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Viscous sheets generated by impinging liquid jets in a quiescent, gaseous medium, desintegrate into droplets whose size range and distribution can be estimated. This work suggests a technique for such a calculation.

### Introduction

The modelling of thin sheets formed by impinging jets and their own inherent stability aspects have been studied by Squire, G.I. Taylor and Hasson, among others.<sup>1,2,3</sup> It is well known that when two equal cylindrical coplanar jets collide they form a sheet whose thickness distribution is a function of the jet radius, the radial distance in the sheet, the impingement angle and of some angular position referred to a pole along the separation streamline.<sup>4</sup>

However, the physics of a viscous liquid sheet breakdown is the same for the several existing injectors, ie, a small instability is amplified until it reaches a critical wavelength and the sheet fragments into ligaments which finally breakdown into drops of several sizes.

Now, the mathematical formulation developed by Dombrowski and Johns for fan-spray nozzles<sup>3</sup> can also be applied to the sheet formed by two impinging jets.

Hence the results obtained by Dombrowski<sup>3</sup> and Hasson<sup>4</sup> can be merged to allow a proper formulation in order to calculate the size and distribution of the droplets formed by colliding jets. Notice however that, for sprays other than the above mentioned, one should refer, for instance, to the works of Levich<sup>5</sup> and Marshall.<sup>6</sup>

### Governing Equations

When two identical cylindrical liquid jets of radius  $R$  and velocity  $v_0$  in a gaseous, quiescent medium, collide obliquely at an angle of  $2\theta$ , a

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sheet is formed, the flow around it being uneven, the flux in the forward flow direction being the largest while the flow in the backward flow direction is the smallest.<sup>4</sup> The sheet thickness distribution,  $h$ , with the radial distance  $r$  (as measured from the sheet stagnation point) and the angular position  $\phi$ , as sketched in Fig. 1, has been calculated by Hasson and Peck,<sup>3</sup> who found

$$\frac{hr}{R^2} = \frac{\sin^3 \theta}{(1 - \cos \phi \cos \theta)^2} \quad (1)$$

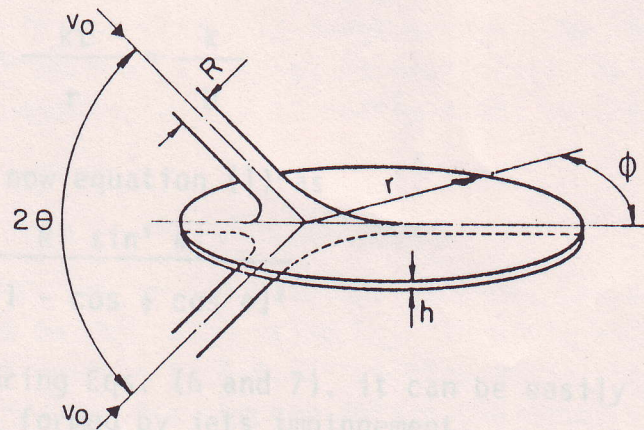


Fig. 1. Sheet formed by impinging jets (After Hasson and Peck<sup>4</sup>).

The thinning sheet thus formed spreads away under the influence of the viscous, inertia, surface tension and pressure forces and, upon reaching a critical situation, desintegrates into fragments that contract themselves by surface tension, forming unstable ligaments which finally break into droplets. This mechanism has been thoroughly discussed by Dombrowski and Johns<sup>3</sup> who found for  $d_L$ , the diameter of those ligaments, the following expression:

$$d_L = 2 \left( \frac{4}{3f} \right)^{1/3} \left( \frac{k^2 \sigma^2}{\rho \rho_L U^2} \right)^{1/6} \left[ 1 + 2.6\mu \sqrt[3]{\frac{k \rho^4 U^3}{6f \rho_L^2 \sigma^5}} \right]^{1/5} \quad (2)$$

where  $f$  is the total wavelength, given by

$$f = \int_0^t \frac{\rho U^2}{\sqrt{2h \rho_L \sigma}} dt \quad (3)$$

and where  $\sigma$  (dyn-cm<sup>-1</sup>) is the liquid surface tension,  $\mu$  (cp) is the



liquid viscosity,  $\rho$  ( $\text{g} - \text{cm}^{-3}$ ) is the density of the surrounding medium (assumed gaseous and quiescent),  $\rho_L$  ( $\text{g} - \text{cm}^{-3}$ ) is the density of the liquid,  $U$  ( $\text{cm} - \text{sec}^{-1}$ ) is the velocity of the radiating sheet (to be discussed later) and  $k$  is a constant obtained by assuming a hyperbolic relationship between the thickness,  $h$ , and the time,  $t$ , ie,

$$ht = k \quad (4)$$

They also showed that for a radiating sheet of uniform velocity the thickness at any point could be given by

$$h = \frac{k}{r} \quad (5)$$

Hence

$$k = ht = \frac{kt}{r} = \frac{k}{U} \quad (6)$$

Rewriting now equation (1) as

$$h = \frac{R^2 \sin^3 \theta}{r (1 - \cos \phi \cos \theta)^2} \quad (7)$$

and, comparing Eqs. (6 and 7), it can be easily seen that in this case of a sheet formed by jets impingement,

$$k = \frac{R^2 \sin^3 \theta}{(1 - \cos \phi \cos \theta)^2} \quad (8)$$

That is, for a given jet impingement configuration (ie, for given  $R$  and  $\theta$ ),  $k$  is now a function of the azimuthal angle  $\phi$ .

Assuming that Eq. (2) still holds for the near cylindrical ligaments formed at the sheet break up and choosing in that equation  $f$  to be equal to 12 (as this is its value at the sheet break up independently of the operating conditions as shown by Dombrowski and Hooper,<sup>7</sup>) and using Eq. (6), then Eq. (2) can be written as

$$d_L = 0.9614 \left[ \frac{k^2 \sigma^2}{\rho \rho_L U^4} \right]^{1/6} \left[ 1 + 2.60 \mu \sqrt[3]{\frac{k \rho^4 U^7}{72 \rho_L \sigma^5}} \right]^{1/5} \quad (9)$$

where  $k$  is given now by Eq. (8).

Then the drop diameter,  $d_d$ , can be estimated using the relation developed by Dombrowski and Johns:<sup>3</sup>



$$d_d = \left( \frac{3\pi}{\sqrt{2}} \right)^{1/3} d_L \left[ 1 + \frac{3\mu}{(\rho_L \sigma d_L)^{1/2}} \right]^{1/6} \quad (10)$$

Notice that  $U$ , taken to be the velocity of the radiating sheet, as already mentioned, was defined by Dombrowski<sup>3</sup> as the mean relative air wave velocity, ie

$$U = \sqrt{(U_1^2 + U_2^2)/2},$$

where  $U_1$  and  $U_2$  are the relative velocities over the upper and lower sheet surfaces, respectively. Here, however, besides the assumption of quiescence of the gaseous medium, one should recall that if  $r$ , the radial distance in the sheet, is large enough, the inward velocity component of the attenuating sheet is negligible. Hence, it follows, from an energy balance, that  $U$  is nearly equal to the jet velocity",  $v_0$ .

### Results and Discussion

Thus, as it was seen in the previous section, the droplet diameter distribution can be estimated using Eq. (10) along with Eq. (8) and (9). Notice that now  $k = k(R, \theta, \phi)$ , so that for a given jet configuration and velocity one can obtain the results shown in Fig. 2.

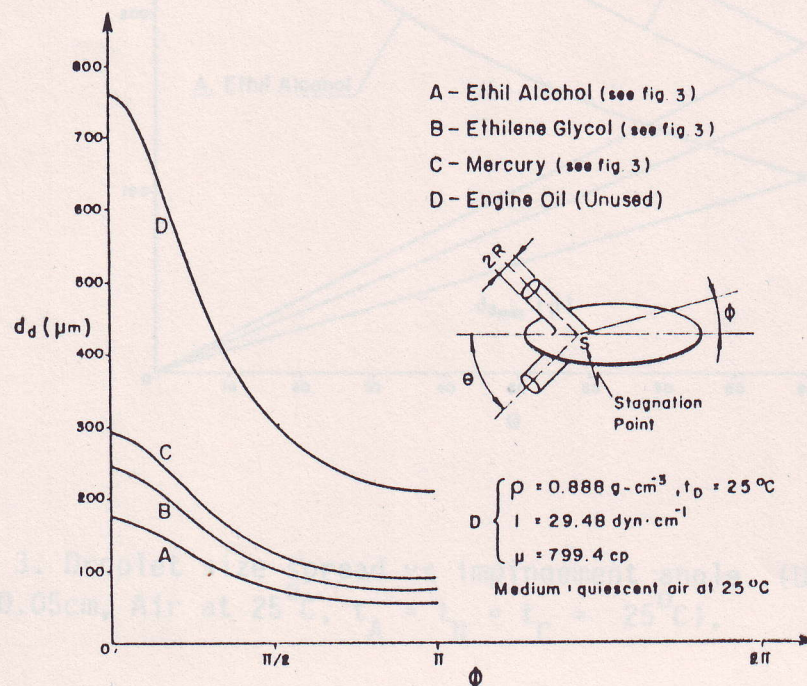


Fig. 2. Azimuthal droplet diameter distribution for a given configuration ( $\theta = 45^\circ$ ,  $U = 3000 \text{ cm sec}^{-1}$ ,  $R = 0.05 \text{ cm}$ ).



Fig. 3 shows the maximum and minimum drop diameters (which occur at  $\phi = 0$  and  $\phi = \pi$  respectively) vs the impingement angle  $\theta$ , for several liquids. Notice that, for increasing  $\theta$ , the droplet size spread decreases (becoming zero in the limit of  $\theta = \pi/2$ , ie, for perfectly opposing jets, when the elliptical sheet turns into a circular one and droplets of the same size are formed and uniformly distributed around the jets axis).

However it is known that, in order to avoid unwanted combustion instabilities, burner designers try to achieve a uniform spray while having as little rearward flying droplets as possible. Therefore a compromise has to be found between these two conflicting factors.

Finally this modelling helps to estimate the size range and distribution of droplets formed by impinging jets in a quiescent atmosphere.

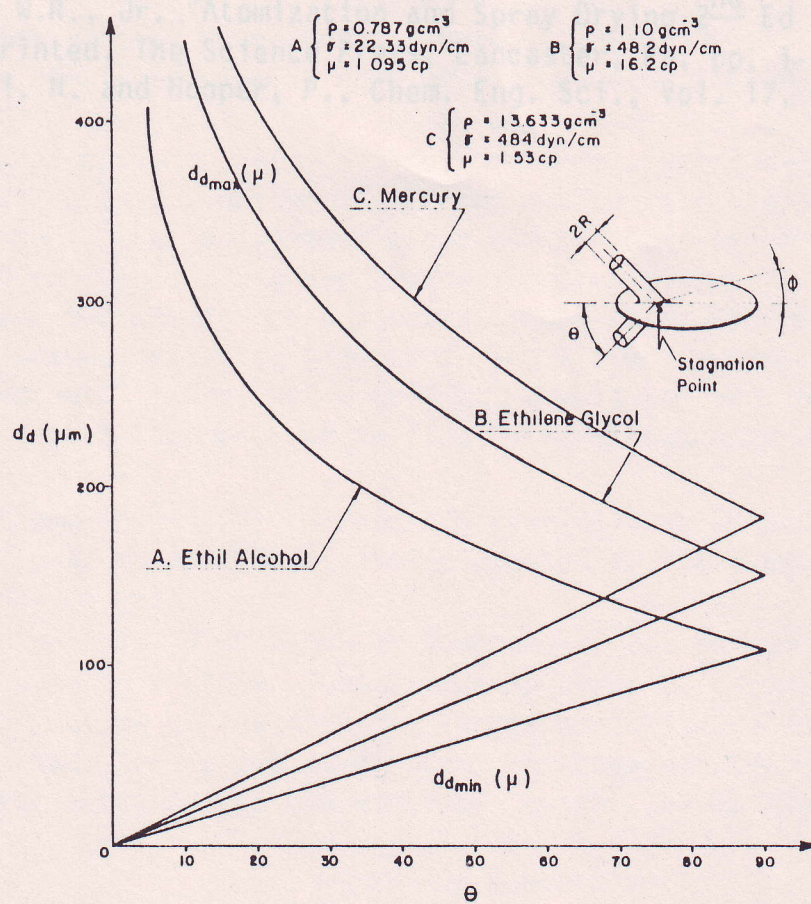


Fig. 3. Droplet size spread vs impingement angle. ( $U = 3000 \text{ cm sec}^{-1}$ ,  $R = 0.05 \text{ cm}$ , Air at  $25^\circ \text{C}$ ,  $t_A = t_B = t_C = 25^\circ \text{C}$ ).



## References

<sup>1</sup>Squire, H.B. "Investigation of the Instability of a Moving Liquid Film". British J. of Applied Physics, Vol. 4, June 1953, pp. 167-169.

<sup>2</sup>Taylor, G.I. "Formation of Thin Flat Sheets of Water". Proc. Roy. Soc. (London), Vol. 259A, Nov. 1960, pp. 01-17.

<sup>3</sup>Dombrowski, N. and Johns, W.R. "The Aerodynamic Instability and Desintegration of Viscous Liquid Sheets", Chemical Engineering Science, Vol. 18, 1963, pp. 203-214.

<sup>4</sup>Hasson, D. and Peck, R.E. "Thickness Distribution in a Sheet Formed by Impinging Jets", A.I. Ch. E Journal, Sept. 1964, pp. 752-754.

<sup>5</sup>Levich, V.G. "Physicochemical Hydrodynamics, Prentice Hall inc, Englewood Cliffs, N.J. (1962), chapter XI.

<sup>6</sup>Marshall, W.R., Jr. "Atomization and Spray Drying 2<sup>nd</sup> Ed AICHE, 1954, Lithoprinted, The Science Press, Lancaster, Pa, pp. 1-122.

<sup>7</sup>Dombrowski, N. and Hooper, P., Chem. Eng. Sci., Vol. 17, 1962, p. 291.

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