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THE EVALUATION OF 3-D INTERPOLATORS FOR DIGITAL ELEVATION MODELS

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THE EVALUATION OF 3-D INTERPOLATORS FOR DIGITAL ELEVATION MODELS

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ABSTRACT

Many times it is necesary to interpolate terrain data into a Digital Elevation Model, DEM, but due to many factors, as for example the use of a not proper interpolator, the result bears no resemblance with the reality. However it is not easy to select the proper interpolator for each case, and estimate its associated errors, especially if the intermediate exact terrain values between the data knots are not known. The objective of this work is to devise a method for the interpolator Beta-spline, in a way that , if its properties, and its input data are known, an estimate of the errors could be calculated. The Beta-splines are suitable surface interpolators, since it is possible to control its tension and bias, to keep the interpolated surface close to the real surface data, also with the desirable shape, by means of the control parameters Beta1 and Beta2. It will be considered a known terrain, and based on the properties of the Beta-splines, the deviations will be evaluated. The algorithm is relatively simple, so the computer code can be made to run fast, even in small machines. An example, with real data, is presented. The calculations were made on an IBM-PC-XT compatible computer, and on a VAX 750. The final results are in graphical form.

INTRODUCTION

When building a Digital Elevation Model, DEM, the ideal situation would be to store the minimum number of data points, from these points to interpolate the model for a finer resolution, and obtain an error free DEM (McCullagh, 1988). Of course, this situation never occurs, but the need for interpolation exists in many cases, especially when satellite images are used.

Many geographic informtion systems, GIS, have built in interpolators, as the Akima interpolator (Akima,1978), that the users freely apply to their models. If care is not taken, the results may be catastrophic, the rendered terrain bearing no resemblance with the real terrain it should model. It is not possible to interpolate without enough knowledge and/or data from the region under study (McCullagh, 1988).

In this work a specific interpolator was evaluated, the Beta-splines (Barsky, 1987), with real terrain data in order to evaluate the its performance.

The elevation data was acquired from contour lines, obtained from an aerial photograph, by standard restitution. The contour lines could have been acquired from satellite data as well. The methodology is described in the next session. The results are presented in graphical form by means of the original data, the interpolated data, and the difference surface, which depicts the difference between the real terrain data and the interpolated values. Conclusions can be drawn, by visualizing the resulting difference between the "true" data and the interpolated surface, for selected sets of the interpolator parameters.

METHODOLOGY

The surface interpolator Beta-spline was chosen due to its main characteristic that allows one to change the shape of the surface without changing the interpolator control points, through the shape parameters beta1, that controls the bias, and beta2, tha controls the tension (Barsky, 1987). Also it is possible to test different settings with the same interpolator, just changing its parameters. Of course, other interpolators could have been used.

From contour lines, obtained by digital restitution, two regular grids were generated: one with 100 meter spacing, and the other with 10 meter spacing. This last one was considered "true" data. The interpolation was made from the 100 meter spacing grid.

When the beta1 and beta2 parameters are changed, the grid spacing is not regular any more, except for beta1 = 1, and beta2 = 0, that is the case in which the beta-spline becomes the B-spline (Barsky, 1987).

In order to produce the error surface, or difference surface, the difference between the "true" surface and the interpolated surface, a new grid for the beta-spline interpolated surface was generated on a VAX computer, using Intergraph software.

The error evaluation itself was computed by averaging the error of reconstruction, e(x,y)

$$e(x,y) = f(x,y) - f'(x,y) \tag{1}$$

where f(x,y) is the real value, and f'(x,y) the interpolated value, at each grid point. For each pair of beta parameters it was computed the e(x,y) surface, the difference surface, the mathematical expectation, or average, and also its standard deviation (Li, 1988). This procedure was made by means of the software STATIGRAPH, running on a IBM-PC-XT-like computer.

The reconstruction of f(x,y) can be done by means of spectral analisys, which is not the case here, since the terrain is not a periodic surface. Zhilin Li (1988) considers several possibilities for digital terrain model accuracy, that deserve further future tests, and concludes that the most comprehensive and best DEM accuracy measure is the Accuracy, Ac, described by him as

$$Ac = u \pm s \tag{2}$$

where u is the mathematical expectation, and s the standard deviation.

RESULTS

Figure 1 presents the terrain contour lines, while Figure 2 shows its three dimensional view, with a grid spacing of 100 meters, and Figure 3 its three dimensional view with 10 meter spacing. Figure 3 is considered the "true" terrain. These results were obtained in a VAX 750 computer, with Intergraph software, as mentioned above.

On the original terrain, it can be seen from bottom to top: a ridge, a river, with some of its tributaries, and a plateau, near the top. Several pair of beta parameters were used, as seen on Table 1, that presents the averages and standard deviations, computed by the STATIGRAPH software on a IBM-PC-XT-like. The sample size was approximately 3000 points for all cases. The exact number for each case is seen in Table 1.

TABLE 1

beta1	beta2	sample size	average (mm)	standard deviation (mm)
1	0	3111	2369	7803
2	0	2900	1864	7531
4	0	3060	1271	7150
8	0	3000	1121	8702
1	5	3060	2427	7567
1	25	3060	2411	7587
1	100	3060	2403	7720
1	10	3060	2420	7539
1e-2	10	3000	2607	7637
1e-7	10	3000	2715	8487
1	-5	3060	2471	9118

Figures 4, and 5, are the beta-spline interpolation, beta1 = 1 (no bias), beta2 = 0 (no tension), or B-spline, and its difference surface, respectively. As seen from Table 1, this result has an average deviation of 2369 mm, and a standard deviation of 7803 mm. Figure 5 is flatter than the original terrain, but the ideal would be an horizontal plane. Figure 4, by visual inspection, is very different from the original terrain.

Still, from Table 1, it is noticed that the best results, as far as average is the case for beta1 = 8, and beta2 = 0, which means no tension and a moderate bias. Figures 6 shows the interpolated surface, and Figure 7 shows the same surface with a regular grid. As far as the standard deviation, the best case is for beta1 = 4, and beta2 = 0, which means again no tension and a slightly less moderate bias. Figures 8 and 9 present the interpolated surface with and without regular grids. The Figures 10 and 11 are the difference surfaces for the above cases, beta1 = 8 beta2 = 0, and beta1 = 4 beta2 = 0,repectively.

For beta1 = 1 and beta2 = 100, a linear interpolation case is recovered, in an inefficient way. However it can be seen by its average of 2403 mm and its standard deviation of 7710 mm that the linear case is not a good fit.

CONCLUSIONS

The beta-spline interpolator still does not represents the terrain as the "true" data. However there is room for improvement, since the same betas were used on all the area, but one could have used a different beta for each major known feature. For example, a pair of betas for the river, a different pair for the plateau, another for the ridge, etc. This procedure, that is more computer time consuming, will be pursued in a future work.

Visually the best results agree with the smaller averages and smaller standard deviations. The measures analised by Li (1988) should also be studied in future work, and the results plotted, in order to check the mathematical accuracy with the visual accuracy, for a wide variety of terrain types.

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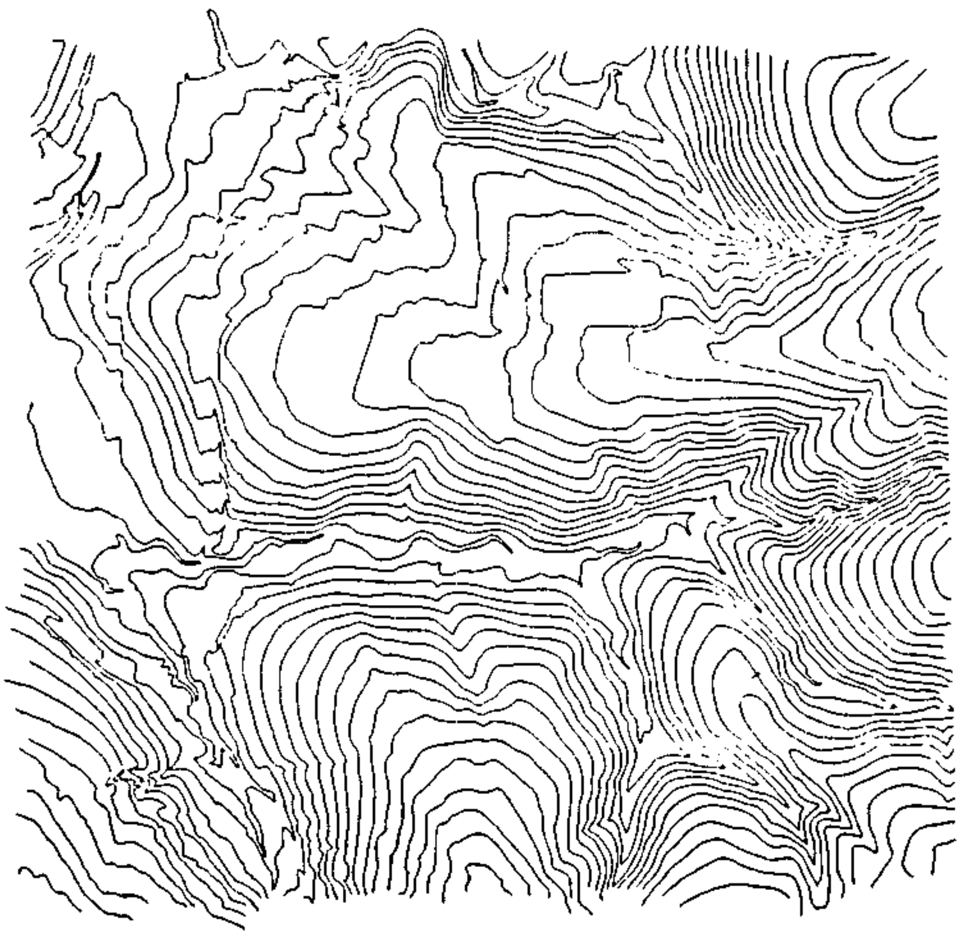

Fig. 1 - Contour line map of the test area.

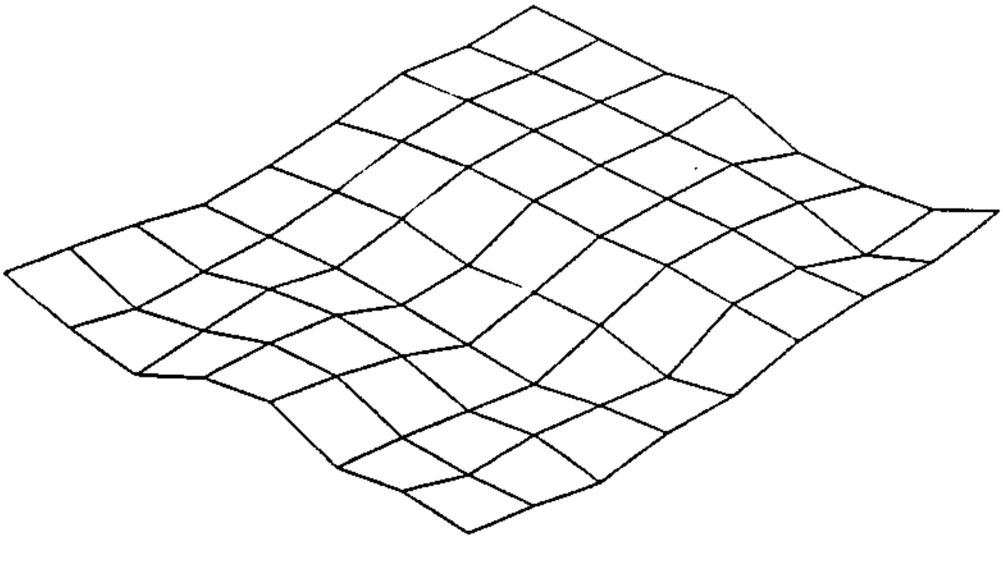

Fig. 2 - DEM of the area, with 100 meter spacing.

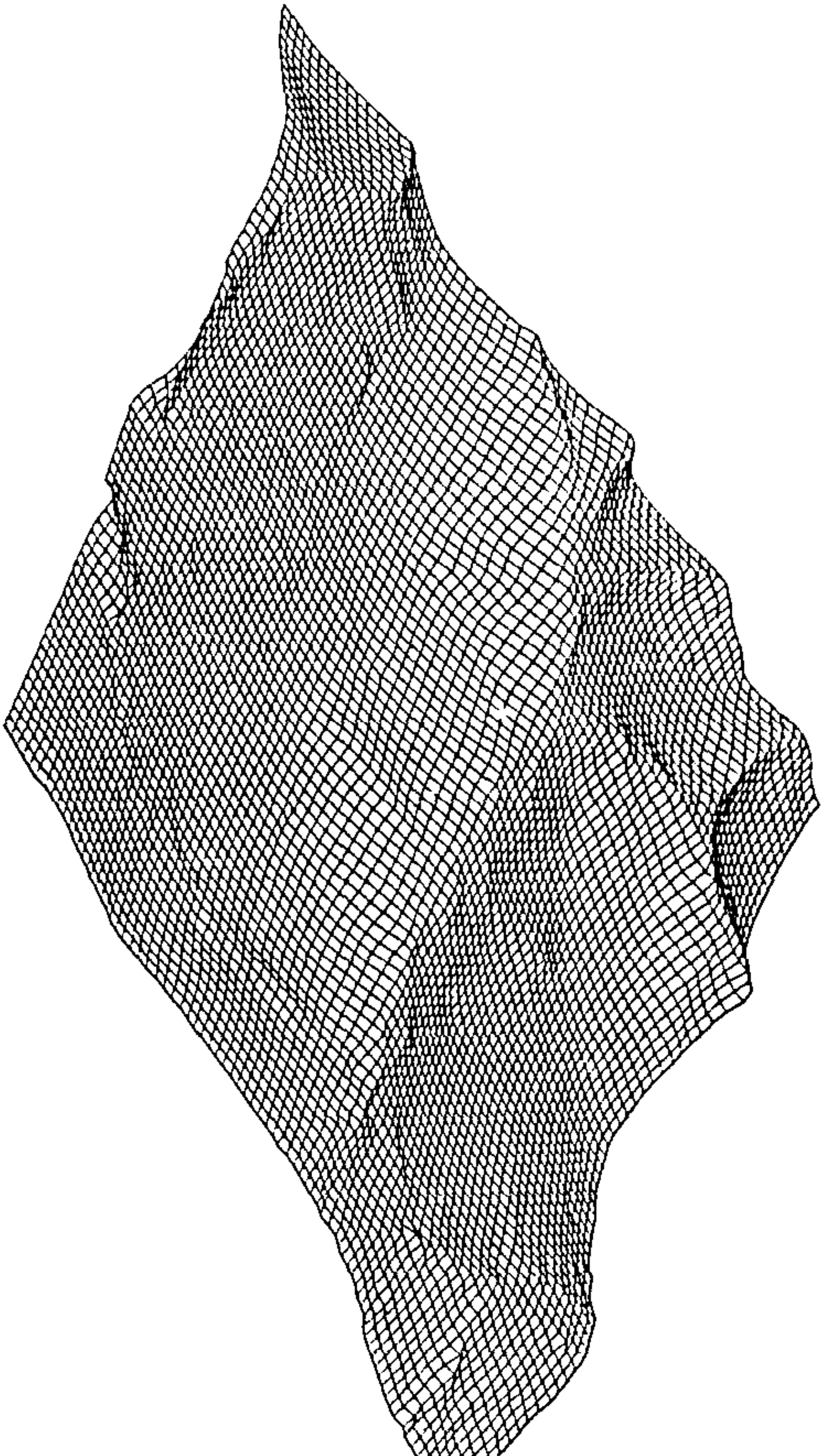

Fig. 3 - DEM of the test area, with 10 meter spacing.
"True" terrain data.

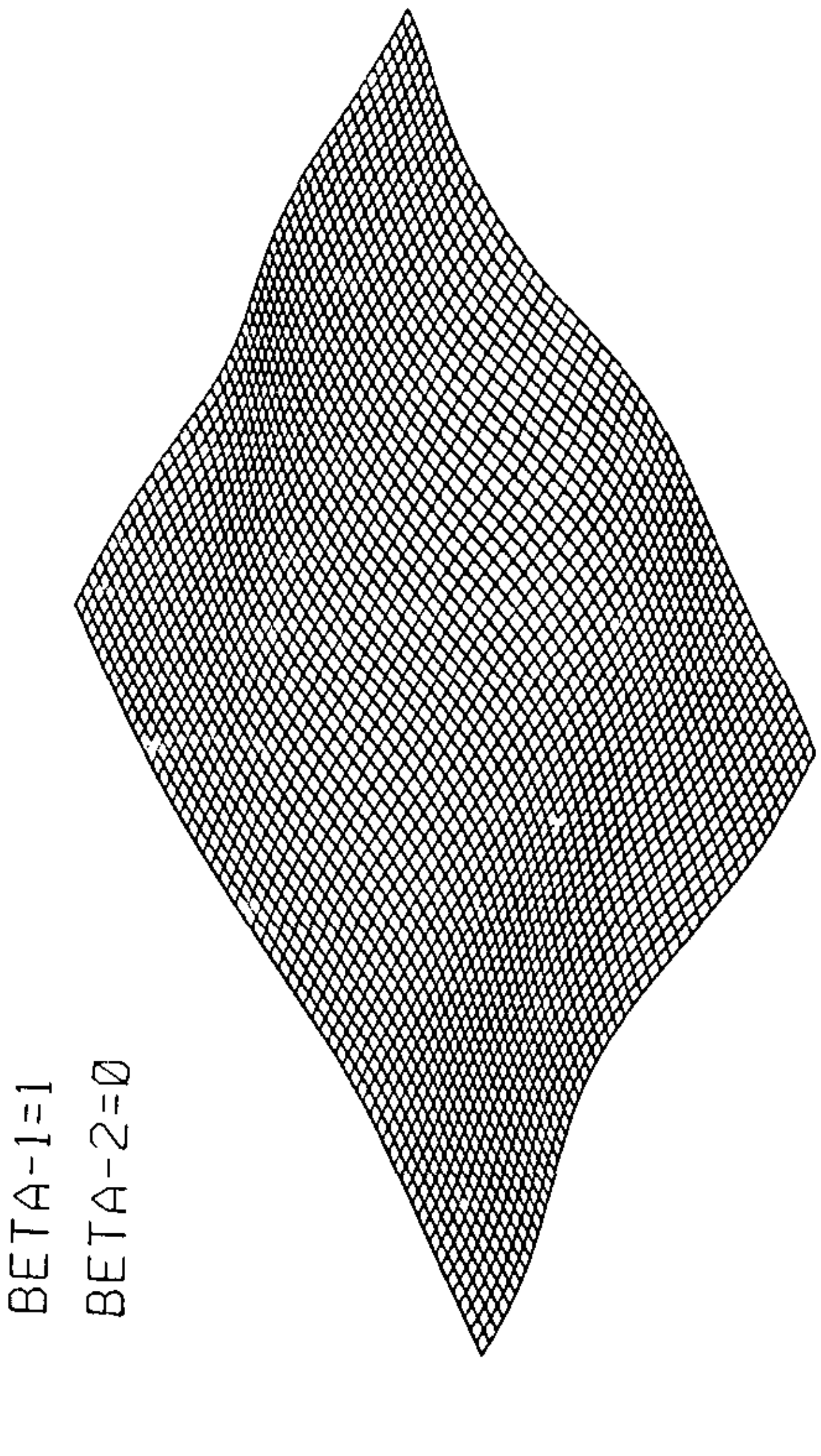

Fig. 4 - Beta - spline interpolated surface: beta1=1, beta2=0, or B-spline interpolation.

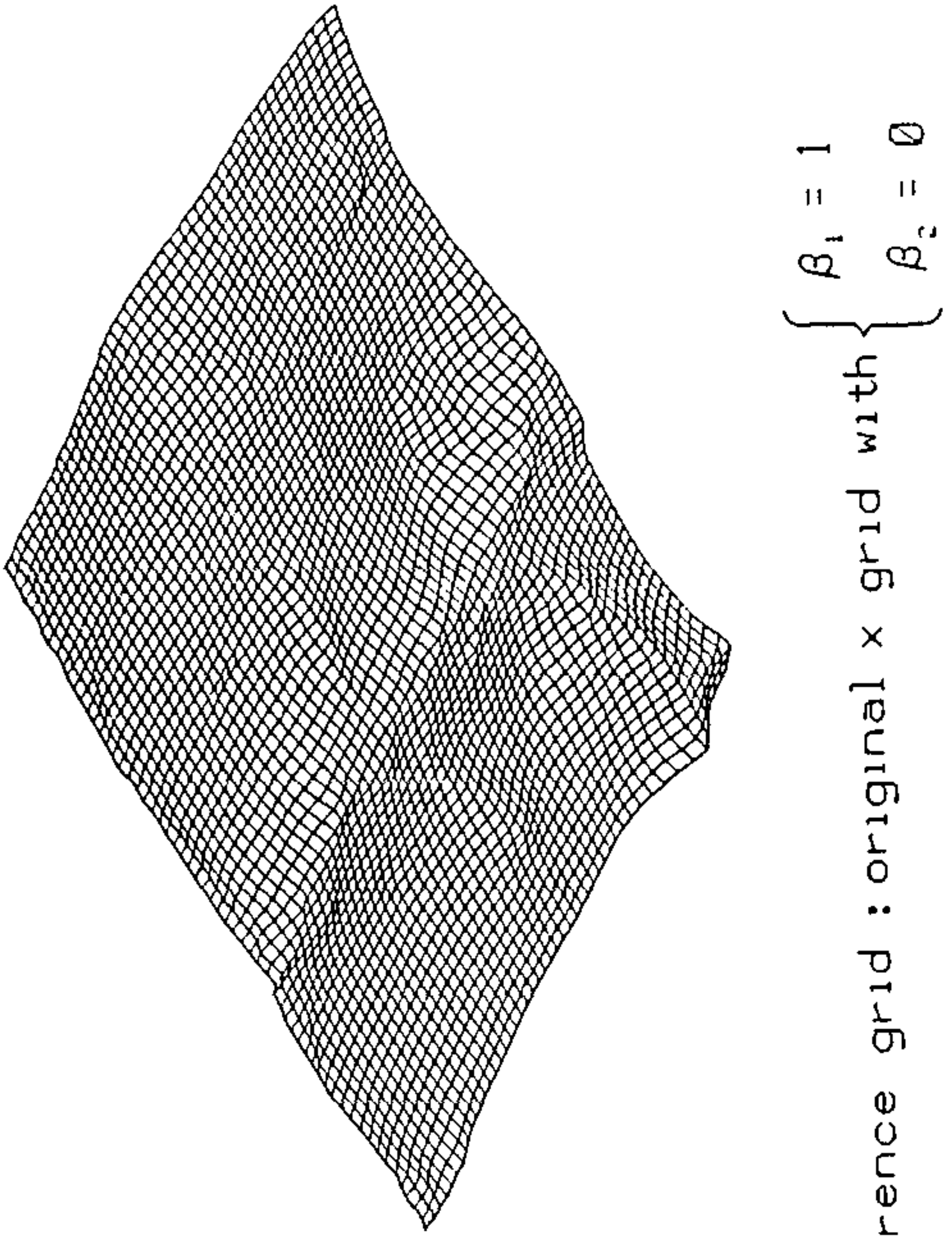

Fig. 5 - Difference surface for B-spline interpolation.
Average = 2369 mm, standard deviation = 7803 mm.

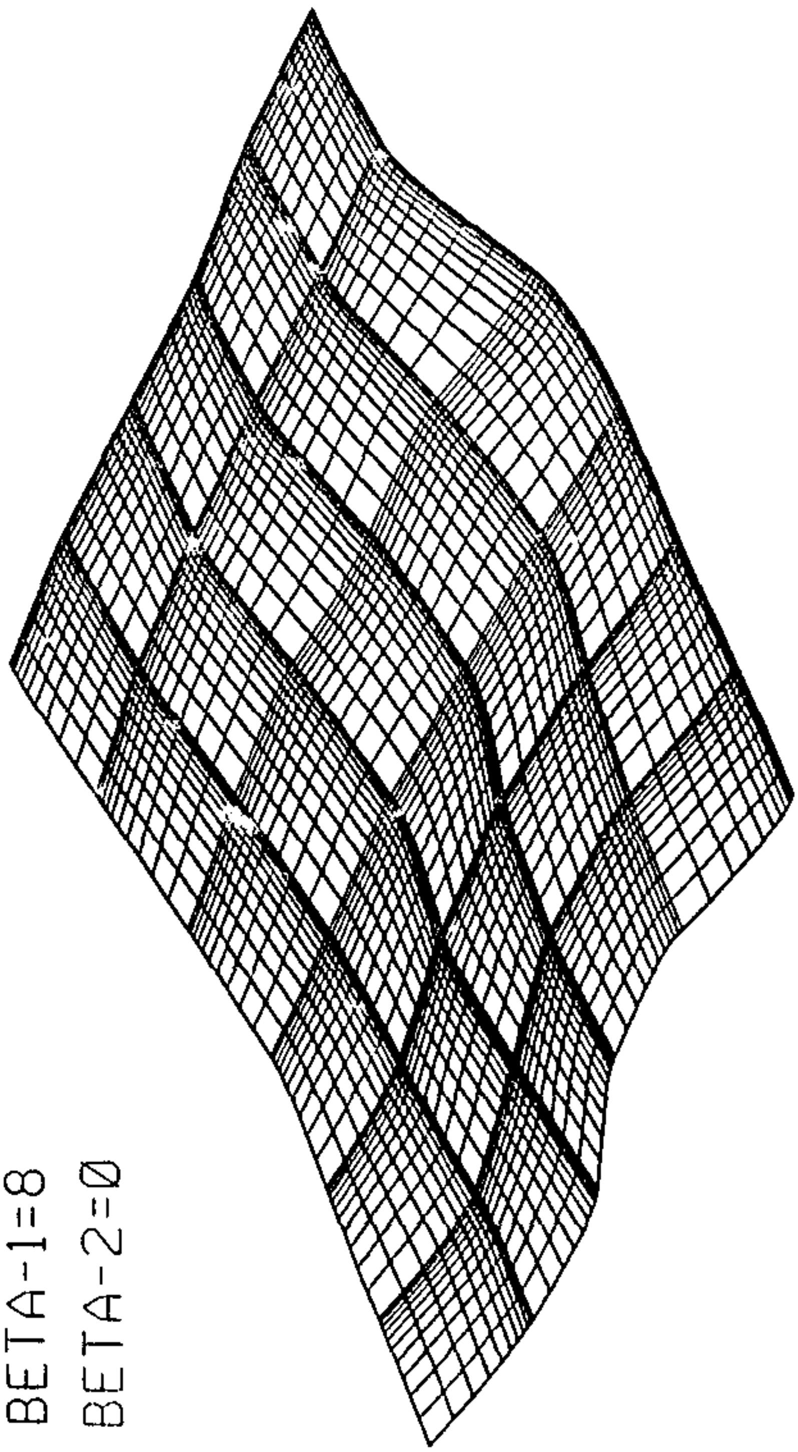

Fig. 6 - Beta-spline interpolated surface, beta1=8, beta2=0.

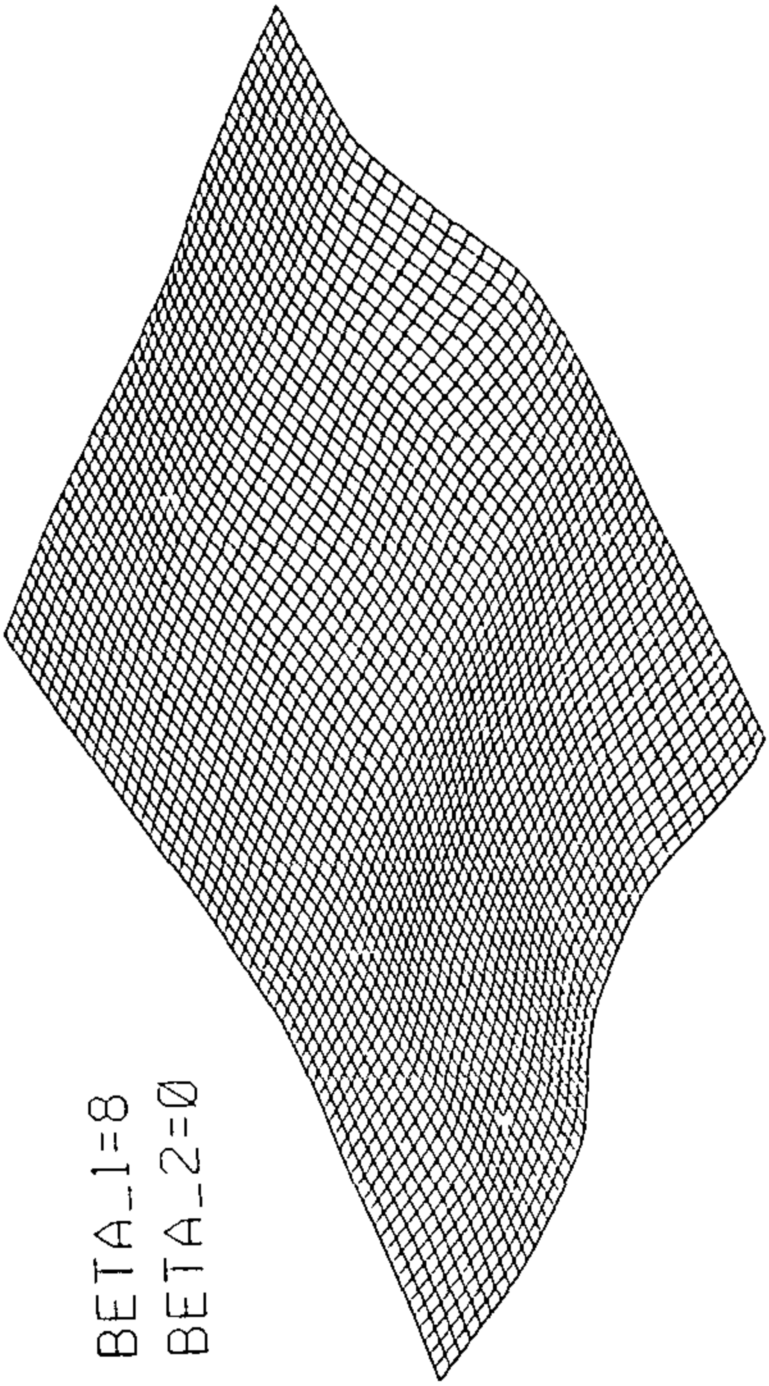

Fig. 7 - Beta-spline interpolated surface, regularized grid.
Beta1 = 8, beta2=0.

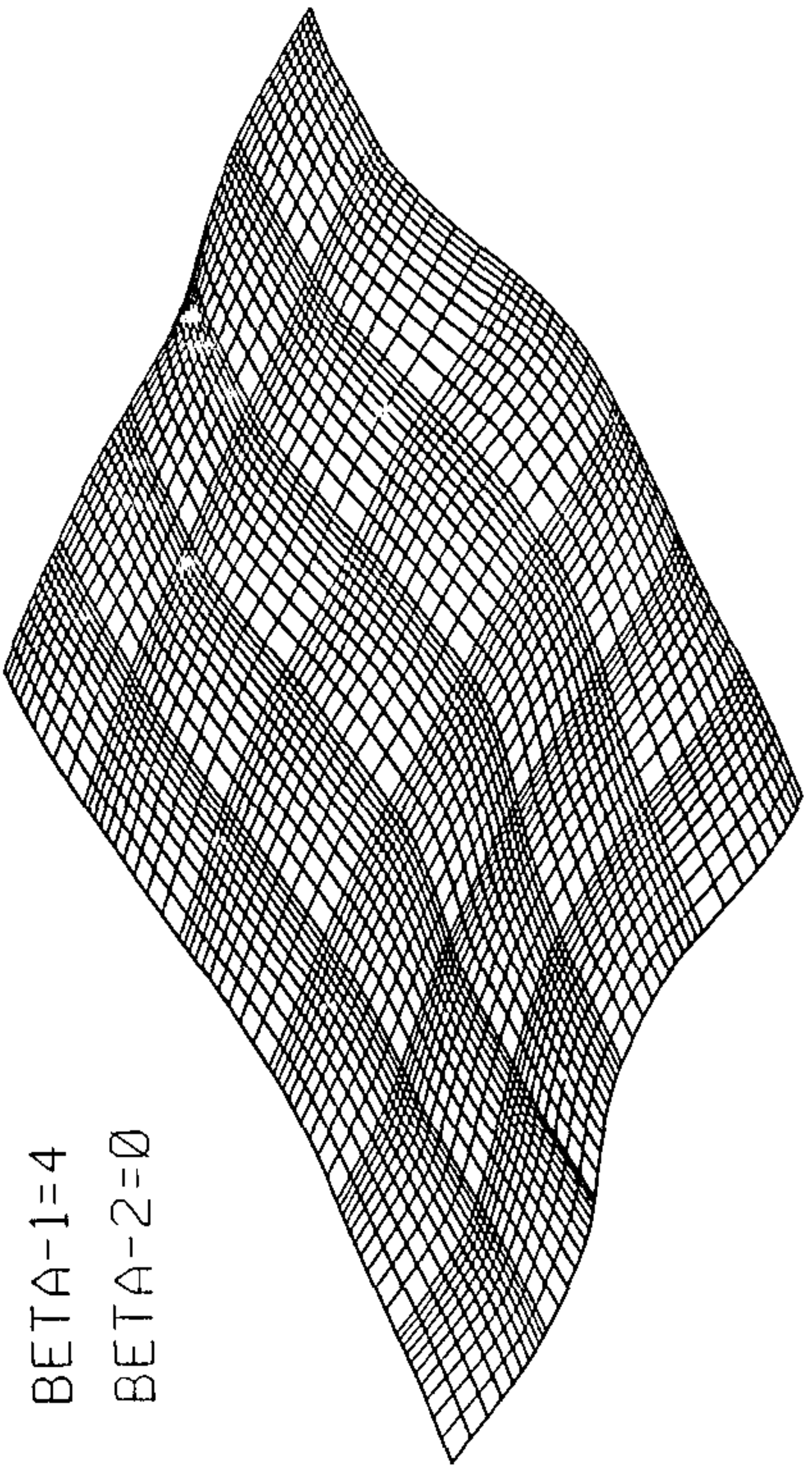

Fig. 8 - Beta-spline interoplated surface, beta1=4, beta2=0.

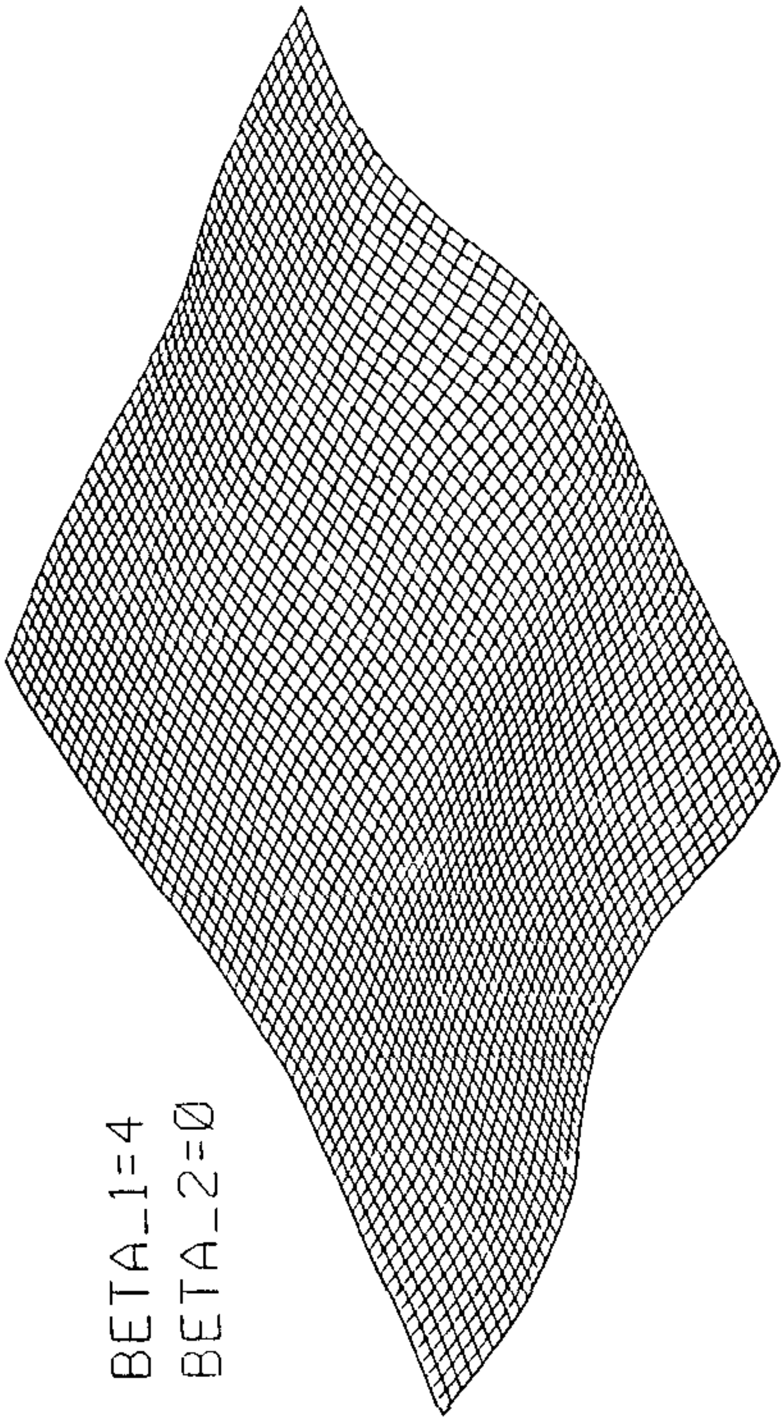

Fig. 9 - Beta-spline interpolated surface, regularized grid.
Beta1 = 4, beta 2 = 0.

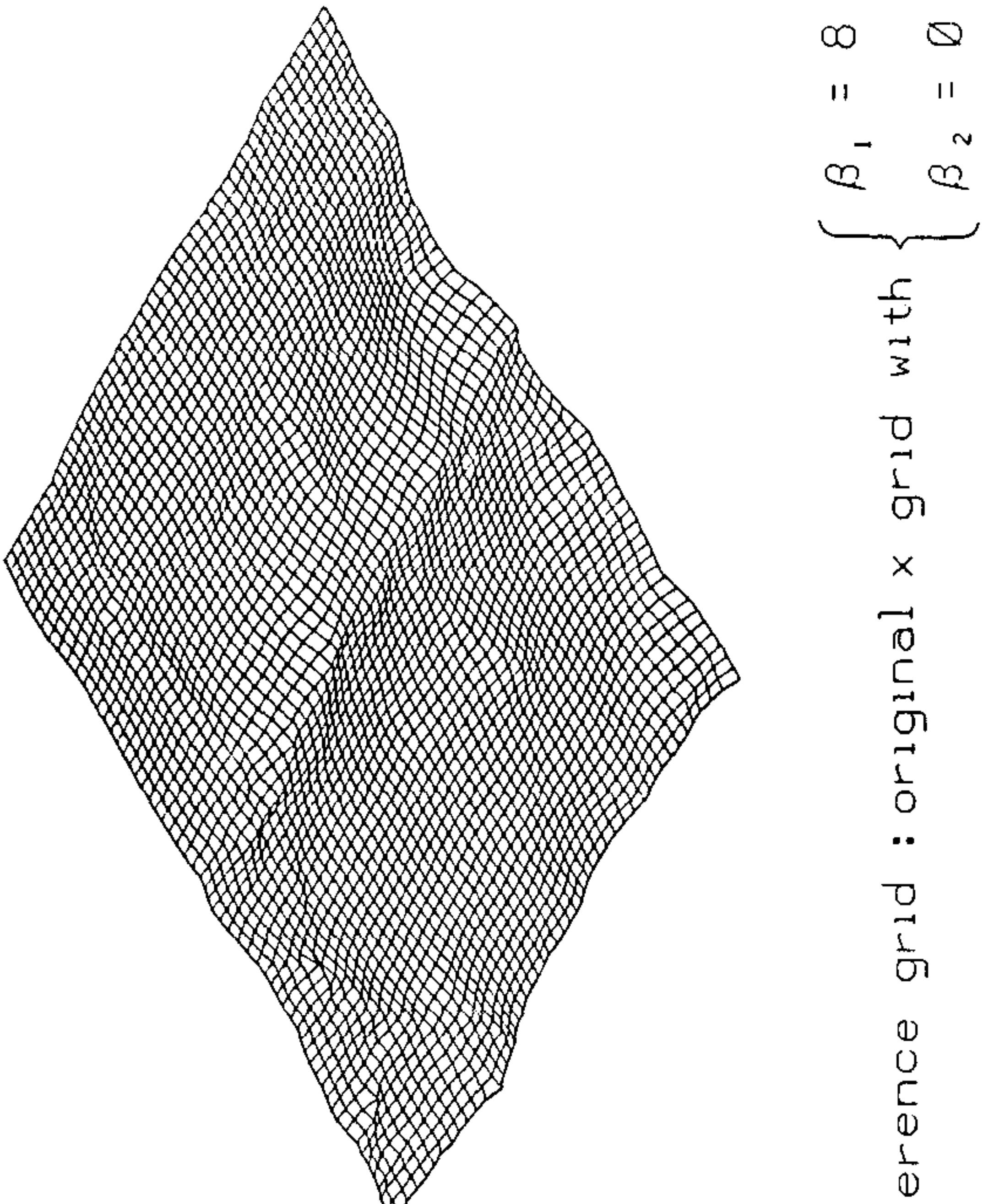

Fig. 10 - Difference surface. Beta1=8, beta2=0.
1121 mm, standard deviation = 8702mm.

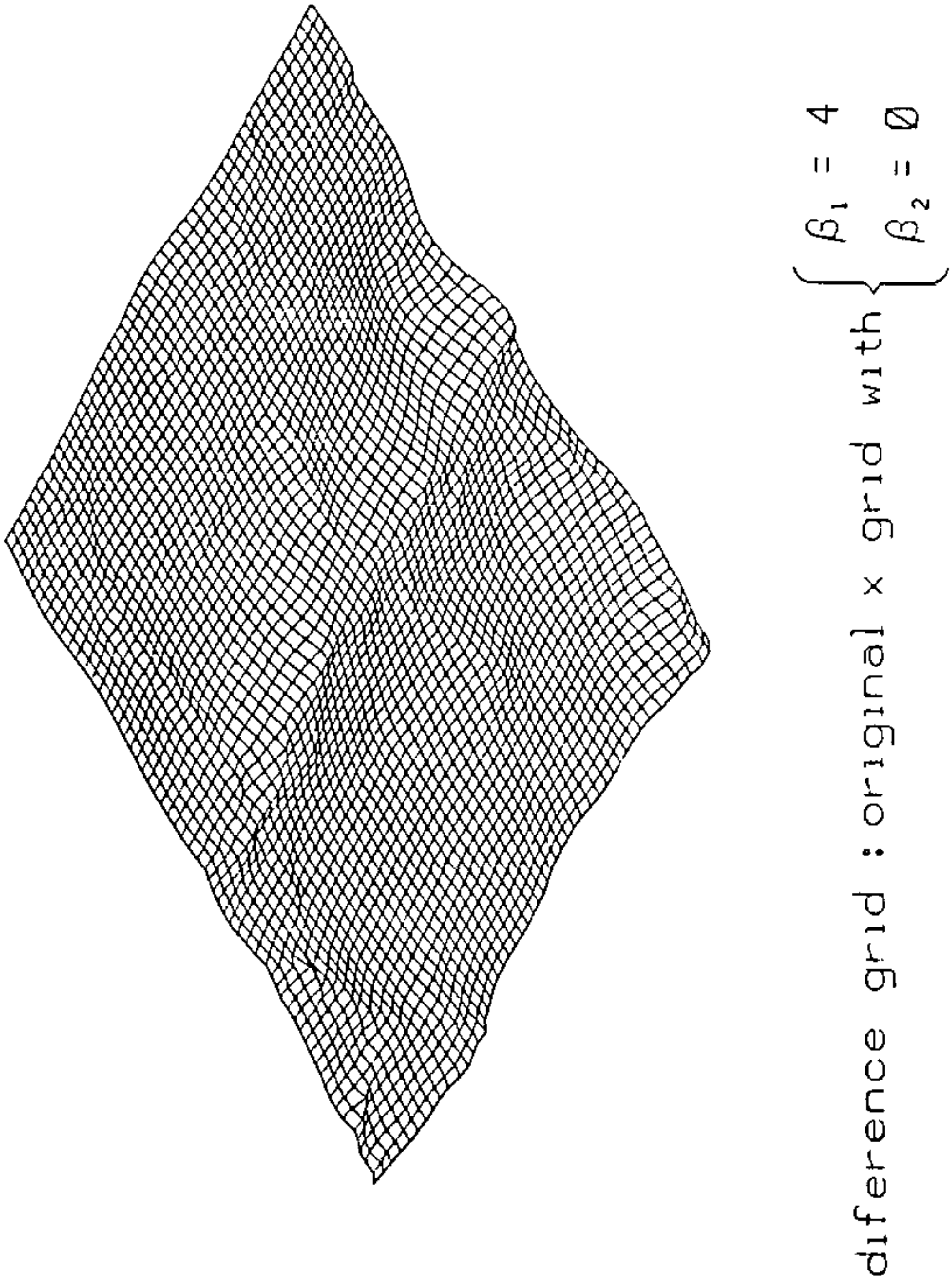

Fig. 11 - Difference surface. Beta1= 4, beta2=0.
Average = 1201 mm, standard deviation=7250mm.

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TÍTULO
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