DESIGN OF SATELLITE CONSTELLATIONS FOR A BRAZILIAN REGIONAL POSITIONING SYSTEM

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Abstract: *This paper presents and discuss three satellite constellation possibilities concerning to the design of a Regional Positioning System - RPS to cover the Brazilian territory. The first one is based only on Low Earth Orbit - LEO satellites with low to moderate orbital plane inclination angles. The second one is based on Medium Earth Orbit - MEO satellites placed in the or near the Equator plane and the third one is composed by geosynchronous satellites. Since Brazil is located near the equator plane, the idea is to design RPS satellite constellations that take advantage of this fact in order to design a cost effective regional system that aims at covering primarily the Brazilian territory. Geometric Dilution of Precision – GDOP [1,2] is used as a metric for the optimized design of three satellite constellations. In the present study, the preliminary survey done in [3] is extended, by performing a full scale optimization design process for the three constellation types under consideration and the performance improvements achieved, in terms of the best GDOP values obtained for each constellation type are reported and analyzed.*

Keywords*: satellite positioning system, satellite constellation design, optimization*

1. Introduction

Global Navigation Satellite Systems - GNSS allow portable receivers located on the ground or near earth's surface to determine their position to within a few meters using position and time information transmitted by radio from satellites. They have become a crucial component of the global information infrastructure [4], since they provide autonomous geo-spatial positioning with global coverage. Nowadays GNSS are a dual-use technology, having significant military and civilian applications, being widely used and a useful tool for a very large (and increasing) variety of applications in a great number of fields. Accurate timing provided by such systems facilitates everyday activities such as banking, mobile phone operations, and even the control of power grids. Around the entire world, professionals from many different areas perform their work more efficiently, safely, economically and accurately.

As an indication of its importance, existing GNSS constellations like the American GPS [5] or the Russian GLONASS [6] systems are continuously being updated with new and improved satellites. Besides, other GNSS and augmentation systems are in the process of deployment, such as the European Galileo satellite system, Japan's Quasi-Zenith satellite system, India's Regional Navigational Satellite System - IRNSS, or China's Compass satellite system [7].

For countries that possess vast territories (like Brazil), there is no doubt about the importance and utility of a satellite positioning system. In this way, it is reasonable to believe that Brazil will develop its own system in the future. Having this idea in mind, in [3], Geometric Dilution of Precision – GDOP [1,2] was used as a metric for the preliminary design of three types of satellite constellations with the purpose of establishing a hypothetical Regional Positioning System – RPS over the Brazilian territory.

Here, the preliminary study formerly done is extended, by performing a full scale optimization design process for the three constellation types previously considered and the performance improvements achieved, in terms of the best GDOP values obtained for each constellation type are reported and analyzed. As already happened in [3], the optimization process is carried out with help of the Generalized Extremal Optimization plus Evolution Strategies GEO + ES hybrid algorithm [8]. GEO + ES is a global optimization algorithm, developed in order to conjugate the good convergence properties of GEO [9] with the self-tuning characteristics present in the ES [10]. The results of the application of the optimization process are presented, analyzed and discussed. A comparison among the three constellations types is also performed.

A second goal of the present study is to find the answer for two questions raised by the results obtained by the preliminary design process [3]. The first one is to find out what is the minimum number of LEO satellites that are needed in order to have good coverage of the entire Brazilian territory, since in [3] even with 120 satellites (the largest satellite number tried then) the constellation coverage was not good. The second question is to observe if the addition of a fifth satellite to the geosynchronous constellation is capable of eliminating the two GDOP peaks that occurred with four satellites, causing a significant deterioration in the performance of the system during the time intervals on which the peaks occur.

The paper is organized as follows: Section 2 gives information about the kinds of constellation one wants to design and the optimization strategy used. In Section 3, the results are presented and commented. In Section 4, the conclusions are presented.

2. Optimization Strategy

The optimization strategy adopted is the same one used and already described in [3]. It consists in optimizing the average Geometric Dilution of Precision – GDOP occurred for a set of 5 hypothetical receivers placed on the Brazilian territory as illustrated in Figure 1, where the visibility circles are for a Low Earth Orbit (~1000km) and for a minimum elevation angle of 10º. The orbit of all satellites in the constellation is numerically propagated using mathematical models that include appropriate orbit perturbations for each orbit type. The GDOP of each receiver is calculated over a simulation period previously chosen and with a predefined time step, resulting in a set of GDOP values covering the whole propagation interval. The average GDOP is calculated considering the whole set of data retrieved from all the receivers considered. The GDOP calculation procedure implemented and used in this article always uses the best four visible satellites and was validated with the help of the STK 8.1 software [11] as described in [3].

Figure 1. Location of the 5 receivers used for the optimization.

Mathematically, the optimization problem can be stated as:

Minimize $\text{GDOP}(\mathbf{X}) = \frac{1}{5}\sum_{i=1}^{5}\left(\frac{1}{\text{NP}}\sum_{k=1}^{\text{NP}}\text{GDOP}_{\text{i,k}}(\mathbf{X})\right)$

Subject to: $\mathbf{X_{MIN}} \leq \mathbf{X} \leq \mathbf{X_{MAX}}$

Where: GDOP(**X**) = Average GDOP over all the 5 receivers on the ground and for all the NP propagation points

NP = Propagation interval / propagation step

$\text{GDOP}_{i,k}(\mathbf{X})$= GDOP calculated for the receiver “i” at the propagation instant “k”

X = Design variables = [$I_{1..NS}$, $\Omega_{1..NS}$, $M_{1..NS}$]

I_1 = Inclination of satellite 1

Ω_1 = Right ascension of the ascending node of satellite 1

M_{NS} = Mean Anomaly of satellite NS

NS = Number of satellites in the constellation

The satellites orbit simulation, with the state vector propagation, is carried out by a computer program written in Fortran and developed specifically to the orbit propagation of multiple satellites. It is based on the orbital dynamics routines developed at INPE by the orbital dynamics group for the propagation of just one satellite. For the Low Earth Orbit constellation the propagation is done considering only the effect of the Earth's gravitational field modeled by a central force field (mass point) plus the second zonal harmonic, J2, which models the poles flatness. For the medium and geosynchronous constellation orbits the propagation is done considering also the gravitational effects of the sun and the moon and the effect of the solar radiation pressure.

For all three constellations some of the keplerian elements at the beginning of the orbit propagation were set as constants and some were defined as design variables of the optimization problem. Table 1 on the following defines each one. Those in bold are design variables while the others are fixed with the values as indicated in the table. It is the task of the optimization algorithm to search for the design variable values that result in the minimum value of the average GDOP.

Table 1 - Values or Optimization Range (X_{MIN} to X_{MAX}) for the Initial Keplerian Elements of the Satellites in each Constellation Type*

Keplerian Element	**Constellation 1** **(Low Earth Orbit)**	**Constellation 2** **(Medium Earth Orbit)**	**Constellation 3** **(Geosynchronous orbit)**
Semi-major Axis (Orbit Altitude)	7,378.139 km (1,000 km)	26,378.139 km (20,000 km)	42,164.139 km (35,786 km)
Eccentricity	0.0	0.0	0.0
Inclination, I	**0° to 60°**	**0° to 60°**	**0° to 60°**
Right Ascension, Ω (of the ascending node)	**0° to 360°**	**0° to 360°**	**0° to 360°**
Perigee Argument	0.0º	0.0º	0.0º
Mean anomaly, M	**0° to 360°**	**0° to 360°**	**0° to 360°**

* = The epoch considered for all the simulations was November 1st 2010 at 0h 0min 0s

For Constellation 2, {10; 12; and 14} satellites were considered. For Constellation 3, {4; and 5} satellites were considered. In order to find an upper bound for Constellation 1, a significant increase in the satellite number (regarding the greatest value of 120 used in [3] without success in achieving good coverage) was used. For this aim, the value 200 was used. After that, the values {180; 160; 140; and 120} were used.

3. Results

The GEO + ES algorithm was run for each Constellation and for each number of satellites described in the previous section. For Constellations 2 and 3, 10 independent runs were used for the search. Each independent run uses a different random seed to start the search. For the more time consuming Constellation 1, only one independent run was used for the search.

For each run, a previously defined number of 1000 generations were allowed to occur within the GEO + ES algorithm and this limit was used as a stopping criterion. Three mutations per variable were used, so $l_j=l=3$ and the number of design variables was L=3*NSAT, for all Constellations, where NSAT is the number of satellites in the constellation. The limits for varying b were set to $b_{MIN}=1.05$ and $b_{MAX}=10$. The values of $\delta=0.0$ and of $\alpha=0.3$ were used. The four parameters just mentioned are internal parameters for GEO + ES. For those moments, during the satellite orbit propagation steps, when there were less than 4 satellites visible to a receiver, the value 10^5 was arbitrarily imposed as being the GDOP of the respective receiver. The simulation period was set to one day (86400s) to all the three constellations. The time step used for the orbit propagation was 150 seconds for Constellations 1 and 2 and 30 seconds for Constellation 3.

3.1. - Results for Constellation 1 (Low Earth Orbit - 1,000km)

The most important result for Constellation 1 was the discovery of the minimum number of satellites for good coverage. This number is 140 satellites. The best average GDOP value found for Constellation 1 with 140 satellites was 3.12. Table 1 gives the solution found for Constellation 1 with 140 satellites, where all values are in degrees.

Table 1. - Constellation 1 solution (140 satellites, GDOP = 3.12)

$I_{1..140}$	$\Omega_{1..140}$	$M_{1..140}$
30.2 49.4 29.8 29.5 30.2 50.9 43.1 31.7 42.6 35.1 0.3 31.1 33.9 20.0 31.6 32.8 40.8 14.0 28.5 41.2 31.9 42.3 34.4 28.5 22.4 47.3 51.4 48.5 35.5 50.2 5.0 32.8 40.7 25.9 19.6 32.3 28.6 59.6 26.1 32.2 38.0 5.2 47.4 33.8 34.1 15.0 43.6 35.3 41.7 0.0 36.9 38.6 37.3 17.2 9.0 49.2 34.8 26.4 31.8 36.0 38.1 34.6 37.8 45.0 50.5 38.4 36.2 28.4 20.0 9.4 18.0 7.6 35.1 11.9 36.7 39.6 35.1 38.9 0.9 51.4 39.2 42.7 55.2 2.3 50.0 13.0 33.4 50.0 37.5 14.2 46.4 10.7 2.2 53.5 36.6 43.7 14.6 0.1 40.2 13.8 36.7 42.2 21.5 28.1 34.1 40.8 37.5 7.8 51.8 26.9 5.7 21.0 51.7 18.0 35.7 11.1 15.2 10.5 39.3 14.0 27.3 11.8 33.0 30.7 44.0 32.3 29.5 43.1 36.5 35.2 31.9 6.6 27.9 39.8 29.4 6.6 31.3 48.8 13.9 5.5	340.3 312.3 293.0 71.1 75.4 130.4 215.4 22.5 249.8 73.5 258.6 195.0 6.4 36.9 144.6 132.1 318.0 221.0 71.5 85.4 356.4 166.7 84.0 1.7 23.3 84.4 247.8 291.5 189.2 127.2 155.2 290.7 190.3 61.5 88.6 289.0 6.9 19.6 208.4 95.0 177.0 127.7 16.6 72.9 135.0 204.7 107.5 120.2 62.8 97.3 40.6 6.6 42.1 272.8 202.5 193.8 7.3 227.9 297.3 133.3 141.1 303.8 115.4 272.3 285.9 349.1 8.9 93.4 313.2 191.2 274.5 287.7 263.8 78.9 7.2 202.9 91.0 356.9 296.6 68.9 172.1 283.0 175.8 311.2 312.1 358.0 264.9 84.6 68.4 223.0 124.1 39.5 210.7 282.5 104.9 112.1 225.8 69.5 193.4 125.8 293.2 179.4 182.7 253.7 197.7 88.6 281.0 126.2 272.5 77.1 85.3 20.3 212.2 313.3 108.0 55.8 285.4 50.6 235.4 30.1 273.5 289.7 128.2 208.9 34.0 274.5 355.4 32.9 255.4 240.9 291.0 214.4 151.1 137.8 216.2 199.8 227.4 267.4 254.8 353.8	30.0 306.8 0.4 125.2 239.8 130.0 42.3 26.3 0.8 111.6 171.9 248.9 68.9 246.9 4.9 204.8 127.6 128.2 139.3 147.1 176.0 143.5 177.0 159.2 316.3 236.4 95.7 9.7 280.1 315.3 127.4 31.0 113.6 0.3 116.3 84.5 350.1 259.3 90.5 186.9 45.4 345.7 82.1 289.0 89.8 219.3 259.5 191.0 331.6 270.2 293.5 287.2 120.7 268.4 50.3 264.5 218.7 300.0 116.8 334.0 60.8 242.0 159.4 228.4 284.6 136.3 128.7 58.9 259.2 192.2 190.4 46.0 136.1 246.8 359.2 194.4 357.9 281.1 18.6 150.5 20.7 271.5 231.2 249.5 81.6 109.7 180.0 16.1 48.8 14.8 16.1 210.3 86.1 6.1 313.4 239.6 285.6 25.1 73.2 318.8 46.6 310.1 329.5 92.0 176.7 266.8 103.3 139.7 173.1 95.8 267.0 10.8 238.7 274.8 59.6 122.6 24.3 353.2 176.8 205.9 31.3 193.0 263.9 339.7 99.4 235.8 253.7 203.4 236.5 335.7 305.3 280.9 255.3 27.7 135.0 15.4 275.6 126.4 16.6 64.1

Figure 2 gives the GDOP values occurred for the 5 receivers during the propagation interval of the best solution found for 140 satellites of Constellation 1.

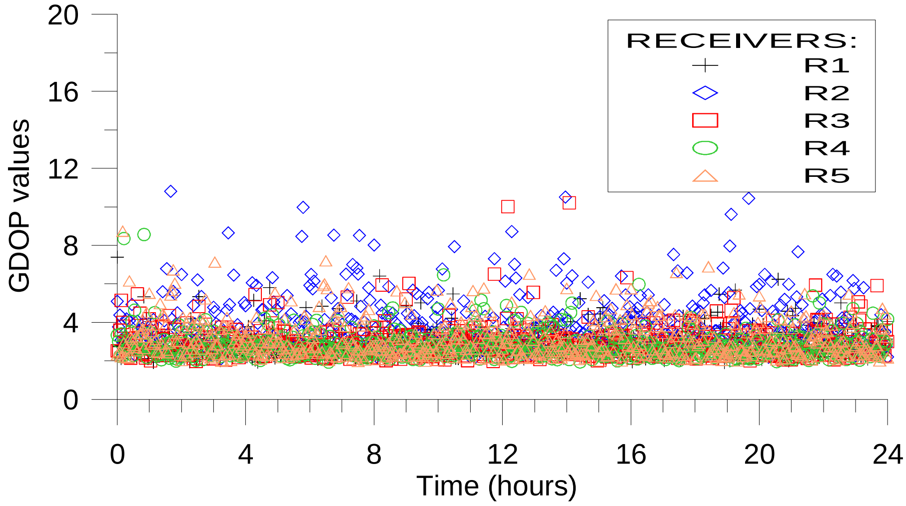

Figure 2. GDOP for Constellation 1 (140 satellites)

Figure 2 shows that with 140 satellites the coverage is good for all the receivers during the whole time, remaining below the value of 10 almost all the time and below 5 most of the time. The percentages of the time interval on which GDOP is less than 10 were {100%; 99.5%; 99.7%; 100%; and 100%} for the receivers {R1, R2, R3, R4, and R5} respectively. The percentages for GDOP < 5 were {97.6%; 84.2%; 97.1%; 98.6%; and 93.8%} for the receivers {R1, R2, R3, R4, and R5} respectively. In terms of computational time for Constellation 1, it was 41 days for 140 satellites on a Intel Core 2 Quad 2.83GHz PC with 2GB RAM.

3.2. - Results for Constellation 2 (Medium Earth Orbit - 20,000km)

The best average GDOP values found for Constellation 2 were { 13,274.2; 5.25; and 3.19} for {10, 12, and 14} satellites respectively. From the GDOP values, it is possible to conclude that 10 satellites were not enough to achieve a good coverage for all the 5 receivers considered. In the case of 12 and 14 satellites, both achieved good average GDOP values. Tables 2 to 4 give the solutions found for Constellation 2 with 10, 12 and 14 satellites, respectively, where all values are in degrees.

Table 2. - Constellation 2 solution (10 satellites, GDOP = 13,274.2)

$I_{1..10}$	$\Omega_{1..10}$	$M_{1..10}$
31.7 9.1 23.2 32.1 33.3 33.0 33.4 0.1 0.1 32.9	157.6 328.4 250.2 290.2 127.6 90.1 359.6 333.3 304.2 99.4	300.1 275.9 72.4 36.1 341.6 60.3 235.9 36.4 72.2 49.2

Table 3. - Constellation 2 solution (12 satellites, GDOP = 5.25)

$I_{1..12}$	$\Omega_{1..12}$	$M_{1..12}$
32.9 0.9 11.7 25.7 6.3 30.4 0.1 44.8 32.7 33.0 46.5 30.3	243.0 211.7 155.8 138.0 39.0 17.6 222.3 288.6 48.0 322.1 138.7 113.2	112.8 316.7 251.3 325.9 185.9 230.4 103.0 203.2 144.5 113.4 145.5 257.2

Table 4. - Constellation 2 solution (14 satellites, GDOP = 3.19)

$I_{1..14}$	$\Omega_{1..14}$	$M_{1..14}$
31.0 21.5 32.7 37.7 0.01 51.3 51.5 15.7 0.05 32.6 0.05 33.0 35.4 24.7	204.1 132.9 337.6 153.0 105.4 23.7 182.3 7.9 276.5 109.2 51.8 58.7 287.4 307.2	165.1 159.5 297.6 21.7 342.9 191.6 231.7 314.0 322.4 6.9 88.9 131.4 115.3 46.6

Figures 3 to 8 give the GDOP values occurred for the 5 receivers during the propagation interval of the best solutions found for 10, 12 and 14 satellites of Constellation 2.

Figures 3 and 4 show that with 10 satellites there are huge gaps in the coverage for the receivers, with the main one occurring around 7h and when there are less then 4 satellites visible to all the receivers simultaneously. Figure 4 also shows that even when there is 4 satellites visible to the receivers the GDOP values for all the receivers are not good (>10) quite frequently. The percentages of the time interval on which GDOP was less than 10 were {5.7%; 23.1%; 12.4%; 11.8%; and 17.7%} for the receivers {R1, R2, R3, R4, and R5} respectively.

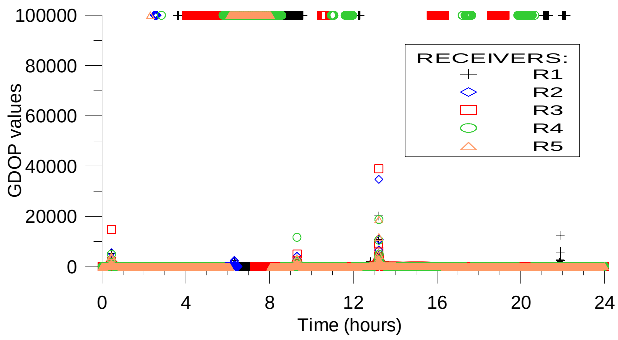

Figure 3. GDOP for Constellation 2 (10 satellites)

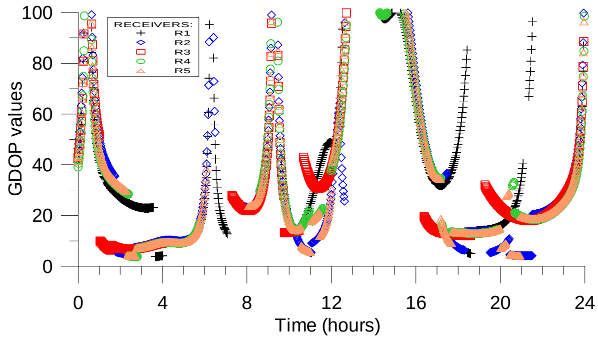

Figure 4. Zoom of Fig. 3.

Figures 5 and 6 show that with 12 satellites there are no gaps in the coverage for the receivers, but there are two big peaks in the GDOP values, affecting some receivers more than others. Figure 11 also shows that, during the peaks, the time interval where the GDOP is greater than 10 lasts around one hour or less, with the possible exception of the peak occurred at 2h. The percentages of the time interval on which GDOP was less than 10 were {93.1%; 100%; 91.1%; 98.4%; and 99.7%} for the receivers {R1, R2, R3, R4, and R5} respectively.

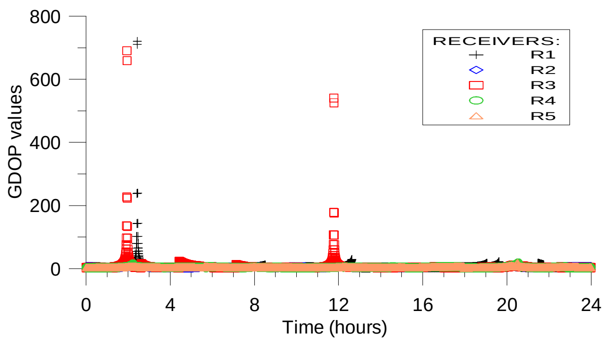

Figure 5. GDOP for Constellation 2 (12 satellites)

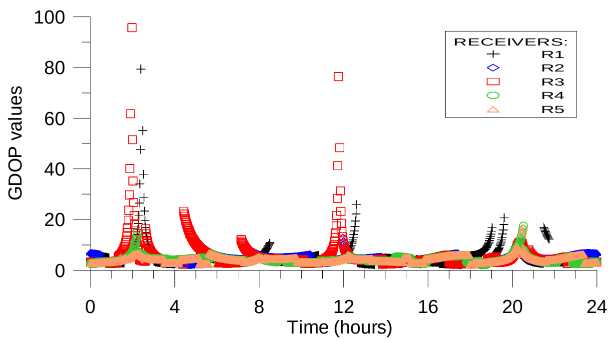

Figure 6. Zoom of Fig. 5.

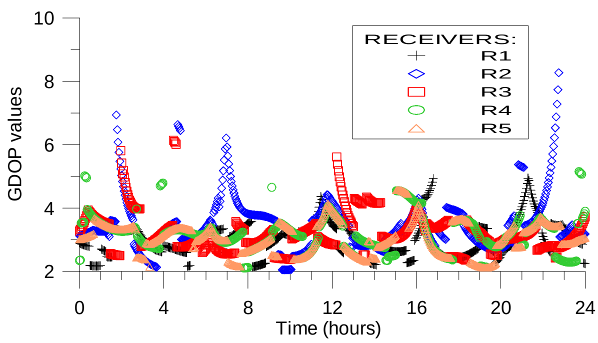

Figure 7. GDOP for Constellation 2 (14 satellites)

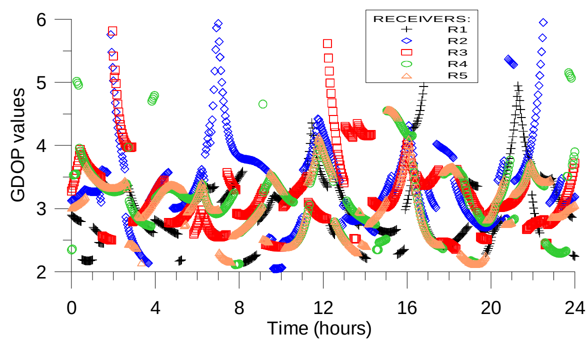

Figure 8. Zoom of Fig. 7.

Figures 7 and 8 show that with 14 satellites the coverage is very good for all the receivers during the whole time, remaining below the value of 10 all the time and below 5 most of the time. The percentages of the time interval on which GDOP is less than 10 were 100.0% for all receivers. The percentages for GDOP < 5 were {100%; 93.4%; 98.1%; 99.1%; and 100%} for the receivers {R1, R2, R3, R4, and R5} respectively. In terms of computational time for Constellation 2, they were 12.7, 18.6, and 29.1 days for 10, 12 and 14 satellites, respectively, on a Intel Core 2 Quad 2.83GHz PC with 2GB RAM.

3.3. - Results for Constellation 3 (Geosynchronous Orbit – 35,786km)

The best solution found for Constellation 3 with 4 satellites is given in the Table 5, with all values in degrees.

Table 5. - Constellation 3 solutions (4 satellites, GDOP = 7.64)

$I_{1..4}$	$\Omega_{1..4}$	$M_{1..4}$
11.8 36.2 37.2 14.1	228.1 28.8 178.8 327.1	168.0 321.7 160.7 330.1

Figures 9 and 10 give the GDOP values occurred for the 5 receivers during the propagation interval with the solution given in Table 5.

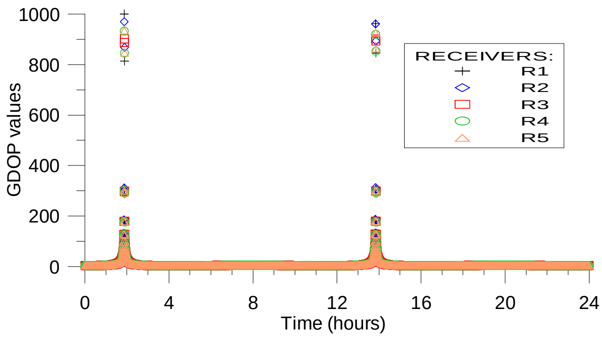

Figure 9. GDOP for Constellation 3 (4 satellites)

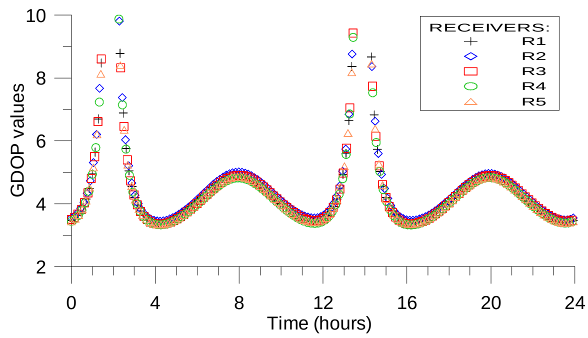

Figure 10. Zoom of Fig. 9.

As can be seen from Fig. 9, there are two big peaks in the GDOP values, equally affecting all five receivers and with a separation time of 12 hours. By its turn, Fig. 10 shows that, during the peaks, the time interval where the GDOP is greater than 10 lasts less than one hour. The time percentages for GDOP < 10 were {93.5%; 93.4%; 93.6%; 93.6%; and 93.6%} for the receivers {R1, R2, R3, R4, and R5} respectively. It is possible to observe that there is uniformity in these percentages, all remaining close to 93.6%. Regarding the two peaks in the GDOP values occurring approximately at hour 02:00 and 14:00 of the period considered in the simulation, it was already shown in [3] that they represent the two moments in which all satellites almost align themselves into one plane, what is a poor geometry for positioning calculations and having, as a consequence, high GDOP values.

The best solution found for Constellation 3 with 5 satellites is given in the Table 6, with all values in degrees.

Table 6. - Constellation 3 solutions (5 satellites, GDOP = 3.11)

$I_{1..5}$	$\Omega_{1..5}$	$M_{1..5}$
57.2 0.01 0.05 59.9 15.1	112.8 352.5 82.6 277.0 207.9	205.9 261.6 275.7 6.2 98.0

Figure 11 gives the GDOP values occurred for the 5 receivers during the propagation interval with the solution given in Table 6.

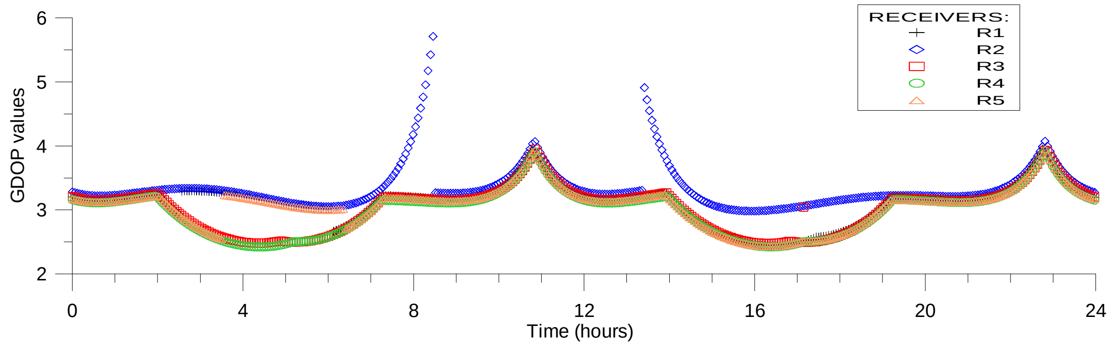

Figure 11. GDOP for Constellation 3 (5 satellites)

As can be seen from Fig. 11, there aren't peaks anymore in the GDOP values, and all five receivers have GDOP values under 6 during the entire propagation interval. The time percentages for GDOP < 4 is 100% for all receivers, but R2, that has GDOP < 4 during 94.9% of the time and GDOP < 5 during 99.1% of the time. These results corroborate the thesis that a fifth satellite in the Constellation 3 solves the GDOP peaks that occur when only 4 satellites are used.

4. Conclusions

In this work, the preliminary design done in [3] was extended, by means of a full scale optimization design process. In both studies, GDOP was used as a metric for the optimized design of three types of satellite constellations with the purpose of establishing a positioning system over the Brazilian territory. The optimization task was performed by the hybrid evolutionary algorithm GEO + ES. The results have shown that for Low Earth Orbit, 140 satellites are needed in order to have good coverage of the entire Brazilian territory. In the case of Medium Earth Orbit, 12 satellites presented good performance and 14 satellites presented excellent performance. In the case of the Geosynchronous Orbit, it was possible to conclude that the addition of a 5^{th} satellite to the constellation has eliminated the peaks observed when only four satellites were used and the resulting constellation presented excellent performance.

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