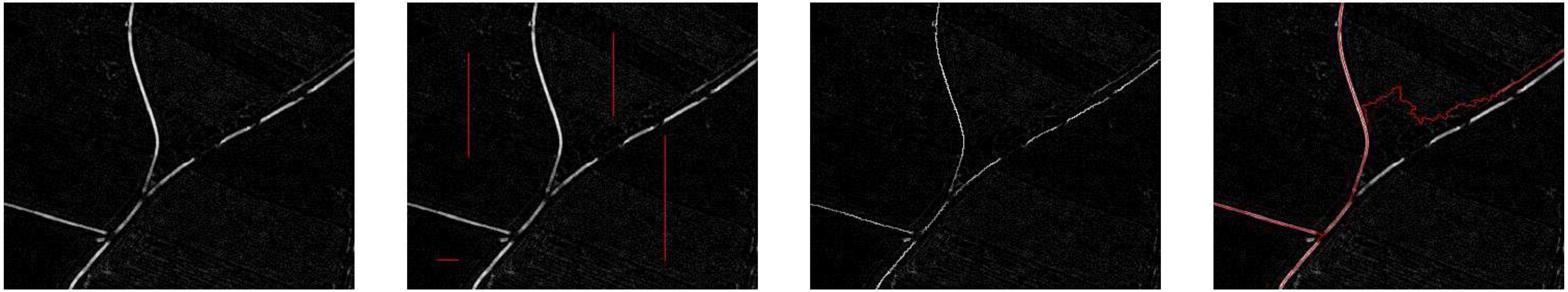
$$\Delta^v(.) = \bigvee_{h \geq 0} h.\delta_{N-h}[\chi_h(.)] \quad \widetilde{\Delta}^v(.) = \bigwedge_{h \geq 0} \delta_{N-h}(.) + h$$

  – Binary case : $f = \chi_1(f) = \chi$
    $\Delta^v(\chi) = \bigvee_{h \geq 0} h.\delta_{N-h}[\chi] = \bigvee_{n=0:N-1} (N-n).\delta_n[\chi] = \sum_{n=0}^{N-1} \delta_n[\chi]$
    * points of set χ are at level N
    * points at distance n from χ are at level $N-n$.
    * classical algorithm for computing the negative distance function from a set in a narrow band of size N.
    * application : thin lines reconnection

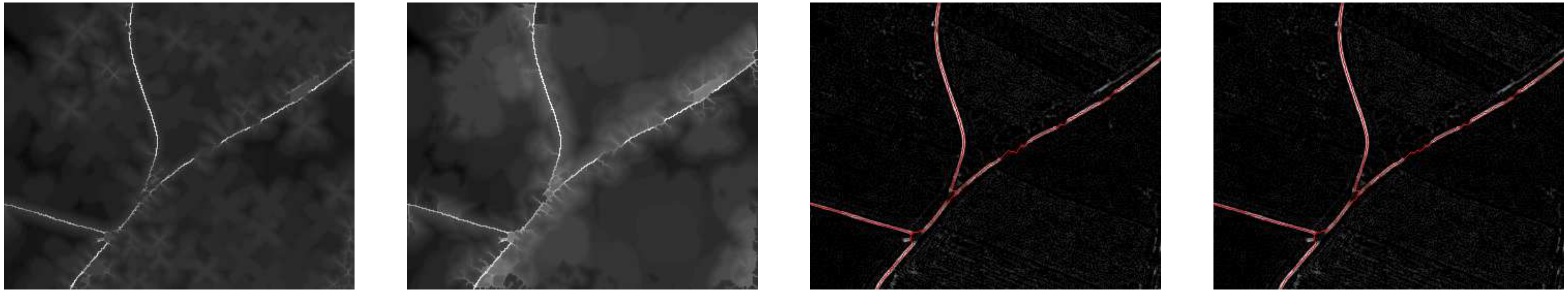
  – Viscous dilations proposed an extension of this algorithm to the case of gray-tone images



VISCOUS DILATIONS AND EROSIONS



Morphological numerical thinning and associated watershed transform



Viscous dilation of the thinned image and associated watershed transform

VISCOUS DILATIONS AND EROSIONS

Standard Watershed Transform

Standard and Viscous Watershed Transform computed after a viscous dilation of the gradient image

- Viscous transforms have been defined in a general manner.
- Viscous closings are used for regularizing crest lines. They lead to the concept of Viscous Watershed Transform.
- Viscous dilations allow the computation of distance maps in the gray-scale case. They can be used for reconnecting crests lines in gray-scale images.
- Future work : study of the properties of the viscous transformations and of their use in others image filtering and segmentation applications.