Determination of the Set of Feasible Investment Rates to Achieve Pre-specified Goals of Forecasted Carbon Emissions

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Abstract

The main idea in this work is to use the constraint satisfaction approach to determine the set of feasible investment rates in reforestation and clean technology, so as to achieve pre-specified goals in terms of carbon emission, as forecasted by a mathematical model. An efficient allocation of resources to reduce the greenhouse effect depends on constraints related to technical and political decisions. In a previous article, the authors proposed to use optimal control theory to provide estimates of the investments needed in land reforestation and in the adoption of clean technologies for an optimum emission and abatement of CO_2. However, the concept of optimality relies on assigning a number to each investment policy via a cost function, which may be difficult to be specified. Here, a different viewpoint is adopted, namely the determination of the set of investment rates that permits goals, represented by inequalities that must be satisfied. More specifically, the goals reflect target values for CO_2 and the times required to attain them. A case study is carried out using published data for the European Union from 1960 up to 1996. The goals are proposed to conform to the *Kyoto Protocol* for European Countries. The investment rates are proportional to the GDP (grow domestic product) and CO_2 concentration and affects the reforestation efforts and the adoption of clean technology. Several scenarios were simulated in order to provide insight in terms of de adopted decisions and effects of the uncertainty in the model parameters.

Keywords: Constrain Satisfaction, Goal Attainment, CO_2 concentration, Forecasting, Dynamic Model, Reforestation, Clean Technology.

1. The Mathematical Model

One can find a variety of mathematical models in the literature describing the dynamics of the GHG - Greenhouse Gases emission (see, for instance, Nordhaus 1991, 1993, 2006). A large class of mathematical model uses ordinary differential equations to relate the production of CO_2 with forest area and GDP, following Caetano *et al*. (2008, 2009). The parameters of the model were adjusted using widely published data, such as those available at UNEP (UNEP GRID ARENDAL, 2007,2008), and the estimates of the required joint investments are determined by solving numerically using closed-loop control (Kirk, 1970; Lewis, 1986). It consists of a system of three coupled ordinary differential equations involving the concentration of atmospheric carbon dioxide *x(t)*, forest region *z(t)* and Gross Domestic Product - GDP, *y(t)*. In what follows, the "dot" notation represents the derivative of a variable with respect to time *t*.

$$\begin{cases} \dot{x} = rx\left(1-\dfrac{x}{s}\right) - \alpha_1 z + (\alpha_2 - u_2)y \\ \\ \dot{z} = u_1 y - hz \\ \\ \dot{y} = \gamma\, y \end{cases} \tag{1}$$

The decision variables are u_1 and u_2, representing the shares of GDP in reforestation and in the adoption of clean technology, respectively. The model parameters (constants) are r, s, h, α_1, α_2 and γ. The relationships among the variables in mathematical model can be visualized in Fig. 1 (Caetano *et al.*, 2008, 2009).

In terms of an intuitive interpretation of the system of equations (1), one can notice that the CO_2 emissions (*x*) are dependent on *r*, the emission rate and s, the carrying capacity of the atmosphere in terms of CO_2. The second term represents the net balance of emission and removal of CO_2 and the contribution of a certain region in terms of removal of CO_2 from the atmosphere is assumed to be proportional to the total forest area. The total area of forest at time t depends on the initial condition (z_0), such as an existing forest and the reforestation effort. The reforestation effort is assumed to be a fraction of the GDP (in countries where there are laws and incentives to promote reforestation) with u_1 representing the intensity of incentives directed to reforestation and u_2 representing the incentives to clean technology considering that the required clean technology is proportional to the GDP. The parameter h is a constant representing the forest depletion rate and amalgamates a variety of factors such as expansion of cattle ranching, fire, commercial logging, shifted cultivators and colonization, among others.

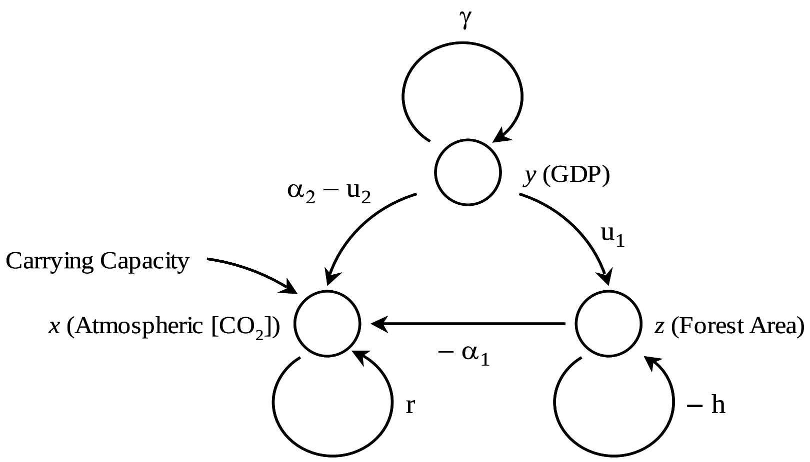

Fig 1. Relationships among the state variables in the adopted model.

The model parameters (Table 1) were adjusted to fit the conifer forest data for the Western Europe region available at UNEP (UNEP, 2007) for the tracking problem. Results presented in Fig. 2 represent the actual data and the numerical simulations using the model defined in (1), show good agreement using numerical values of parameters from Caetano *et al.*(2008). The initial conditions for the Western Europe in 1960 (UNEP, 2007) are $x(0)$ = 398 million tons of CO_2, as for the , $z(0)$ = 43 million m^3 in 1960 and $y(0)$ = 2,787 billion international dollars in 1960, GDP value for 1995 (The World Economy, 2007).

Table 1. Model Parameters Fitted for Western Europe.

Parameters	**Values**
r	0.15
s	700
h	0.01
u_1	0.0002
u_2	0.0008
α_1	0.15
α_2	0.00005

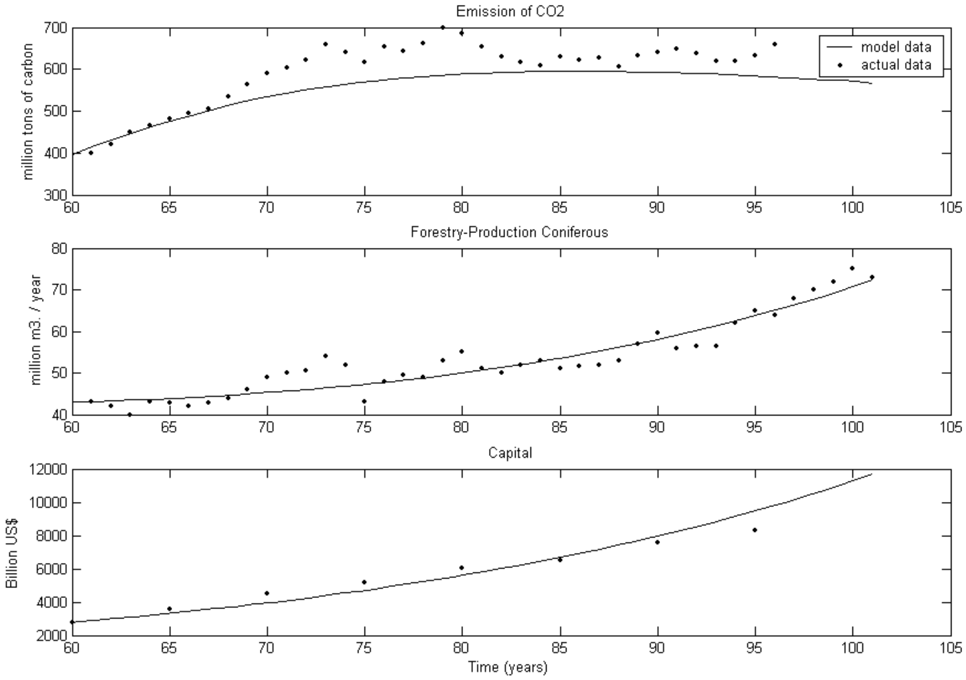

Fig 2. Comparison between the actual data and the simulation using the estimated non linear model.

3. The Closed-Loop Control

According to the *Kyoto Protocol*, the recommended collective reduction of Greenhouse Gases (carbon dioxide, methane, nitrous oxide, sulfur hexafluoride, hydrofluorocarbons, and perfluorocarbons) for industrialized countries is 5.2%, averaged over the period of 2008-2012, taking as reference the year of 1990. The European Union (EU) has accepted a quantitative absolute reduction of 8% of its GHG emissions and the projected welfare cost may vary across countries between 0.6% and 5% of GDP (excluding favorable terms of trade effects), assuming that no climate policies are implemented (Viguier *et. al*, 2003).

The initial condition for the state variables in our model correspond to the actual data for 1996, starting with *x(0) = 659* million tons of CO_2, *z(0) =68* million m^3 and *y(0) = 8,306* billion international dollars. The final target for Western Europe CO_2 emission, according to the *Kyoto Protocol* is 590 million tons.

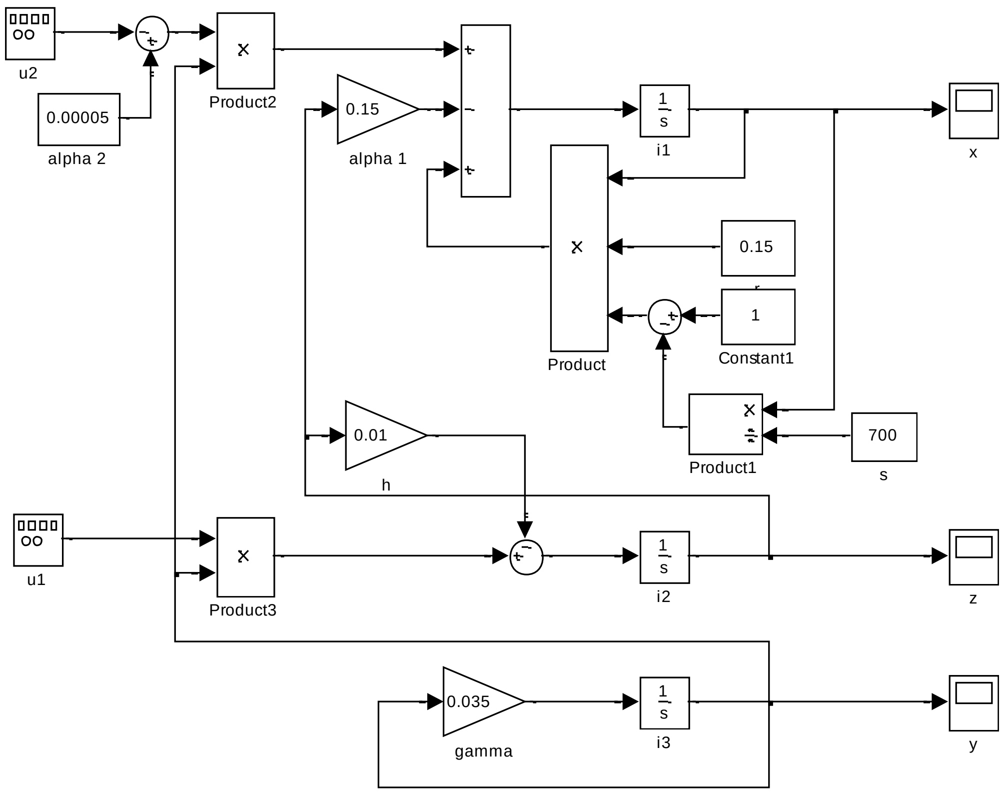

Fig 3. Schematics for Simulation of the Dynamic System given in (1)

The figure 3 shows the simulation schematics for the dynamic system given in (1) where u_1 and u_2 are the control variables. The control u_1 is made proportional to the percentage p_x from CO_2 emission

$$u_1 = \beta_1 p_x x \qquad (2)$$

and the second control u_2 is a percentage p_y of the GDP

$$u_2 = \beta_2 p_y y \qquad (3)$$

where β_1 and β_2 are assumed constant in this work. If β_1 and β_2 were time variant, one would need to solve an optimal control problem.

The system (1) under the controls u_1 and u_2 operates in closed-loop configuration and the dynamics now depends on the values of fixed investment rates (percentages) p_x and p_y, that can be chosen in such a way as to satisfy political, economic and technological constraints. The state equations then becomes

$$\begin{cases} \dot{x} = rx\left(1-\frac{x}{s}\right) - \alpha_1 z + \left(\alpha_2 - \beta_2 p_y y\right)y \\ \dot{z} = (\beta_1 p_x x)y - hz \\ \dot{y} = \gamma\, y \end{cases} \tag{4}$$

where p_x and p_y are terms used to represents small parts transferred from CO_2 emission and GDP to closed-loop control.

The problem of constraint satisfaction is to determine values of p_x and p_y so as to attain a state where

$x(Final\ time) \geq$ Inferior Limit for CO_2

$x(Final\ time) \leq$ Superior Limit for CO_2

Inferior Limit for years $\leq$ *Final time* $\leq$ Superior Limit for years

4. Results

The numerical solutions of the constraint satisfaction problem were obtained by combining a search-in-a-grid procedure for p_x and p_y and using the *ode45.m* on Matlab 6.5 to simulate the closed loop system given in (4). Pairs (p_x,p_y) that satisfy the constrains constitute a region in the plane $\{p_x,p_y\}$. Once the region in characterized, the decion maker can choose the most convenient pair (p_x,p_y) using secondary criteria.

The values of β_1 and β_2 that were chosen are β_1 = 0.00001 and β_2 = 0.000001. The term related to the removal of CO_2 from the atmosphere is assumed to be α_1=15% in this first scenario.

The limits in the constraints were chosen according to the *Kyoto Protocol*. For the CO_2 emissions the adopted values are 550 and 600 million tons of CO_2. For the final time the values are 12 years and 14 years. The figure 4 shows the region in the plane $\{p_x,p_y\}$ where x-axis represents % of GDP (p_y) and y-axis represents % of CO_2 (p_x). The white circles are the possible pairs (p_x,p_y) that satisfy the constraint for the CO_2 emission. The filled circles are those that satisty both, CO_2 and Final time criteria. Then, the admissible region is represented by $p_y \in [0\%, 10\%]$ of GDP and $p_x \in [0\%, 6\%]$ of $[CO_2]$.

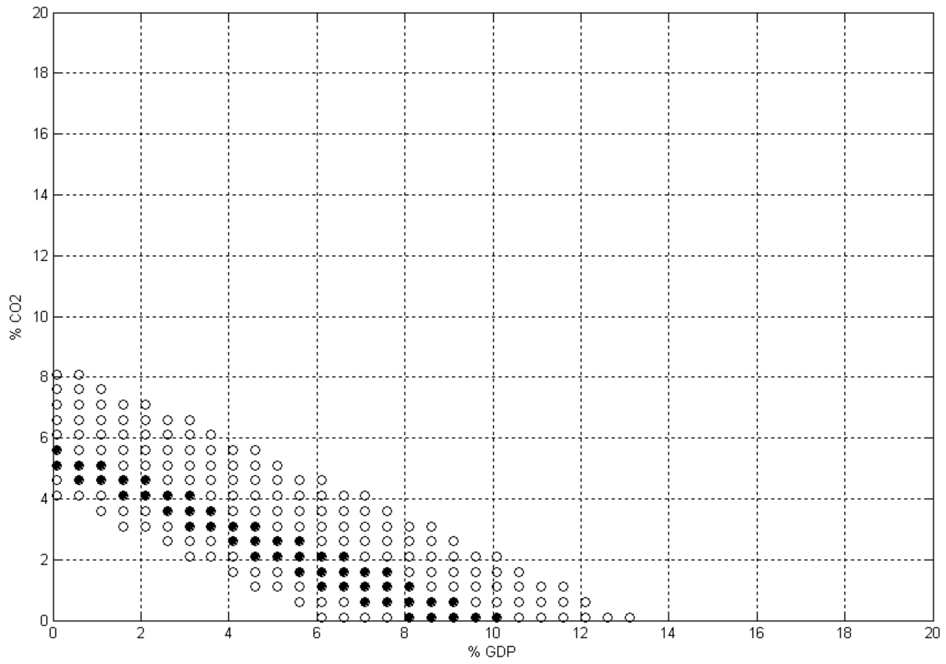

Fig 4. Feasible regions for CO_2 emissions between 550 and 600 millions tons of CO_2. White circle $550 \leq CO_2 \leq 600$. Black circle represents second constraint $12 \leq$ *Final time* ≤ 14 years considering $\alpha_1 = 15\%$.

In figure 5, one can see the results corresponding to $p_x = 2\%$ and $p_y = 7\%$. As mentioned before, other feasible pairs could have been chosen in the region marked with filled circles in figure 4. In the same figure, it is possible to notice that z increases to 75 million m^3 and y for 14 trillion American dollars. In top graph of figure 5, it is possible to notice that both, the emission and time constraints are satisfied.

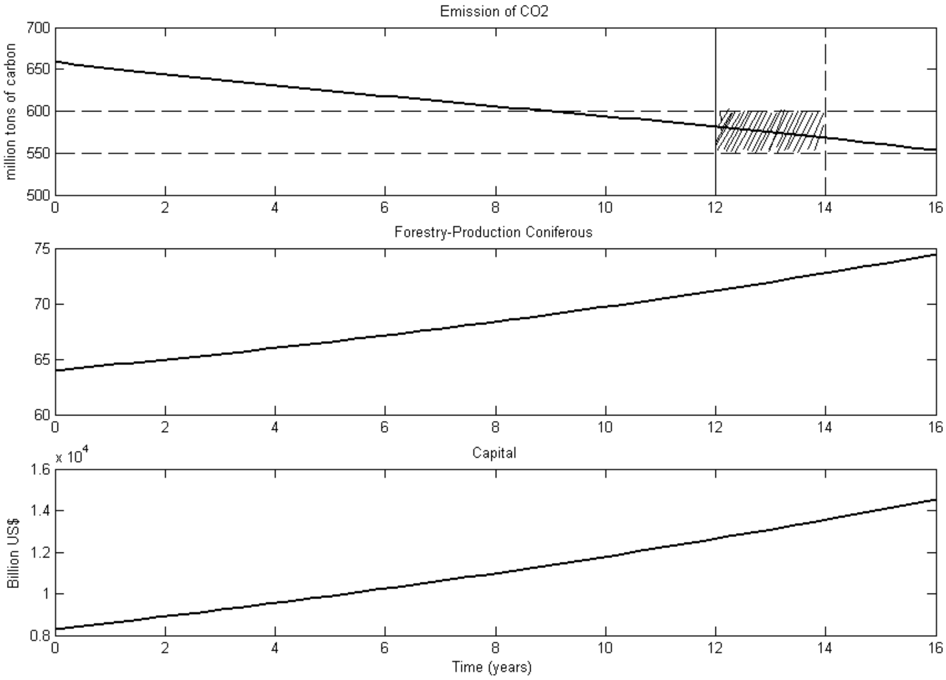

Fig 5. Solution for multi-objectives for CO_2 emissions

In figure 6, it is possible notice that the strategy to control CO_2 emissions is to start with high incentives in reforestation that decrease in successive years as the investments in technology increase. Using this strategy it is possible to achieve the aimed goal in an acceptable time lapse.

For a second scenario, it is interesting consider the case where to rate of the removal of CO_2 *(α_1)* is smaller, i.e., let α_1=10%.

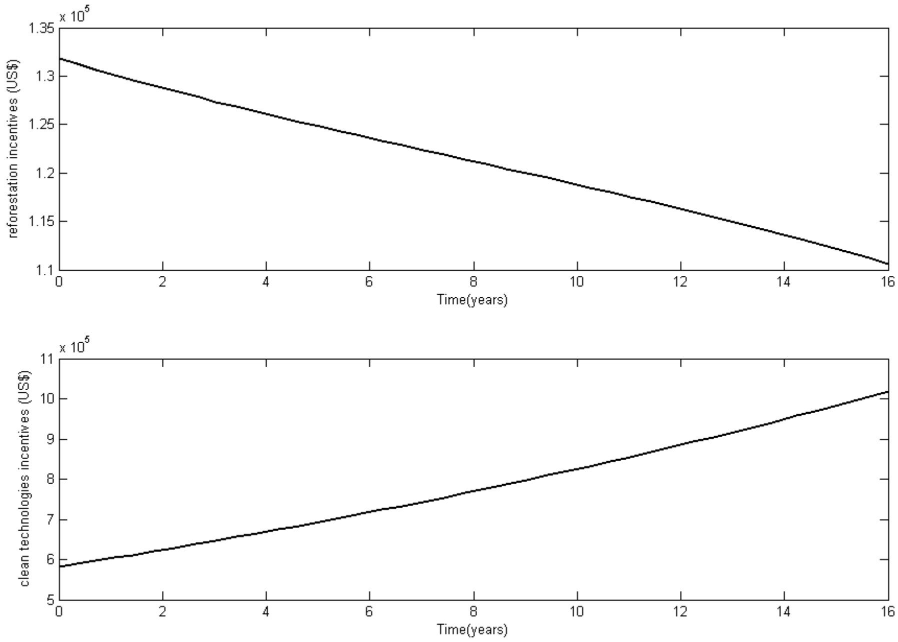

Fig 6. Controls u_1 and u_2 used in the closed-loop

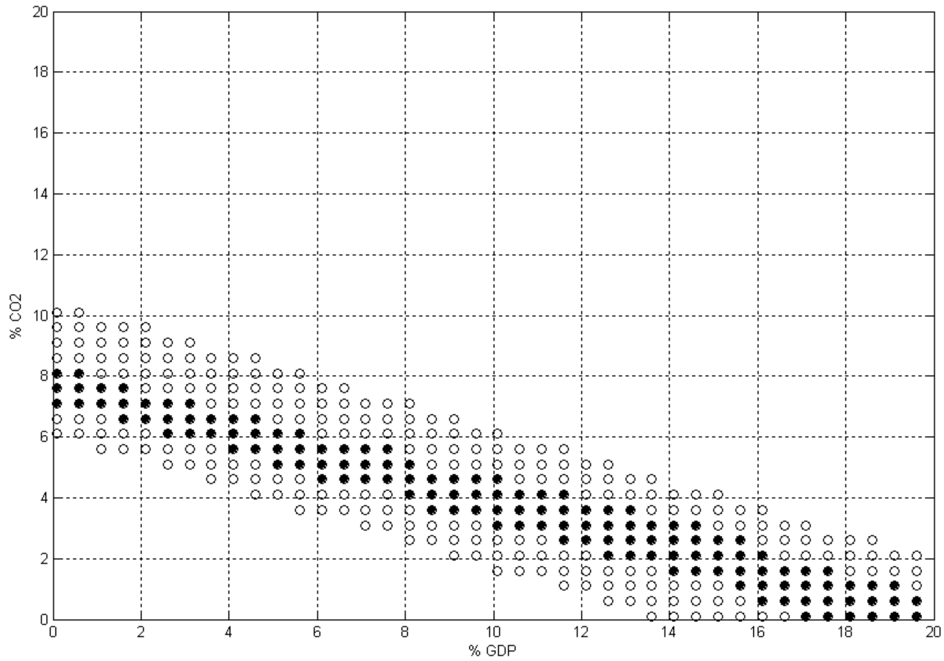

Fig 7. Feasible regions for CO_2 emissions between 550 and 600 millions tons of CO_2. White circle $550 \leq CO_2 \leq 600$. Black circle represents second constraint $12 \leq$ *Final time* ≤ 14 years considering α_1=10%.

In the figure 8, one can see that it is still possible to attain the goal in terms of $[CO_2]$ reduction in 12-14 years but as one can see in figure 9, the costs are higher.

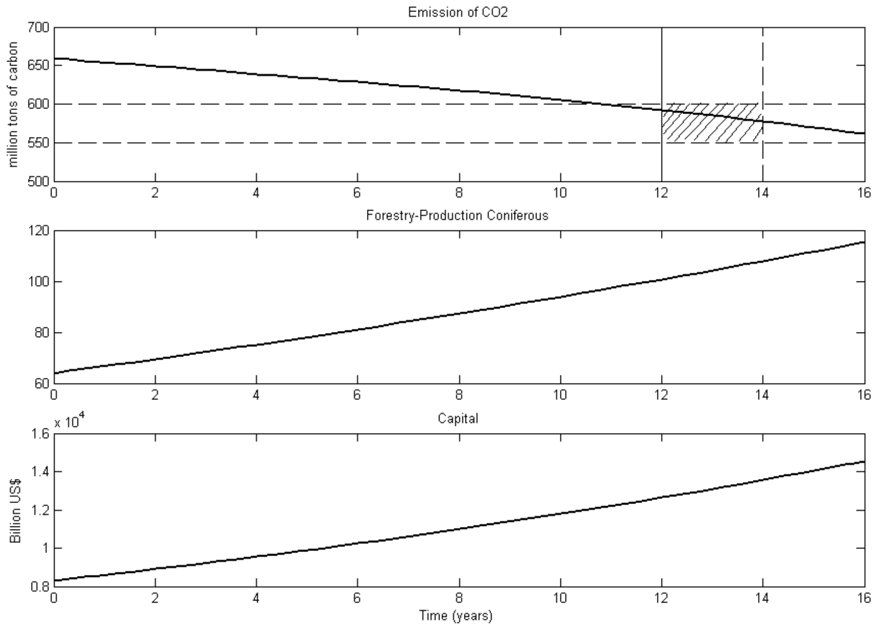

Fig 8. Solution for multi-objectives for CO_2 emissions with α_1 = 10%

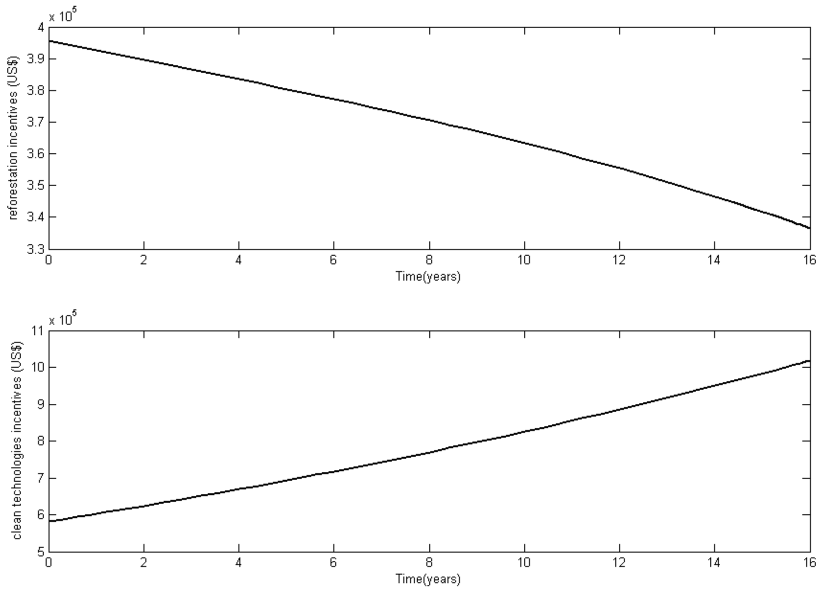

Fig 9. Control closed-loop considering α_1 = 10%

The last scenario considers a smaller *Final-time* (11 to 13 years) and to achieve a more demanding goal in terms of CO_2 emissions (500-550 million tons of [CO_2]). In this case, the possibilities are very little because the feasible region is significantly reduced (region marked with filled dots in figure 10). In fact, only 6 points (p_x,p_y) are feasible. For the choice of the par p_x=10.5% and p_y=4.5% the simulation results is figure

11 shows that the graph of [CO_2] is tangent to the rectangle x $\in$ [500,550] and *Final-time* $\in$ [11,13] . This point is in the limit of desirable region.

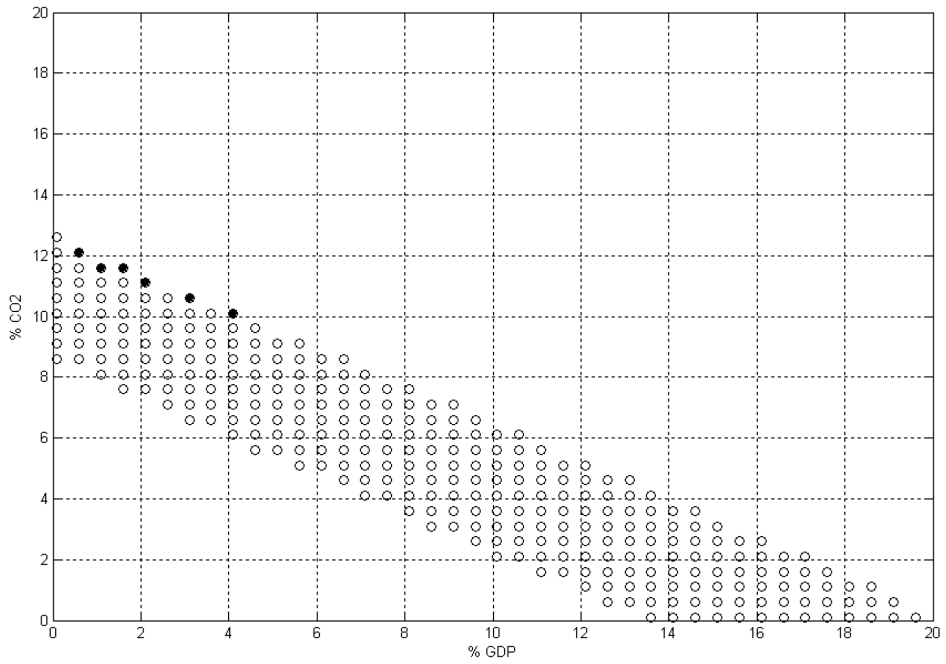

Fig 10. Feasible regions for CO_2 emissions between 500 and 550 millions tons of CO_2. White circle $500 \le CO_2 \le 50$. Black circle represents second constraint $11 \le$ *Final time* ≤ 13 years considering α_1=15%.

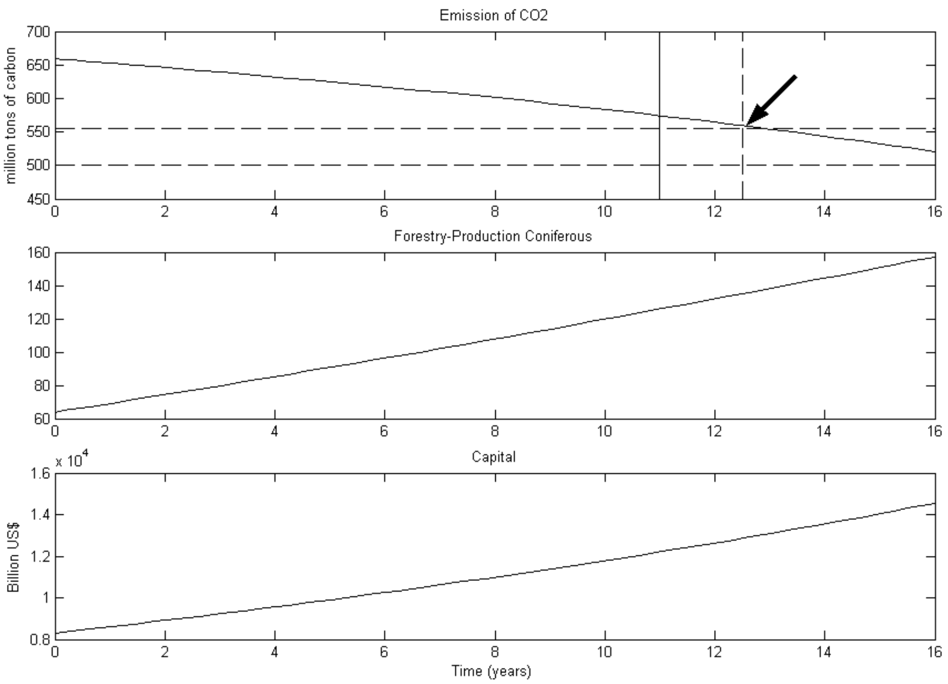

Fig 11. Solution for multi-objectives for CO_2 emissions with α_1 = 15% and final time between 11 and 13 years.

5. Conclusions

The present work proposes the use of constraint satisfaction method to aid the decision makers in the problem of reducing the global warming effect by considering investments in clean technology and reforestation. The method is shown to allow a much flexible resource allocation strategies than our previous work in which the optimal control theory was used. The main idea is to determine the investments as percentages (p_x,p_y) of the relevant variables in order to force the concentration of CO_2 to satisfy the levels recommended by the *Kyoto Protocol*. The model was assumed to be time-invariant, so that economical crises, such as the sub-prime crash that initiated in 2007, may lead to the necessity of re-estimating new parameters. Moreover, the case study was applied to Western Europe as a whole and it is a fact that not all countries follow the same policy in terms of emission of GHG. The results show with three different scenarios that is possible use these type of strategies to help the decision making process in public policy to adjust the incentives to control of CO_2 emission and contribute to reduce the global warming.

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