Nonlinear programming methods for solving problems: a recent bibliographic review

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Abstract: A great number of mathematical-programming applications are cast naturally as linear programs. Linear programming assumptions or approximations may also lead to appropriate problem representations over the diversity of decision variables being measured. At other times, however, nonlinearities in the form of either nonlinear objective functions or nonlinear constraints are critical for representing an application properly as a mathematical program. In mathematics, nonlinear programming (NLP) is the process of solving a system of equalities and inequalities, collectively termed constraints, over a set of unknown real variables, along with an objective function to be maximized or minimized, where some of the constraints or the objective function are nonlinear. This paper aims to list several nonlinear programming methods for solving problems that have been published at last 10 years.
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1. Introduction

The models produced by linear programming are, as its name implies, linear (both the objective function, and restrictions). This fact is, without doubt, "most of the restrictions" imposed on a model of programming. In most applications, linear models reflect only approximations of real models. Physical or economic phenomena are usually better represented by nonlinear models.

Most nonlinearities encapsulated in a programming model is within two main categories:
1) Relations empirically observed, such as changes in non-proportional costs, results and process quality characteristics.
2) Structurally derived relations, which include physical phenomena inferred mathematically and administrative rules.
In general, the models employed in Nonlinear Programming are like:

$$\text{Max (or Min)} \quad f(x)$$

$$\begin{cases} gi\,(x) \leq bi \\ x \geq 0 \end{cases} \qquad \text{for i=1, 2, ... , m}$$

With:
$X = (x_1, x_2, \ldots, x_n)$
$f(.)$ and $g_i(.)$ are nonlinear functions

The methods for solving problems of Nonlinear Programming can be divided into two groups: 1) Models without restrictions and 2) Models with restrictions. The main concept involved in Nonlinear Programming is the rate of change. The major problem that hinders the achievement of the optimal solution of the problems Nonlinear Programming is the minimum and maximum (extreme) local of the objective function.

According to [1], the practical problems of optimization often involve nonlinear behavior, which must be taken into consideration. Sometimes it is possible to reformulate these nonlinearities to fit a linear programming format. However, the best approach often is to use a nonlinear programming formulation. In most models is that real problems do not have some degree of linearity.

When studying computer science, or science in general is essential to seek to understand what the state of the art research theme. For this, the literature review becomes essential because through it we know what is being produced in the academic environment and, therefore, brings ideas and explanations for the researcher.

Thus, this paper attempts to show some work on nonlinear programming produced recently, with the objective of join, organize and present a systematic manner such publications.

2. Bibliographic Review

The propose of [2] was a new approach to solving nonlinear optimization problems with discrete variables using continuation methods. His focus was on pure integer nonlinear optimization problems with linear equality constraints (ILENP) but he showed how the technique can be extended to more general classes of problems such as those involving linear inequality and mixed integer constraints.

He showed the effectiveness of the approach by applying it to a number of real problems and also test problems taken from the literature. These include the binary unconstrained quadratic problem, the frequency assignment problem and the quadratic assignment problem. The results were compared to those from alternative methods, indicating that the new approach was able to produce good-quality solutions for diverse classes of nonlinear discrete optimization problems.

[3] presented a nonlinear model predictive control (NMPC) for multiple autonomous helicopters in a complex environment. The NMPC provides a framework to solve optimal discrete control problems for a nonlinear system under state constraints and input saturation. Their approach combines stabilization of vehicle dynamics and decentralized trajectory generation, by

including a potential function that reflects the state information of possibly moving obstacles or other vehicles to the cost function.

Various realistic scenarios were presented by [3] which show that the integrated approach outperforms a hierarchical structure composed of a separate controller and a path planner based on the potential function method. The proposed approach is combined with an efficient numerical algorithm, which enables the real-time nonlinear model predictive control of multiple autonomous helicopters.

[4] studied an integrated overview and derivation of mixed-integer nonlinear programming (MINLP) techniques, Branch and Bound, Outer-Approximation, Generalized Benders and Extended Cutting Plane methods, as applied to nonlinear discrete optimization problems that are expressed in algebraic form. Numerical comparisons were presented on a small process network problem to provide some insights to confirm the theoretical properties of these methods.

The objective of the work of [5] was to develop non-linear programming models for land grading to be applied in irregular shaped areas and that minimize soil movement. The GAMS (General Algebric Modeling System) software was used for calculations and the models were compared with the Method of Generalized Minimum Squares developed by [6], using as evaluation parameter the volume of moved soil. It was concluded that the non-linear programming models developed in this study were shown suitable for application to irregular shaped areas and provided smaller values of soil movement when compared with the method of minimum squares.

3. Conclusions

A bibliographical review on nonlinear programming is extremely important when performing research in this area. Thus, this work has brought a brief presentation on some works that use this form of programming to solve the most diverse problems.

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