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A NOTE ON THE MINIMIZATION OF THE NUMBER OF CUTTING CYCLES PROBLEM

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Resumo

Neste artigo abordamos o problema de minimização do número de ciclos de corte de máquina dentro de um processo de corte. Mostramos que quando o custo do tempo de máquina é dominante comparado com outros custos, o problema é equivalente ao problema de corte de estoque em que as demandas são reduzidas por um fator de escala. Apresentamos também heurísticas simples para gerar uma solução viável para este problema.

Palavras chave: minimização de ciclos de corte, corte de estoque, redução de padrões

Abstract

In this article we address the problem of minimizing the number of machine cutting cycles problem within a cutting process. We show that when the cost of the machine time is dominant compared to the other costs, then the problem is equivalent to a cutting stock problem where the demands are scaled by a factor. We also present some simple heuristics to generate a feasible solution to this problem.

Keywords: minimization of cutting cycles, cutting stock, pattern reduction

1. Introduction

Cutting problems arise in many industries such as the paper, glass, furniture, metallurgy, plastics and textile industries. Their practical application and the computational difficulty in solving them motivate the researchers in the search for improved solution methods.

Cutting problems have been studied by many researchers (see, for instance, Hinxamn,1980; Dyckhoff and Waescher, 1990; Lirov, 1992; Dowsland and Dowsland, 1992; Sweeney and Paternoster, 1992; Dyckhoff and Finke, 1992; Martello, 1994a, 1994b; Bischoff and Waescher, 1995; Mukhacheva, 1997; Dyckhoff et al., 1997); Arenales et al., 1999; Wang and Waescher, 2002; Hifi, 2002; and ESICUP, 2007). Many different cutting problems have been addressed in the literature, each one with its particularities. Hence, a taxonomy for these problems has been proposed (see for instance, Dyckhoff, 1990; Wascher et al., 2006), so that readers can correctly identify the problem been treated in an article.

We address here the cutting stock problem that consists in cutting large pieces (denoted by objects), usually available in stock, into a set of smaller pieces (denoted items) in order to fulfill their demands, optimizing a certain objective function such as minimization of the number of objects cut, minimization of waste, minimization of production costs.

Let us consider a cutting stock problem (CSP) with a single type of object with the objective of minimizing the total number of objects used to cut all the required items. A possible mathematical model for this problem is:

Model CSP

$$z_0 = \text{Minimize } f(y) = c\sum_{j=1}^{n} x_j \quad (1)$$

$$\text{subject to:} \quad \sum_{j=1}^{n} \alpha_{ij} x_j \geq d_i \qquad i = 1,..., m \quad (2)$$

$$x_j \geq 0 \qquad \text{and integer, } j = 1,\ldots, n \quad (3)$$

where

n is the total number of different cutting patterns;

m is the total number of different items;

$\alpha_j = (\alpha_{1j}, \alpha_{2j}, ..., \alpha_{Mj})$ is an m non-negative integer valued vector that represents pattern j, where each element α_{ij} is the quantity of items i contained in pattern j, $j = 1, 2, ..., n$;

c is the cost of an object;

d_i is the requirement of item i, $i = 1, 2, ..., m$;

x_j is the decision variable that gives the number of times (the frequency) that pattern j is to be cut, $j = 1, 2, ..., n$.

Consider a industrial setting with high demand of items where an important component of the production costs is the time of the saw machine. The machine is able to cut many objects, all at once, by just stacking them one on top of the other. The maximum amount p of objects that the machine can cut at a time is determined by the width of each object, say w, and it is given by $p = \lfloor H / w \rfloor$, where H is the maximum height of the stack of objects that the machine can handle.

If the cost of the machine time is dominant compared with the other costs of the production, a desired objective in this high demand setting is to reduce the number of the machine cutting cycles assuming the machine takes about the same time to cut one object or p objects.

The problem of minimizing the number of cutting cycles (MCC) can be formulated as

Model MCC

$$z_1 = \text{Minimize } \sum_{i=1}^{n} \lceil x_i / p \rceil \quad (4)$$

$$\text{subject to:} \quad \sum_{j=1}^{n} \alpha_{ij} x_j \geq d_i \qquad i = 1,..., m \quad (5)$$

$$x_i \geq 0 \qquad i = 1, 2, ..., n \text{ and integer} \quad (6)$$

The problem of minimizing the number of cutting cycles (MCC) has not been addressed much in the literature. We are aware of the heuristic proposed to solve it in Yanasse et al (1993) and, more recently, its variation suggested by Mosquera and Rangel (2007) .

In the next section we discuss some features of the MCC problem (4)-(6).

2. The cutting problem with reduction on the number of cycles

In (4)-(6), if x_i is a multiple of p for all i, then models CSP and MCC are equivalent except for a constant. However, x_i be a multiple of p for all i is unlikely to happen, unless $p = 1$.

Consider the case where p ≠ 1. Model MCC can be rewritten as

Model 1

$$z_1 = \text{Minimize } \sum_{i=1}^{n} y_i \tag{7}$$

Subject to

$$\sum_{j=1}^{n} \alpha_{ij} x_j \geq d_i \qquad i = 1,\dots, m \tag{8}$$

$$x_i \geq 0 \qquad i = 1, 2, \dots, n \text{ and integer} \tag{9}$$

$$y_i \geq x_i / p \qquad i = 1, 2, \dots, n \tag{10}$$

$$y_i \qquad \text{integer, } i = 1, 2, \dots, n \tag{11}$$

where

y_i is the minimum number of cutting cycles required to cut x_i times the pattern i,

Since z_0 is the minimum number of objects necessary to cut all the required items, then a lower bound for z_1 is $\lceil z_0 / p \rceil = \left\lceil \sum_{i=1}^{n} x_i^* / p \right\rceil$, where $\mathbf{x}^*$ is an optimal solution of (1)-(3). An upper bound for z_1 is $\sum_{i=1}^{n} \lceil x_i^* / p \rceil$ since $y_i = \left| x_i^* / p \right|$, $i = 1, 2, \dots, n$, is a feasible solution to (7)-(12).

Let $\mathbf{y}^*$ be an optimal solution to (7)-(11). We have from (10) that

$$x_i \leq p\, y_i \qquad i = 1, 2, \dots, n$$

and since

$$\sum_{j=1}^{n} \alpha_{ij} x_j \geq d_i \qquad i = 1,\dots, m$$

then

$$\sum_{j=1}^{n} \alpha_{ij}\, p y_j^* \geq d_i \qquad i = 1,\dots, m$$

and, therefore,

$$\sum_{j=1}^{n} \alpha_{ij} y_j^* \geq d_i / p \qquad i = 1,\dots, m \tag{12}$$

Since α_{ij} is integer for all i and j, y_i^* is integer for all i, and p is integer, then the left hand side of the inequalities (12) are integer, therefore, without modifying the optimality of the solution $\mathbf{y}^*$, we can replace the right hand side of (12) by $\lceil d_i / p \rceil$, $i = 1,\dots, m$.

Let us define Model 2 as

Model 2

$$z_2 = \text{Minimize } \sum_{i=1}^{n} y_i \qquad (13)$$

Subject to

$$\sum_{j=1}^{n} \alpha_{ij} y_j \geq \lceil d_i / p \rceil \qquad i = 1,..., m \qquad (14)$$

$$y_i \geq 0 \qquad i = 1, 2, \ldots, n \text{ and integer} \qquad (15)$$

Proposition 1: Models 1 and 2 are equivalent (that is, an optimal solution $\mathbf{y}^*$ of model 1 is an optimal solution of model 2 and vice-versa; and the optimal solution values are equal, that is $z_1 = z_2$).

Proof of Proposition 1: Let ($\mathbf{x}^*$, $\mathbf{y}^*$) be an optimal solution to Model 1. Therefore, we have from (10) that

$$x_i^* \leq p\ y_i^* \qquad i = 1, 2, ..., n.$$

From (8) we have that

$$\sum_{j=1}^{n} \alpha_{ij} x_j^* \geq d_i \qquad i = 1,..., m,$$

therefore

$$\sum_{j=1}^{n} \alpha_{ij}\, p y_j^* \geq d_i \qquad i = 1,..., m$$

and, then,

$$\sum_{j=1}^{n} \alpha_{ij} y_j^* \geq d_i / p \qquad i = 1,..., m.$$

Since α_{ij} is integer for all i and j, y_i^* is integer for all i, and p is integer, then the left hand side of the previous inequalities are integer, so, it must be larger or equal to $\lceil d_i / p \rceil$, $i = 1,..., m$. Therefore, $\mathbf{y}^*$ is a feasible solution to Model 2 and $z_2 \leq z_1$.

Consider now an optimal solution $\mathbf{y}^*$ for Model 2. From (14) we have that

$$\sum_{j=1}^{n} \alpha_{ij} y_j^* \geq \lceil d_i / p \rceil \qquad i = 1,..., m,$$

and, therefore,

$$\sum_{j=1}^{n} \alpha_{ij} y_j^* \geq d_i / p \qquad i = 1,..., m$$

or

$$\sum_{j=1}^{n} \alpha_{ij}\, p y_j^* \geq d_i \qquad i = 1,..., m.$$

Let $x_i^* = p\ y_i^*$, $i = 1, 2, ..., n$. Then we have that

$$\sum_{j=1}^{n} \alpha_{ij} x_j^* \geq d_i \qquad i = 1,..., m,$$

and x_i^*, i = 1, 2, ..., n satisfy (10), therefore, it is feasible for Model 1. Therefore, (**x***,**y***) is a feasible solution to Model 1 and $z_1 \leq z_2$. Consequently, an optimal solution $\mathbf{y}^*$ of model 1 is an optimal solution of model 2 and vice-versa; and $z_1 = z_2$.

From the previous results we observe that for the case where the cost of machine time to cut the objects is dominant compared with the other costs involved in the production, the problem of minimizing the number of cycles is equivalent to solve a CSP with scaled demands.

Let us consider next the case where the cost of the material (objects) is also relevant. We have

Model 3

$$z_3 = \text{Minimize } c\sum_{j=1}^{n} x_j + t \sum_{i=1}^{n} y_i \tag{16}$$

Subject to

$$\sum_{j=1}^{n} \alpha_{ij} x_j \geq d_i \qquad i = 1,..., m \tag{17}$$

$$x_i \geq 0 \qquad i = 1, 2, ..., n \text{ and integer} \tag{18}$$

$$y_i \geq x_i / p \qquad i = 1, 2, ..., n \tag{19}$$

$$y_i \qquad \text{integer, } i = 1, 2, ..., n \tag{20}$$

Lower bound for z_3

We have that

$$z_3 \geq z_0 + t\, z_1 \tag{21}$$

since z_0 and $t\, z_1$ are the smallest values we can obtain for $c\sum_{j=1}^{n} x_j$ and $t \sum_{i=1}^{n} y_i$, respectively, in (16), constrained to (17)-(20).

Simple heuristics to determine a solution to Model 3

Heuristic 1

Let **x*** be an optimal solution to Model CSP. Let **y**** be obtained from **x*** by rounding up to the nearest integer the elements of **x*** divided by p. (**x***, **y****) is a feasible solution to Model 3, therefore,

$$z_3 \leq z_0 + t\,(\sum_{i=1}^{n} \lceil x_i^* / p \rceil).$$

Heuristic 2

Let **y*** be an optimal solution to Model 2. Let **x**** be obtained by the following iterative procedure:

Step 0: Set $\mathbf{x} \leftarrow p\, \mathbf{y}^*$

Step 1: For $i = 1, 2, \ldots, m$, determine $\delta_i = \sum_{j=1}^{n} \alpha_{ij} x_j - d_i$

Step 2: For $j = 1, 2, \ldots, n$ such that $y_j^* > 0$, determine $h_j = \min_i \{ \delta_i / \alpha_{ij}$, with $\alpha_{ij} > 0\}$.

Step 3: Let $s = h_k = \max_j \{ h_j$, with $y_j^* > 0\}$.

Step 4: If $s = 0$, set $\mathbf{x^{**}} \leftarrow \mathbf{x}$ and stop, else set $x_k \leftarrow x_k - s$ and return to Step 1.

($\mathbf{x^{**}}$, $\mathbf{y^*}$) is a feasible solution to Model 3, therefore,

$$z_3 \leq c \sum_{j=1}^{n} x_j^{**} + t\, z_1.$$

So, using Heuristics 1 and 2, we can obtain feasible solutions to Model 3 and choose the best. An idea of the quality of the solution can be obtained with the lower bound given in (21).

Heuristic 3

In Model 3, when all the values of x_i, $i = 1, 2, ..., n$ are smaller or equal to p (for instance, when we have low demands for the items) then the variables y_i, $i = 1, 2, ..., n$ are binary. Therefore, the model reduces to the problem of minimizing the number of cutting patterns which has been also studied by several researchers (see, for instance, Haessler (1975, 1991); Farley and Richardson (1984), Foerster and Wäscher (2000), Diegel et al. (1993), Umetami et al. (2003), Vanderbeck (2000), McDiarmid (1999), Yanasse and Limeira (2006)). The problem of minimizing the number of different patterns arises when there is a significant cost involved with the set up of new cutting patterns.

If we have a good solution method for the problem of minimizing the number of different cutting patterns, we can generate a feasible (hopefully good) solution to Model 3 by the following procedure.

Let $\mathbf{x^*}$ be an optimal solution to Model CSP. Let $\mathbf{y}$ be obtained from $\mathbf{x^*}$ by rounding down to the nearest integer the elements of $\mathbf{x^*}$ divided by p. Let $\mathbf{z} = p\,\mathbf{y}$. For $i = 1, 2, \ldots, m$, determine $\delta_i = d_i - \sum_{j=1}^{n} \alpha_{ij}\, p y_j$. Observe that for all i, $\delta_i < p$. Therefore, we propose to obtain a solution by solving the residual problem as the following minimization of the number of different cutting patterns problem (MNCP)

Model MNCP

$$\text{Minimize } c \sum_{j=1}^{n} x_j + t \sum_{i=1}^{n} y_i$$

Subject to

$$\sum_{j=1}^{n} \alpha_{ij} x_j \geq \delta_i \qquad i = 1, \ldots, m$$

$$x_i \geq 0 \qquad i = 1, 2, ..., n \text{ and integer}$$

$$M\, y_i \geq x_i \qquad i = 1, 2, ..., n$$

$$y_i \quad \text{binary} \qquad i = 1, 2, ..., n.$$

If we obtain solution (**x****, **y****) for this problem, we can construct the solution (**z** + **x****, **y** + $\sum_{i=1}^{n} \lceil x_i^{**} / p \rceil$) that is feasible to Model 3.

MNCP is a difficult problem to solve. We can obtain a solution to it using a heuristic, for instance, the ones proposed in Yanasse and Limeira (2006).

3. Remarks

We presented some features of the problem of minimizing the number of cycles within a cutting process. We also present three simple heuristics to get a feasible solution to this problem and a lower bound for the optimal solution value.

It is worth observing that we analysed the case where overproduction of items is allowed. Scaling of the demands may not be valid in the case where the demands of the items have to be met exactly.

Recently Ranck Jr. et al. (2008), proposed the following model (Model 4) for the minimization of cycles problem where the demands have to be met exactly:

Model 4:

$$\text{Minimize } \sum_{j=1}^{n} \sum_{a=1}^{p} x_{ja}$$

Subject to

$$\sum_{j=1}^{n} \sum_{a=1}^{p-1} \alpha_{ij} x_{ja} (p-a) = b_i \qquad i = 1,\dots, m$$

$$x_{ja} \geq 0 \qquad j = 1, 2, \dots, n,\ a = 1, \dots, p-1, \text{ and integer}$$

where:

x_{ja} is the number of cycles cut of pattern j with a object stacked together.

The extension of this model to include the cost of the objects becomes:

Model 5:

$$\text{Minimize } c \sum_{j=1}^{n} \sum_{a=1}^{p-1} x_{ja} (p-a) + t \sum_{j=1}^{n} \sum_{a=1}^{p} x_{ja} =$$

$$\text{Minimize } \sum_{j=1}^{n} \sum_{a=1}^{p-1} x_{ja} [c(p-a)+t]$$

Subject to

$$\sum_{j=1}^{n} \sum_{a=1}^{p-1} \alpha_{ij} x_{ja} (p-a) = b_i \qquad i = 1,\dots, m$$

$$x_{ja} \geq 0 \qquad j = 1, 2, \dots, n,\ a = 1, \dots, p-1, \text{ and integer}$$

We hope the features, heuristics and models presented can be useful to better understand this problem and in the development of more efficient algorithms to solve it.

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