Predictability for a Chaotic CHAOTIC Solar Plasma System

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PREDICTABILITY FOR A CHAOTIC SOLAR PLASMA SYSTEM

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Abstract. *The problem of unpredictability in a physical system due to the incomplete knowledge of the evolution laws is addressed. Predictability is an indication of the instability of the underlying flow computed from a numerical model, where small errors in the initial conditions (or imperfections in the model) grow to large amplitudes in finite times. Bred vectors are the difference between two nonlinear model integrations, periodically rescaled to avoid nonlinear saturation of the instabilities of interest. The technique of breeding vectors is applied to the Lorenz model, as an example, and a three coupling waves model for solar activities connected to the space weather process. The bred vector growth can be used to reliably predict which will be the last orbit in each of the two regimes and how long will the next regime last. The purpose of this paper is to describe the breeding method that explores chaotic model predictability and its results.*

Keywords: *chaotic dynamics, bred vector, solar three coupled wave model.*

1. INTRODUCTION

The ability to predict the future state of a system, given its present state, stands at the foundations of scientific knowledge with relevant implications from an applicative point of view in geophysical and astronomical sciences. Major interest is devoted to the analysis of error amplification in chaotic systems with many characteristic times and scales. Using the concepts of dynamical systems theory, there has been some progress made in understanding the growth of an uncertainty during the time evolution.

By definition, chaotic dynamical systems display sensitive dependence on initial conditions: two initially close trajectories will diverge exponentially in the phase space with a rate can be given by the leading Lyapunov exponent (Boffetta, 1998). The prediction of the future state of a system knowing its initial conditions is a fundamental problem with obvious applications in geophysycal flows (Dalcher, 1987). The predictability of weather and climate forecasts is determined by the projection of uncertainties in both initial conditions and model formulation onto flow-dependent instabilities of the chaotic climate attractor. Since it is essential to be able to estimate the impact of such uncertainties on forecast accuracy, no weather or climate prediction can be considered complete without a forecast of the associated flow-dependent predictability. Earths climate is a prototypical chaotic system (Lorenz, 1963), however an appreciation of the importance of quantifying the role that initial error plays in limiting the accuracy of weather predictions (Thompson, 1957). With the understanding of how the solar wind influences the magnetosphere (related do space weather), it became possible to make quantitative predictions of magnetic activity.

The space weather refers to conditions on the sun and in the solar wind, magnetosphere and thermosphere that can influence the performance and reliability of space-borne and ground-based technological systems (Cole, 2003). With the understanding of how the solar wind influences the magnetosphere, it became possible to make quantitative predictions of magnetic activity. Then, the predictability of the Magnetosphere is also fundamental to understanding the its dynamics.

The use of ensemble forecasting and data assimilation shows the importance of local predictability properties of the atmosphere in space and in time (Toth, 1997). The local/regional loss of predictability is an indication of the instability of the underlying flow computed from a numerical model, where small errors in the initial conditions (or imperfections in the model) grow to large amplitudes in finite times.

Chaotic dynamics implies also that rate of growth of initial error is itself a function of the initial state (Palmer, 1993). Forecast errors can originate from errors in the initial conditions that, due to the chaotic nature of the atmosphere grow with time, or from model deficiencies. Because the error growth is not uniform, but is associated with instabilities of the background flow, forecast errors tend to be dominated by relatively large errors intermittent in space and in time. Bred vectors are the difference between two nonlinear model integrations, periodically rescaled.

The method of applying small perturbations in chaotic systems has been applied to a variety of physical applications for some purposes. A perturbation initialization method is used to quantify error growth due to inaccuracies of the forecast model. We review the **breeding method** to generate and the properties of bred vectors in Section 2. In Section 3, we show that breeding growth rates provide reliable forecast rules for regime transition in the Lorenz (1963) 3-variable model. This method was applied in a nonlinear three-wave interactions involving Langmuir, Whistler, and Alfvén waves in the planetary magnetosphere, and that growth rates also provide the predictability of that chaotic dynamic system.

2. BREEDING METHOD

The breeding method is a well-established and computationally inexpensive method for generating perturbations for ensemble integrations. Breeding was developed as a method to generate initial perturbations for ensemble forecasting in numerical weather prediction at the National Centers for Environmental Prediction (NCEP) (Toth, 1997).

The method involves simply running the nonlinear model used for the *control* a second time. Periodically subtracting the control from the perturbed solution, and rescaling the difference so that it has the same size as the original perturbation. The rescaled difference (a bred vector) is added to the control run and the process repeated. Their growth rate is a measure of the local instability of the flow. The stability properties of evolving flows have been studied using Lyapunov vectors (Alligood, 1996), and with bred vectors more recently (Kalnay, 2002). Bred vectors are a nonlinear generalization of leading Lyapunov exponents, that rate presents the differences of two initially close trajectories of chaotic dynamical systems.

In the context of data assimilation, the rescaled difference is added to the analysis (an appropraited combination of the predicted fields and observed fields) – se Figure 1. Their growth rate is a measure of the local instability of the flow.

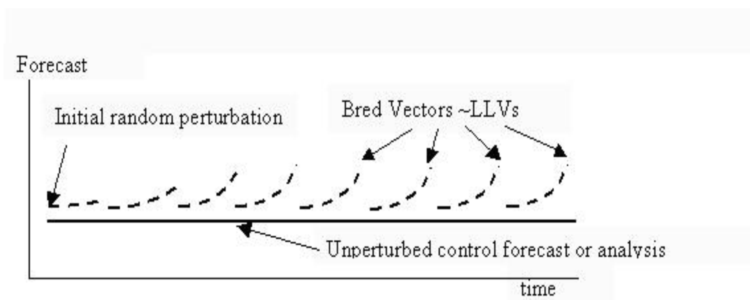

Figure 1: Schematic of the growth of bred vectors.

Bred Vectors (BVs) are computed as follows (Kalnay and Cai, 2002):

1. Start with an arbitrary initial perturbation $\delta f(x,t)$ of size A defined with an arbitrary norm. This initialization step is executed only once. The size of A is essentially the only tunable parameter of breeding.

2. Add the perturbation to the basic solution, integrate the perturbed initial condition with the nonlinear model, and subtract the original unperturbed solution from the perturbed nonlinear integration

$$\delta f(x, t+\Delta t) = M[f(t+\Delta t)] \quad (1)$$

3. Measure the size $A+\delta A$ of the evolved perturbation $\overline{\delta f(x,t+\Delta t)}$, and divide the perturbation by the measured amplification factor so that its size remains equal to A:

$$\delta f(x, t+\Delta t) = \overline{\delta f(x,t+\Delta t)} \sim A/(A+\delta A) \quad (2)$$

Steps 2 and 3 are repeated for the next time interval and so on. It has been found that after a short transient time of the order of the time scale of the dominant instabilities. In practical applications, bred vectors are intrinsically local in space and time, and they are finite amplitude, finite time vectors – see Fig.1.

3. NUMERICAL EXPERIMENTS

The application of the bred vector methodology as a scheme to drive us to formulate the predictability for a chaotic system will be illustrated with Lorenz system. From such example, the procedure will be applied to produce some rules for the chaotic three coupled waves in the solar dynamics.

3.1 Lorenz System

Like an Example to study how to apply *Bred Vectors* in chaotic dynamical ssystems, we reproduce the *Research Internships in Science and Engineering (RISE)* (Evans, 2004) experiment with the 3-variable Lorenz model that indicate that orthogonalized the bred vectors can result in significantly improved performance. This experiment showed that the regime changes in Lorenzs model are predictable.

Explore the predictability using breeding a Lorenz (1963) model, this algorithm was chosen because of its simplicity. The Lorenz Model equations are:

$$\frac{dx}{dt} = -\sigma x - y \tag{3}$$

$$\frac{dy}{dt} = -\rho x - y - xz \tag{4}$$

$$\frac{dz}{dt} = xy - \beta z \tag{5}$$

where $\sigma = 10, \rho = 28, \beta = 8/3$ are its parameters and they are chosen by Lorenz results in chaotic solutions . This model has been very widely used as a prototype of chaotic behavior (Fig. 2). The model was integrated with a 4th order Runge-Kutta numerical scheme. We used two sets of the Lorenz equations starting with different initial condition. The Lorenz attractor have two regimes, which we could denote as two seasons (”warm” and ”cold”), but it is hard to identify the changes in regimes will happen, and how long will they last.

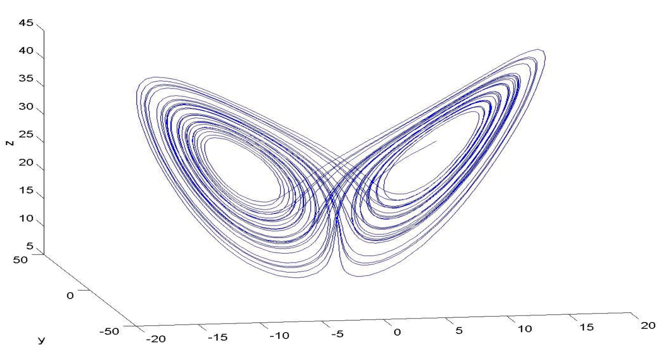

Figure 2: Solutions of the Lorenz model equations showing two chaotic regime

The breeding is performed on the Lorenz model integrated with time steps $\Delta t = 0.01$, and a second run started from an initial perturbation $\delta x_0 = (\delta x_0, \delta y_0, \delta z_0)$ added to the control at time t_0. The Every 8 times steps we take the difference δx between the perturbed and the control run; rescale it to the initial amplitude and add it to the control. The growth rate of the perturbation was measured per time step as (Evans, 2004)

$$g = \frac{1}{n} * (|\delta x| / |\delta x_0|) \ .$$

The Figure 3 presents the attractor with that simple procedure which allows us to estimate the stability of the attractor. Moreover, the growth rate measured by breeding provides remarkably precise forecasting rules, illustrate in (Fig. 4), that could be used by a forecaster living in the Lorenz attractor to make extended range forecasts about when will the present regime end, and how long will the next regime last. The presence of a red star shows bred vector growth in the previous 8 steps was greater than 0.064, it can be used to forecast that the next orbit will be the last one in the current regime. The blue stars indicate a negative growth rate, meaning that the perturbations are actually decaying. The results shown in Figure 3 suggested that the bred vector growth would allow estimating of high and low predictability. We found that plotting the growth rates on the evolution of the variable $x(t)$ provides a means to predict when the model will enter a new regime, and also how long the new regime will last.

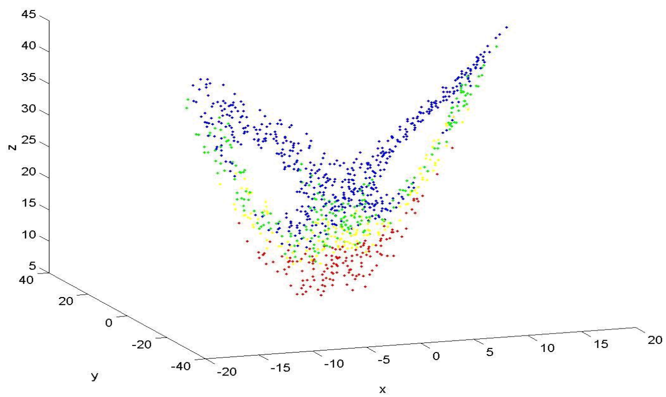

Figure 3: The Lorenz "butterfly" attractor colored with the bred vector growth

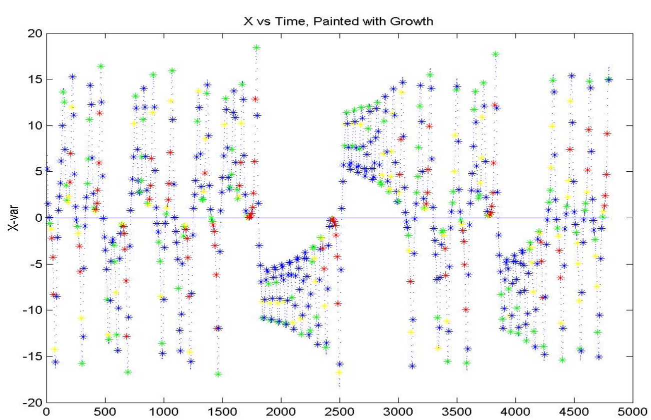

Figure 4: X(t) for the Lorenz model with red stars providing "forecasting rules".

3.2 Nonlinear Three-wave coupled model

Nonlinear three-wave coupling is of general interest in many branches of physics such as nuclear fusion, space geophysics, astrophysics, nonlinear optics, and fluid mechanics. For example, it causes the stimulated scattering and anomalous absorption of laser beams in inertial fusion experiments (Chow, 1992) and appears in the plasma edge region of a magnetic fusion device during radio-frequency heating experiments (Hidalgo, 1993); it is responsible for the generation and modulation of plasma waves in the planetary magnetosphere and solar wind (Chian, 1994); and other applications. A nonlinear analysis of auroral Langmuir, whistler and Alfven (LAW) events in the planetary magnetosphere was carried out by Lopes and Chian (1996) (Lopes, 1996), under the assumption that all three interacting waves are linearly damped. In the satured regime of this model, chaotic solutions can be found, in this case, the wave solutions can evolve from order to chaos via various routes such as period-doubling or intermittent (Chian, 2000).

The theoretical modeling of this experiment based on Lorenz equation is in good agreement with the experimental resulting. The simplest model for describing the temporal dynamics of resonant nonlinear coupling of three waves can be obtained assuming terms in the wave amplitudes. Moreover, the waves may be assumed monochromatic, with the electric fields written in the form: $E_\alpha(x,t) = \frac{1}{2}A_\alpha(x,t)\exp\{i(k_\alpha x - \omega t)\}$, where $\alpha = 1, 2, 3$ and the time scale of the nonlinear interactions is much longer than the periods of the linear (uncoupled) waves.

In order for three-wave interactions to occur, the wave frequencies ω_α and wave vectors k_α must satisfy the resonant conditions

$$\omega_3 \cong \omega_1 - \omega_2, k_3 = k_1 - k_2 \tag{6}$$

Under these circumstances, the nonlinear temporal dynamics of the system can be governed by the following set of three first-order autonomous differential equations written in terms of the complex slowly varying wave amplitude (Meunier, 1982):

$$\begin{aligned} dA_1/d\tau &= v_1 A_1 + A_2 A_3 && (7)\\ dA_2/d\tau &= i\delta A_2 + v_2 A_2 A_1 A_3^* && (8)\\ dA_3/d\tau &= v_3 A_3 A_1 A_2^* && (9) \end{aligned}$$

where the variable $\tau = \chi t$, with χ is a characteristic frequency: $\delta = (\omega_1 - \omega_2 - \omega_3)/\chi$ is the normalized linear frequency mismatch and $v_\alpha = v_\alpha/\chi$ give the linear wave behaviors on the long time scale. The wave A_1 is We assumed linearly unstable ($v_1 > 0$) and the other two waves, A_2 and A_3, are linearly damped ($v_2 = v_3 \equiv -v < 0$) and henceforth it is set $\chi = v_1$ (Meunier, 1982; Lopes, 1996). The system admits both periodic and chaotic waves. The Figure 5 shows the chaotic attractor. One regime is characterized by the straight line, and other regime is identified as a curve line.

The breeding method was applied in three-wave model like first experiment: first perform breeding on the three-wave model integrated with time steps $\Delta t = 0.001$, and a second run started from an initial perturbation δx_0 added to the control at time t_0 . Every 8 times steps, the difference δx was also taken between the perturbed and the control run; rescale it to the initial amplitude and add it to the control, then get the growth rate and plot the trajectory of wave A_1 (Langmuir wave).

Two regimes were also noted in three-wave attractor, one is at straight line and other were noted in a curve line (see Figure 6), we suppose the rules like the first experiment, two or more red stars together predict the change of regime and it lasts a long time, two yellow stars predict a change of regime at short time, one or more separated stars with blue stars contain weak chaos but predict a change of regime too. The same rules are applied at waves A_2 and A_3. See Figures 7 to 9.

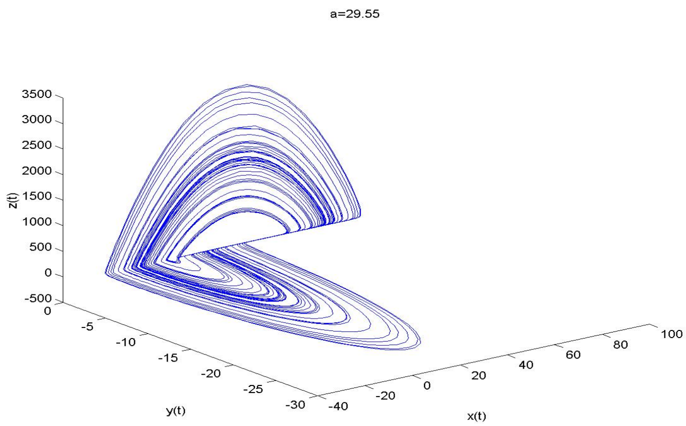

Figure 5: Solutions of the Three-wave model equations showing two chaotic regimes.

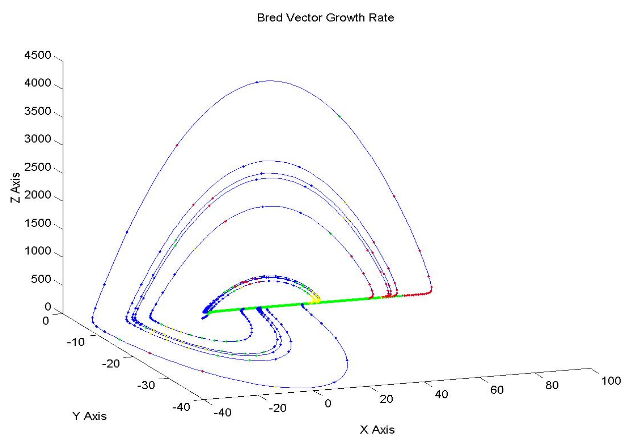

Figure 6: Langmuir wave for the coupled three-wave model with ”forecasting rules”.

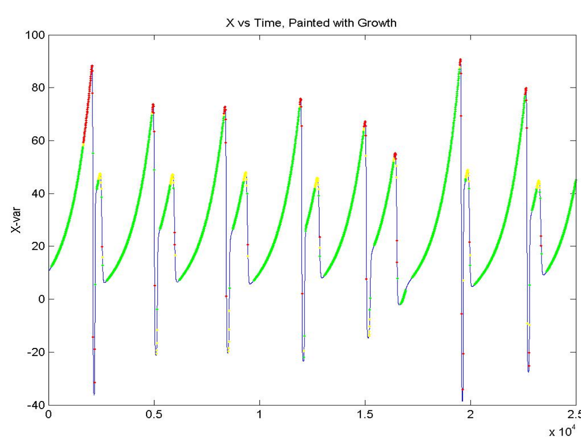

Figure 7: Langmuir wave for the coupled three-wave model with ”forecasting rules”.

4. Conclusion

This paper presents examples with models that indicate that with simple breeding, we can make accurate ”long-range forecasts” of regime changes. The chaotic behaviors of nonlinear

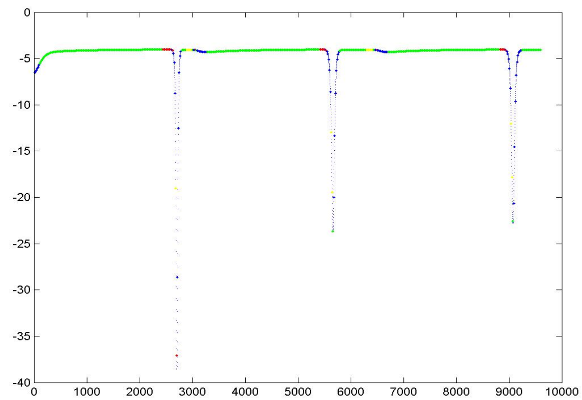

Figure 8: Whistler wave for the coupled three-wave model with ”forecasting rules”.

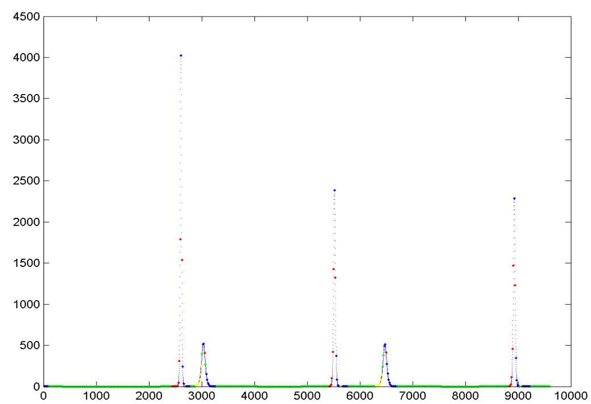

Figure 9: Alfvn wave for the coupled three-wave model with ”forecasting rules”.

interactions in three wave model can be predicted by the analysis of bred vectors too. This is a simple and effective method of predict changes in experiments in plasma physics.

Bred Vectors have been used as well-adapted ensembles of initial conditions. It has explained by a phenomenological argument saying that they carry some of the underlying dynamics, and errors are spacialy distributed according to this dynamics.

In spatio-temporal chaotic systems predictability strongly depends on the spatial correlation of initial conditions. Correlated ensembles have errors, which implies a longer prediction time of variables. The prediction range of numerical models can be adjusted. Hence a method to obtain ensembles with varying amplitude and correlation by means of breeding method is feasible like a predictive method.

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