

Structural Damage Assessment through the Adjoint Method

Leonardo D. Chiwiacowsky¹, Haroldo F. de Campos Velho¹ and Paolo Gasbarri²

1: Laboratório Associado de Computação e Matemática Aplicada (LAC)
Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos, SP, Brazil

2: Dipartimento di Ingegneria Aerospaziale e Astronautica
Università Degli Studi di Roma - “La Sapienza”, Rome, Italy

E-mail: ¹ [leodc, haroldo]@lac.inpe.br, ²paolo.gasbarri@uniroma1.it

Abstract

The present investigation is focused on the solution of a dynamic inverse problem for damage identification in structures from measured data. The inverse problem is formulated as an optimization problem. It is solved using the Adjoint Method, adapted by frequency domain measurements, coupled with the Genetic Algorithm method. The damage estimation has been evaluated using noiseless and noisy synthetic experimental data, considering three different structures: a simple spring-mass system, a truss structure and a beam-like structure.

Keywords: Inverse vibration problem, damage identification, variational approach, genetic algorithm.

1 Introduction

The direct solution of forced vibration problems are concerned with the determination of the system displacement, velocity and acceleration at time t when the initial conditions, external forces, and system parameters, such as stiffness coefficients and damping coefficients, are specified. On the other hand, the inverse vibration problems are concerned with the estimation of such quantities (stiffness or damping coefficients, external forces) from the measured vibration data, i.e., natural frequency and/or mode shapes, or displacement measurements.

The techniques of inverse problems have been applied in many different areas of engineering research, such as in thermal science, where some of them have been presented good results. In general the inverse problem, i.e. the ill-posed problem, is presented as a well-posed functional form, whose solution is obtained through an optimization procedure. Such thermal inverse problems have been addressed in the literature by using regularization approaches [16, 3], the conjugate gradient method with the adjoint equation [1, 12, 3], and a regularized solution through the genetic algorithm

method [3, 5].

Regarding the inverse vibration problem, considerable research and effort over the last few decades has taken place in the field of damage detection and damage classification in structures. A variety of experimental, numerical and analytical techniques has already been proposed to solve the damage identification problem, and has received notable attention due to its practical applications [8].

The structural damage detection is displayed as an inverse vibration problem, since the damage evaluation is achieved through the determination of the stiffness coefficient variation, or the stiffness coefficient by itself. Recently this type of problem has already been solved employing the Variational technique, i.e. the adjoint equation method, where the results have been reported concerning lumped-parameter systems with a small number of degrees-of-freedom (DOFs) [10] or with a higher number of DOFs [11, 4]. Also, in some works more realistic structures have been considered such as truss and beam-like structures [6, 7, 2]. In the cited works the time-history of the displacement data have been adopted as the available experimental data.

In this work, the adjoint equation method has been applied to the inverse vibration problem of damage assessment in three different structures: a lumped-parameter system represented by an spring-mass system, a naturally discrete structure of truss and a distributed-parameter system represented by a beam-like structure, Figures 1, 2 and 3, respectively. In all cases natural frequencies measurements have been assumed as the available experimental data.

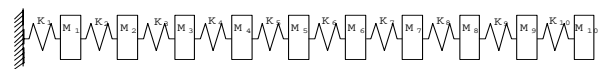


Figure 1: The 10-DOF spring-mass system considered in this work.

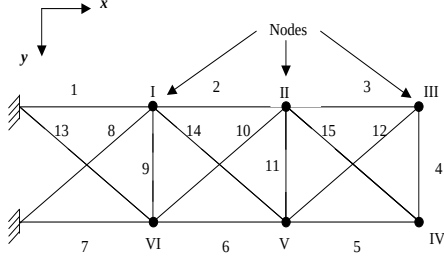


Figure 2: The three bay structure considered in this work.

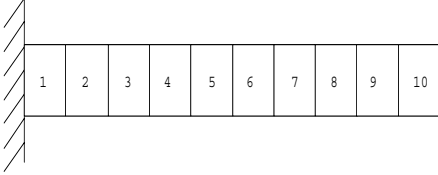


Figure 3: The 20-DOF beam structure considered in this work.

2 The Inverse Analysis

The goal is to recover the unknown stiffness coefficients from the synthetic system frequency measurements of three different structures. The inverse analysis with the conjugate gradient method involves the following steps [1, 12]: (i) the solution of direct problem; (ii) the solution of sensitivity problem; (iii) the solution of adjoint problem and the gradient equation; (iv) the conjugate gradient method of minimization; (v) the stopping criteria.

Next, a brief description of the basic procedures involved in each of these steps is presented.

2.1 The Direct Problem

The undamped free vibration of a N -DOF structural system gives rise to the matrix eigenvalue problem,

$$\mathbf{K}\phi = \lambda\mathbf{M}\phi, \quad (1)$$

which will be considered as the direct eigenproblem in the frequency domain; being $\mathbf{K}$ and $\mathbf{M}$ the stiffness and mass matrices, λ are the eigenvalues (natural frequencies squared), and ϕ are the eigenvectors.

2.2 The Sensitivity Problem

Since the problem involves N unknown stiffness parameters, which constitute the elements of the stiffness matrix $\mathbf{K} = f[\mathcal{K}]$, where $\mathcal{K} = [K_1, \dots, K_N]$, in order to derive the sensitivity problem for each unknown function K_i , each unknown stiffness coefficient should be perturbed at a time. Supposing that the K_i is perturbed by a

small amount $\Delta K_{ij} \delta(i-j)$, where the $\delta(\cdot)$ is the Dirac-delta function and $j = 1, \dots, N$, it results in a small change in frequencies and mode-shapes by the amounts $\Delta \lambda_{ij}(t)$ and $\Delta \phi_{ij}(t)$, respectively. Then, the sensitivity problem is obtained by replacing in the eigenvalue problem $\mathbf{K}_i$ by $\mathbf{K}_i + \Delta \mathbf{K}_{ij} \delta(i-j)$, λ_i by $\lambda_i + \Delta \lambda_{ij}$, ϕ_i by $\phi_i + \Delta \phi_{ij}$, and is given by

$$[(\mathbf{K}_i + \Delta \mathbf{K}_j) - (\lambda_i + \Delta \lambda_{ij})\mathbf{M}](\phi_i + \Delta \phi_{ij}) = \mathbf{0}. \quad (2)$$

Rearranging the terms of the above equation and subtracting from the resulting expression the original eigenvalue problem (1), after some algebraic manipulations and neglecting the second-order terms, the following expression yields

$$\phi_i^T (\Delta \mathbf{K}_j - \Delta \lambda_{ij} \mathbf{M}) \phi_i = \mathbf{0} \Rightarrow \Delta \lambda_{ij} = \frac{\phi_i^T \Delta \mathbf{K}_j \phi_i}{\phi_i^T \mathbf{M} \phi_i}, \quad (3)$$

which provides a sensibility analysis where the eigenvectors ϕ have been obtained when the updated stiffness matrix is considered.

2.3 The Adjoint Problem and the Gradient Equation

The inverse problem is to be solved as an optimization problem, requiring that the unknown function $\mathcal{K}$ should minimize the functional $\mathbf{J}[\mathcal{K}]$ defined by

$$\mathbf{J}[\mathcal{K}] = \|\lambda^{exp} - \lambda\|_2^2, \quad (4)$$

where λ and λ^{exp} are the computed and measured frequencies, respectively. For solving the minimization problem (4), the *Lagrange* multipliers ψ are usually used to associate the constraints (1) to the functional form.

$$\mathbf{J}(\lambda, \mathcal{K}, \psi) = \|\lambda^{exp} - \lambda\|_2^2 - \psi^T (\mathbf{K} - \lambda \mathbf{M}) \phi. \quad (5)$$

The variation $\Delta \mathbf{J}_j[\mathcal{K}]$ of the functional is obtained by perturbing $\mathbf{K}$ by $\Delta \mathbf{K}_j$ which causes a small disturbance in the frequencies λ by $\Delta \lambda_j$ and also in the mode-shapes ϕ by $\Delta \phi_j$ in Eq. (5). Subtracting from the resulting expression the original Eq. (5), after some algebraic manipulations and neglecting the second-order terms, the following expression yields

$$\psi^T \mathbf{M} \phi = -2(\lambda^{exp} - \lambda), \quad (6)$$

which is defined as the adjoint problem which is used for the determination of the *Lagrange* multipliers. Applying the variational theory [17], the left term is employed to determine the gradient $\mathbf{J}'[\mathcal{K}]$, which is given by

$$\mathbf{J}'[\mathcal{K}] = -\psi^T \Delta \tilde{\mathbf{K}} \phi, \quad (7)$$

where $\Delta \tilde{\mathbf{K}}_j$ refers to the j th perturbed stiffness matrix, i.e. $\Delta \tilde{\mathbf{K}}_j = \partial[\Delta \mathbf{K}]/\partial K_j$.

2.4 The conjugate gradient method and the Stopping Criteria

The iterative procedure based on the conjugate gradient method is used for the estimation of the unknown stiffness coefficients $\mathcal{K}$ given in the form [12]:

$$\mathcal{K}^{n+1} = \mathcal{K}^n - \beta^n \mathbf{P}^n, \quad n = 0, 1, 2, \dots, \quad (8)$$

$$\mathbf{P}^n = \mathbf{J}'^n + \gamma^n \mathbf{P}^{n-1}, \quad \text{with } \gamma^0 = 0, \quad (9)$$

where β^n is the step size vector, $\mathbf{P}^n$ is the direction of descent vector and γ^n is the conjugate coefficient vector. The step size vector β^n , appearing in Eq. (8), is determined by minimizing the functional vector $\mathbf{J}[\mathcal{K}^{n+1}]$ given by Eq. (4) with respect β^n . The discrepancy principle [15] for the stopping criterion is taken as

$$\mathbf{J}[\mathcal{K}^{n+1}] < \epsilon^2. \quad (10)$$

where $\epsilon^2 = N \sigma^2$, and σ is the standard deviation of the measurements errors.

2.5 The Solution Algorithm

The procedure for the adjoint method can be summarized as:

- Step 1:** Choose an initial guess $\mathcal{K}^0$ – for example, $\mathcal{K}^0 = \text{constant}$.
- Step 2:** Solve the direct eigenvalue problem [Eq. (1)], to obtain λ .
- Step 3:** Solve the adjoint problem [Eq. (6)], to obtain the *Lagrange* multiplier vector ψ .
- Step 4:** Knowing ψ , compute the gradient function vector $\mathbf{J}'(\mathcal{K})$ from Eq. (7).
- Step 5:** Compute the conjugate coefficient vector γ^n .
- Step 6:** Compute the direction of descent vector $\mathbf{P}^n$ from Eq. (9).
- Step 7:** Setting $\Delta \mathbf{K} = \mathbf{P}^n$, solve the sensitivity problem [Eq. (3)], to obtain $\Delta \lambda$.
- Step 8:** Compute the step size β^n .
- Step 9:** Compute $\mathcal{K}^{n+1}$ from Eq. (8).
- Step 10:** Test if the stopping criteria, Eq. (10), is satisfied. If not, go to step 2.

Concerning the detection and assessment of damage in a specific structure, it has been observed that the application of the standard Variational Method is employed providing excellent estimation results only when both the structure presents a small number of degrees of freedom and small size damages are to be evaluated.

When these conditions are not satisfied, it has been noticed that the initial guess choice becomes more decisive and according the choice this iterative procedure could not converge. To overcome this difficulty a new approach has been proposed, where the Genetic Algorithm (GA) method is used to generate a primary solution which is employed as the initial guess for the conjugate gradient method. This new approach could be inserted in the above procedure as the new **Step 1**.

2.6 The Stochastic Method – Genetic Algorithm

Genetic algorithms are essentially optimization algorithms whose solutions evolve somehow from the science of genetics and the processes of natural selection – the Darwinian principle. They differ from more conventional optimization techniques since they work on whole populations of encoded solutions, and each possible solution is encoded as a gene.

The most important phases in standard GAs are selection (competition), reproduction, mutation and fitness evaluation. Selection is an operation used to decide which individuals to use for reproduction and mutation in order to produce new search points. Reproduction or crossover is the process by which the genetic material from two parent individuals is combined to obtain one or more offsprings. Mutation is normally applied to one individual in order to produce a new version of it where some of the original genetic material has been randomly changed. Fitness evaluation is the step in which the quality of an individual is assessed [9].

The application of GA method to solve the problem of damage identification is also a minimization problem, as well as the gradient conjugate method. The same functional form, or fitness function, given by Eq. (4), is employed

$$\mathbf{J}[\mathcal{K}] = \|\lambda^{exp} - \lambda\|_2^2.$$

In this GA implementation, the algorithm operates on a fixed-sized population which is randomly generated initially. The members of this population are fixed-length and real-valued strings that encode the variable which the algorithm is trying to optimize ($\mathcal{K}$). The fixed population size has been taken as 100 individuals, the mutation probability has been taken as 25% and a fixed maximal generation number of 5000 has been adopted as stopping criteria for the GA method.

Next, the evolutionary operators employed in this work are presented.

– Tournament Selection [14]

```

bigger:=rand; smaller:=rand; val:=0.75;
if (rand < val) then
    position:=smaller;
else
    position:=bigger;
endif

```

where *rand* is a random number from [0,1] with uniform distribution, *smaller* is the better fitness individual and *bigger* is the worse fitness individual.

– Arithmetic Crossover [13]

A crossover operator that linearly combines two parent chromosome vectors, x and y , to produce two new offspring according to the following equations:

$$\begin{aligned} z_1 &= a \times x + (1 - a) \times y, \\ z_2 &= (1 - a) \times x + a \times y, \end{aligned} \quad (11)$$

where a is a real weighting factor in [0,1] which has been taken as $a = 0.5$ in this work.

– Non-uniform Mutation [13]

This mutation operator is defined by

$$x'_i = \begin{cases} x_i + \Delta(t, l_{sup} - x_i) & \text{if a digit is 0,} \\ x_i - \Delta(t, x_i - l_{inf}) & \text{if a digit is 1,} \end{cases} \quad (12)$$

such that

$$\Delta(t, y) = y \left[1 - rand^{(1 - \frac{t}{T})^b} \right],$$

where *rand* is a random number from [0,1] with uniform distribution, T is the maximal generation number, t is the current generation number, and b is a system parameter determining the degree of non-uniformity.

– Epidemical Strategy [3]

This operator is activated when a specific number of generations is reached without improvement of the best individual. When it is activated, all the individuals are replaced, and only the best-fit individuals remain. At the present work, the new individuals have been generated randomly, just as the initial population, and 80% of the current population have been substituted, preserving only 20% of this original population, the best individuals.

3 Results and Discussion

In this work an alternative hybrid approach has been used to solve the damage identification problem involving the estimation of the unknown stiffness parameters. In order to evaluate the capability of this new approach three different structures have been considered: a 10-DOF spring-mass system, a 3-bay truss structure (12-DOF) obtained from a finite element model of bar and a 20-DOF beam structure obtained from a finite element model of beam. In order to generate a damaged structure a reduced value of some stiffness parameters have been imposed on the discretization model.

The experimental data (*measured frequencies*) have been obtained from the exact solution of the eigenvalue problem (noiseless data) and also by adding a random perturbation (noisy data),

$$\lambda^{exp} = \lambda (1 + \sigma \mathcal{R}), \quad (13)$$

where σ is the standard deviation of the errors and $\mathcal{R}$ is a random variable from a normal distribution $\mathcal{R} \sim \mathcal{N}(\text{Normal}(0;1))$. For numerical purposes, it has been adopted $\sigma = 1\%$. The stopping criterion has been set by using $\epsilon^2 = 10^{-10}$ for the noiseless case. The stopping criterion has been set by using Eq. (10) for the noisy case.

Also, the comparison between the estimated and exact values has been done through the use of the damage factor, defined as

$$DF_i = \frac{K_i^u - K_i^d}{K_i^u} \quad i = 1, \dots, N \quad (14)$$

where K_i^u and K_i^d are the undamaged and damaged parameters, respectively. The numerical results have been obtained considering no prior information about the functional form of the unknown quantities, and the initial guess has been provided by the GA method.

3.1 The Spring-mass system

The parameters which define the undamaged configuration of the spring-mass, Figure 1, are taken as: $M_i = 10.0$ kg, and $K_i = 2 \times 10^5$ N/m, where $i = 1, \dots, 10$. The following damage configuration has been considered: a 10% damage over the element 1; 25% over the element 3; 15% over the element 4; 5% over the element 5; 30% over the element 7; 20% over the element 8 and a 10% damage over the element 10. All the others elements have been assumed as undamaged for the generation of the experimental data.

Figure 4 shows estimated and exact damage factor values, for (a) noiseless and (b) noisy experimental data, respectively. Perfect damage estimations are obtained

for noiseless data and even for noisy data very good estimations are obtained.

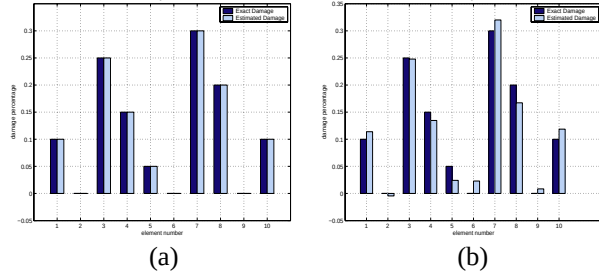


Figure 4: Estimated damage factor for the spring-mass system: (a) noiseless data, (b) noisy data.

3.2 The Truss Structure

The truss structure considered here, Figure 2, is composed by 12 aluminum bars ($\rho = 2700 \text{ kg/m}^3$ and $E = 70 \text{ GPa}$) with a square cross section area $A = 9.0 \times 10^{-4} \text{ m}^2$, where the nondiagonal elements are 1.0 m long. For this numerical example it has been used the finite element method to calculate the mass and the stiffness matrices that appear in Eq. (1), note that for this example one finite element for each bar has been used. The following damage configuration has been considered: a 15% damage over the element 2; 5% over the element 4, 30% over the element 7; 10% over the element 10 and a 20% damage over the element 12. All the others elements have been assumed as undamaged.

Figure 5 shows estimated and exact damage factor values, for (a) noiseless and (b) noisy experimental data, respectively. Perfect damage estimations are obtained for noiseless data and even for noisy data very good estimations are obtained.

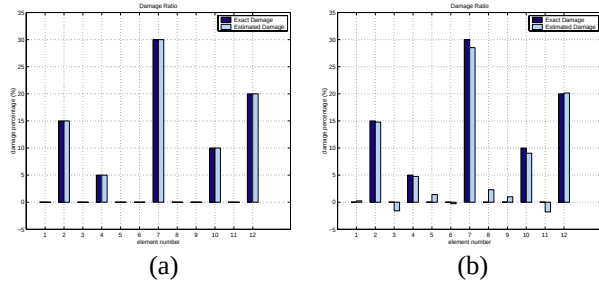


Figure 5: Estimated damage factor for the 3-bay truss structure: (a) noiseless data, (b) noisy data.

3.3 The Beam Structure

The numerical example considered here is a beam-like structure, Figure 3, modeled with 10 beam finite elements and clamped at one end. The aluminum beam

($\rho = 2700 \text{ kg/m}^3$ and $E = 70 \text{ GPa}$) presents the following properties: rectangular cross section area $A = 4.5 \times 10^{-5} \text{ m}^2$, length $L = 0.43 \text{ m}$ and inertial moment $I = 3.375 \times 10^{-11} \text{ m}^4$. For this numerical example it has been used the finite element method to calculate the mass and the stiffness matrices that appear in Eq. (1), note that for this example one finite element for each bar has been used. The following damage configuration has been considered: a 20% damage over the element 2; 10% over the element 5; 15% over the element 9 and a 5% damage over the element 10. All the others elements have been assumed as undamaged.

Figure 6 shows estimated and exact damage factor values, for (a) noiseless and (b) noisy experimental data, respectively. Perfect damage estimations are obtained for noiseless data and good estimations are obtained when noisy experimental data is used.

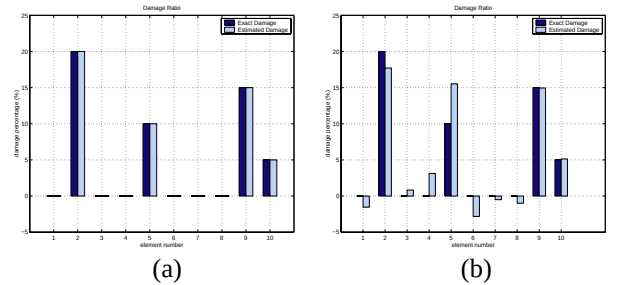


Figure 6: Estimated damage factor for the beam structure: (a) noiseless data, (b) noisy data.

4 Final Remarks

The evaluation of the conjugate gradient method with the adjoint equation on the estimation of stiffness coefficients (damage identification) has been considered. Three simple dynamical system have been adopted to verify the feasibility of the variational approach considering damage scenarios and employing synthetic noiseless and noisy frequencies measurements. Perfect reconstructions have been achieved for the noiseless data, and satisfactory estimations for the noisy data.

Futute works include more realistic structures, as well as other inverse vibration problems, such as damping matrix identification.

5 Acknowledgements

The authors are gratefully indebted to FAPESP (Proc. 00/09488-6) and CAPES (Proc BEX 2086/02-6), Brazilian agencies for research support.

References

- [1] Alifanov, O.M., "Solution of an inverse problem of heat conduction by iteration methods", *Journal of Engineering Physics*, 1974, **26**(4), 471-476.
- [2] Castello, D.A. & Rochinha, F.A., "On the identification of constitutive parameters of viscoelastic materials by means of a time domain technique", *Proceedings of the 4th International Conference on Inverse Problems in Engineering*, Rio de Janeiro/RJ, Brazil, 2002.
- [3] Chiwiacowsky, L.D. & Campos Velho, H.F., "Different Approaches for the Solution of a Backward Heat Conduction Problem", *Inverse Problems in Engineering*, 2003, **11**(6), 471-494.
- [4] Chiwiacowsky, L.D.; Campos Velho, H.F. & Gasbarri, P., "The Damage Identification Problem: A Hybrid Approach", *2nd Thematic Congress on Dynamics and Control (DINCON 2003)*, São José dos Campos/SP, Brazil, 2003.
- [5] Chiwiacowsky, L.D.; Campos Velho, H.F.; Preto, A.J. & Stephany, S., "Identifying Initial Condition in Heat Conduction Transfer by a Genetic Algorithm: A Parallel Approach", *Proceedings of the XXIV Iberian Latin-American Congress on Computational Methods in Engineering*, Ouro Preto/MG, Brazil, 2003.
- [6] Chiwiacowsky, L.D.; Campos Velho, H.F. & Gasbarri, P., "A solution for the structural stiffness estimation problem through the variational approach", *XVII Congresso Nazionale AIDAA (Associazione Italiana di Aeronautica e Astronautica)*, Rome, Italy, 2003, **2**, 1403-1414.
- [7] Chiwiacowsky, L.D.; Campos Velho, H.F. & Gasbarri, P., "A variational approach for solving an inverse vibration problem", *Inverse Problems, Design and Optimization Symposium (IPDO)*, 17-19 March, Rio de Janeiro (RJ), Brazil, 2004, Proceedings in CD-Rom, paper code IPDO-078.
- [8] Doebling, S.W.; Farrar, C.R.; Prime, M.B. & Shevitz, D.W. *Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review*, Los Alamos National Laboratory, report LA-13070-MS, USA, 1996.
- [9] Goldberg, D.E. *Genetic algorithms in search, optimization, and machine learning*, Addison-Wesley Publishing Company Inc., USA, 1989.
- [10] Huang, C.H., "A non-linear inverse vibration problem of estimating the time-dependent stiffness coefficients by conjugate gradient method", *International Journal of Numerical Methods in Engineering*, 2001, **50**, 1545-1558.
- [11] Huang, C.H., "An inverse vibration problem for simultaneously estimating the time-dependent stiffness coefficients", *Proceedings of the 4th International Conference on Inverse Problems in Engineering*, Rio de Janeiro, Brazil, 2002.
- [12] Jarny, Y.; Özisik, M.N. & Bardon, J.P., "A general optimization method using adjoint equation for solving multidimensional inverse heat conduction", *International Journal of Heat and Mass Transfer*, 1991, **34**(11), 2911-2919.
- [13] Michalewicz, Z. *Genetic algorithms + data structures = evolution programs*, Springer-Verlag, USA, 1996.
- [14] Mitchell, M. *An introduction to genetic algorithms*, MIT Press, USA, 1996.
- [15] Morozov, V.A., "On the solution of functional equations by method of regularization", *Soviet Math. Dokl.*, 1966, **7**, 414.
- [16] Muniz, W.B.; Ramos, F.M. & Campos Velho, H.F., "Entropy and a Tikhonov regularization techniques applied to the backwards heat equation", *Computers & mathematics with Applications*, 2000, **40**(8/9), 1071-1084.
- [17] Woodbury, K.A. *Inverse Engineering Handbook*, CRC Press, USA, 2002.