Nonlinear response scaling of the two-dimensional *XY* spin glass in the Coulomb-gas representation

Enzo Granato
Laboratório Associado de Sensores e Materiais, Instituto Nacional de Pesquisas Espaciais, 12201 São José dos Campos, São Paulo, Brazil

(Received 7 April 1999)

Vortex critical dynamics of the two-dimensional *XY* spin glass is studied by Monte Carlo methods in the Coulomb-gas representation. A scaling analysis of the nonlinear response is used to calculate the correlation length exponent ν of the zero-temperature glass transition. The estimate, $\nu=1.3(2)$, is in agreement with a recent estimate in the phase representation using the same analysis and indicates that the relevant length scale for vortex motion is set by the spin-glass correlation length and that spin and chiralities may order with different correlation length exponents.

It is well known that vector spin glasses, such as the *XY* spin glass, have a chirality order parameter with Ising-like symmetry in addition to the continuous degeneracy associated with global spin rotation.[1] Chirality arises from quenched in vortices due to frustration effects in each elementary cell of the lattice which contains an odd number of antiferromagnetic bonds. The interplay between spin and chiral variables has always received considerable attention because of the possibility of separate spin-glass and chiral-glass ordering due to the freezing of spins and chiral variables, respectively.[2–6] The possible separation of spin and chiral variables also arises in the frustrated *XY* model with weak disorder.[7,8] While in three dimensions the existence of a finite-temperature transition is under current investigation,[9–11] in two dimensions there is a consensus that the transition only occurs at zero temperature. Associated with the zero-temperature transition there is a correlation length which increases with decreasing temperatures as $\xi \propto T^{-\nu}$. However, the possibility of different spin- and chiral-glass short-range correlation lengths ξ_s and ξ_c with different critical exponents ν_s and ν_c, has not been resolved satisfactorily.

The first evidence that spin- and chiral-glass correlation length exponents are different in two dimensions were reported by Kawamura and Tanemura[2] from domain-wall calculations. Various estimates of these exponents give approximate values[2–5] of $\nu_s\sim 1$ and $\nu_c\sim 2$ but the errorbars are usually quite large and a single exponent scenario may not be ruled out which would be consistent with an analytical work for a particular type of disorder distribution[6] and more recent numerical work[12] on domain-wall scaling behavior at zero temperature. These calculations are usually performed in a representation of the *XY* spin-glass model in terms of the orientational angle of the two-component *XY* spins. In this representation, spin-glass order can be directly identified as a long-range order in appropriate phase correlations, while the chiral variables are built from nearest-neighbor phase correlations. In numerical simulations, the dynamics of the chiral variables are then determined by the phase variables and equilibration problems may prevent an adequate study of the vortex correlations in the system.

An alternative approach which allows us to study the vortex dynamics directly can be obtained from the Coulomb-gas representation.[13] Recently, Bokil and Young[4] used Monte Carlo simulations in the vortex representation to obtain an estimate of the chiral-glass correlation length exponent and found $\nu_c=1.8\pm 0.3$. This agrees with previous estimates within the errorbar. It also supports the scenario in which spin- and chiral-glass variables order with different critical exponents if one accepts earlier estimates of the spin-glass exponent $\nu_c\sim 1$, obtained in the phase representation. However, a determination of the spin-glass correlation length exponent from simulations in the vortex representation should be of interest as it is not completely clear how such a length scale shows up in the dynamical behavior of vortices. In particular, since the *XY* spin-glass model has currently been used as a model for granular high-T_c superconductors containing π junctions,[9,14,15] which leads to quenched in vortices even in the absence of external magnetic field, a natural question arises as to which correlation length, ξ_s or ξ_c, is actually probed by transport measurements. In the measurements, the response of the vortices to an applied force can be observed as the voltage response to an applied driving current which acts as a Lorentz force on the vortices.[13,16] The vortex response, or resistive behavior, is therefore determined by vortex mobility and the current-voltage scaling is expected to be controlled by the relevant divergent length scale,[13] which could be either ξ_s or ξ_c.

The question of the relevant correlation length for the vortex response is also of interest for the three-dimensional *XY* spin glass. Recent simulations of the vortex dynamics[10] in three dimensions showed evidence of a resistive phase transition at finite temperatures which was attributed to glass ordering of chiralities while the spins remain disordered. This would be consistent with the scenario of a finite-temperature chiral glass transition in the absence of a spin-glass transition which has been proposed previously[9] from calculations in the phase representation of the *XY* spin-glass model. This interpretation, however, is only justified if the relevant length scale for vortex dynamics is determined by the chiral-glass correlation length ξ_c. On the other hand, since it is well known that vortex motion leads to phase incoherence, one expects that vortex dynamics should probe the spin-glass correlation length ξ_s and therefore, the resistive transition should correspond instead to a spin-glass transition at finite temperatures. In fact, this is supported by a more recent domain-wall calculation suggesting a spin-glass transition at finite temperatures in three dimensions.[11] The present study of the vortex response in two dimensions may help us to clarify this point as it is well known that in this case both spin- and chiral-glass ordering only occurs at zero temperature and so the scaling analysis involves less unknown parameters.

In the absence of a precise agreement among the various studies of the *XY* spin-glass model, as mentioned above, and in view of its relevance for vortex dynamics, the additional numerical results presented below may help to settle some issues.

In this work, we study the vortex critical dynamics of the two-dimensional *XY* spin glass by Monte Carlo methods in the Coulomb-gas representation.[13] A scaling analysis of the nonlinear response is used to calculate the correlation length exponent ν of the zero-temperature glass transition. The estimate, $\nu=1.3(2)$, is in agreement with a recent estimate in the phase representation[17] using the same analysis and indicates that the relevant length scale for vortex motion is set by the spin-glass correlation length ξ_s and that spin and chiralities may order with different correlation length exponents.

We consider the *XY* spin glass on a square lattice defined by the Hamiltonian

$$H=\sum_{\langle ij\rangle} J_{ij} s_i \cdot s_j = -J\sum_{\langle ij\rangle} \cos(\theta_i-\theta_j-A_{ij}), \qquad (1)$$

where θ_i is the phase of a two-component classical spin of unit length, $s_i=(\cos\theta_i,\sin\theta_i)$, $J>0$ is a coupling constant, and A_{ij} has a binary distribution, 0 or π, with equal probability, corresponding to a coupling constant $J_{ij}=-J$ or J, respectively, between s_i and s_j spins. The sum is over all nearest-neighbor pairs. This Hamiltonian also describes an array of Josephson junctions where there is a phase shift of π across a fraction of the junctions as in models of *d*-wave ceramic superconductors.[9,14,15]

To study the vortex dynamics it is convenient to rewrite the above Hamiltonian in the Coulomb-gas representation

$$H_{cg}=2\pi^2 J\sum_{r,r'} (n_r-f_r)G'_{rr'}(n_{r'}-f_{r'}), \qquad (2)$$

which can be obtained following a standard procedure[18] in which Eq. (1) is replaced by a periodic Gaussian model separating spin-wave and vortex variables. The Coulomb interaction is given by $G'_{rr'}=G(r-r')-G(0)$, where G is the lattice Green's function:

$$G(r)=\frac{1}{L^2}\sum_k \frac{\exp(ik\cdot r)}{4-2\cos k_x-2\cos k_y}, \qquad (3)$$

and L is the system size. $G'(r)$ diverges as $2\pi\ln|r|$ at large separations r. The vortices are represented by integer charges n_r at the sites r of the dual lattice and the frustration effects of A_{ij} by quenched random charges f_r given by the directed sum of A_{ij} around the plaquette, $f_r=\Sigma A_{ij}/2\pi$. The charges are constrained by the neutrality condition $\Sigma_r(n_r-f_r)=0$. For the *XY* spin glass, the charges f_r have a correlated random distribution of integer and half integer values. Other random distributions can represent different models. A uniform distribution describes the gauge glass model[13,16] where A_{ij} in Eq. (1) is a continuous variable in the range $[0,2\pi]$ and an uncorrelated continuous distribution can describe arrays of superconducting grains with random flux[7] or arrays of mesoscopic metallic grains with random offset charges.[19]

We study the nonequlibrium response of the vortices in the *XY* spin glass by Monte Carlo simulations of the Coulomb gas under an applied electric field.[13] An electric field E represents an applied force acting on the vortices and gives an additional contribution to the energy in Eq. (2) as $\Sigma_r E n_r x_r$ for E in the x direction. A finite E sets an additional length scale[13] in the problem since thermal fluctuations alone, of typical energy kT, leads to a characteristic length $l\sim kT/E$ over which single charge motion is possible. Thus, increasing E will probe smaller length scales. Crossover effects are then expected when l is of the order of the relevant correlation length for independent charge motion. As vortex motion leads to phase incoherence we thus expect that the scaling behavior of the nonlinear response will probe the spin-glass correlation length ξ_s of the original model and allow an estimate of the thermal critical exponent ν_s. This dynamical approach complements previous equilibrium calculations in the vortex representation of the *XY* spin glass where only the chiral-glass correlation length was studied.[4]

In the dynamical simulations, the Monte Carlo time is identified as the real time t and we take the unit of time $dt=1$ corresponding to a complete Monte Carlo pass through the lattice. A Monte Carlo step consists of adding a dipole of unit charges and unit length to a nearest-neighbor charge pair (n_i,n_j), using the Metropolis algorithm. Choosing a nearest-neighbor pair i,j at random, the step consists of changing $n_i\rightarrow n_i-1$ and $n_j\rightarrow n_j+1$, corresponding to the motion of a charge by a unit length from i to j. If the change in energy is ΔU, the move is accepted with probability $\min\{1,\exp(-\Delta U/kT)\}$. The external electric field E biases the added dipole, leading to a current I as the net flow of charges in the direction of the electric field if the charges are mobile. The current I is calculated as

$$I(t)=\frac{1}{L}\sum_i \Delta Q_i(t) \qquad (4)$$

after each Monte Carlo pass through the lattice, where L is the lattice size and $\Delta Q_i(t)=1$ if a charge at site i moves one lattice spacing in the direction of the field E at time t, $\Delta Q_i(t)=-1$ if it moves in the opposite direction and $\Delta Q_i(t)=0$ otherwise. Periodic boundary conditions are used. Most calculations were performed for $L=32$ and compared to a smaller system of $L=16$ but size dependence was not significant in the temperature range studied. The current density J is defined as $J=I/L$. The linear response is given by the linear conductance $G_L=\lim_{E\rightarrow 0} J/E$ which can be obtained from the fluctuation-dissipation relation as

$$G_L=\frac{1}{2kT}\int dt\langle I(0)I(t)\rangle \qquad (5)$$

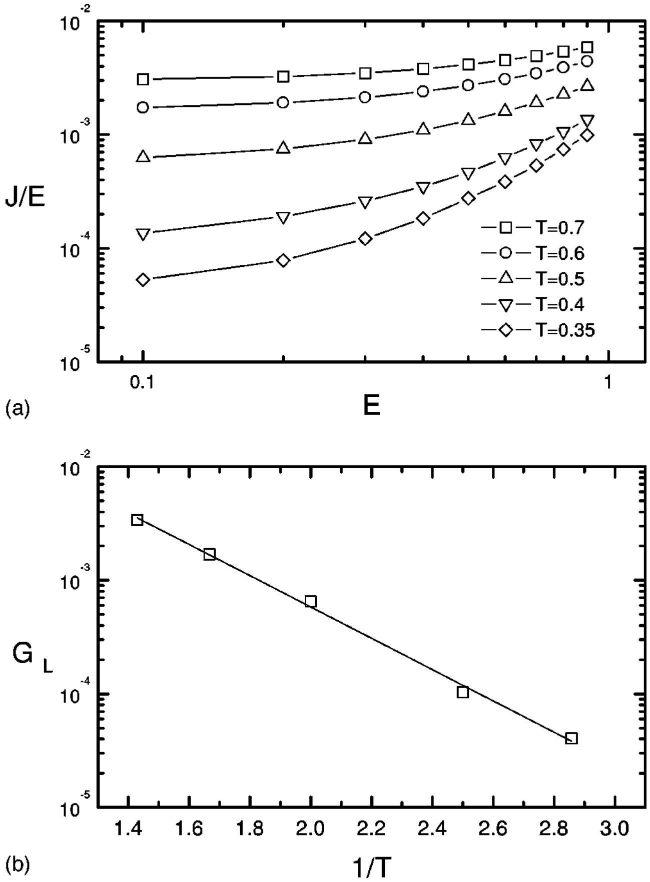

FIG. 1. (a) Nonlinear conductance J/E as a function of temperature T. (b) Arrhenius plot for the temperature dependence of the linear conductance G_L.

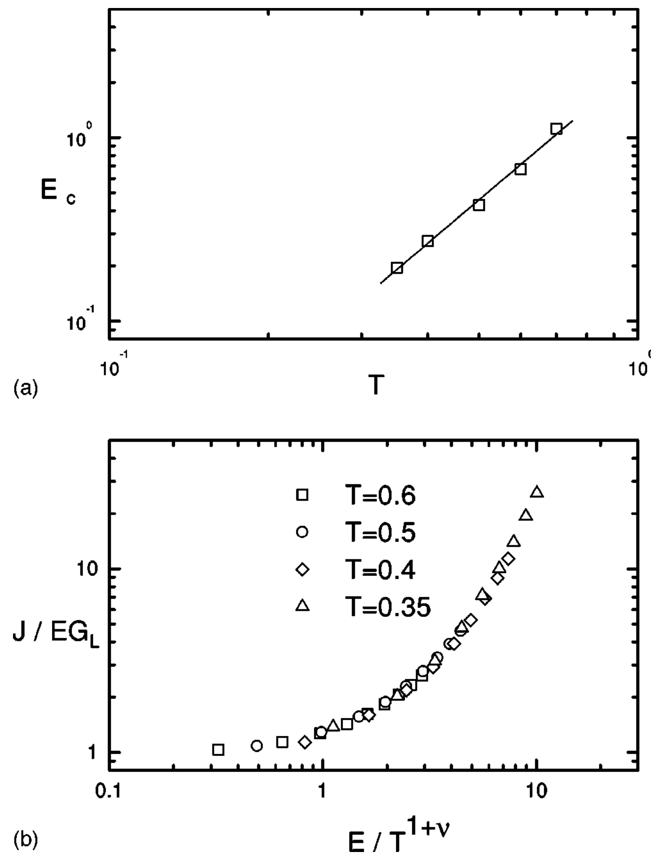

FIG. 2. (a) Crossover field E_c as a function of temperature. (b) Scaling plot $J/EG_L \times E/T^{1+\nu}$ for $\nu = 1.3$.

without imposing the external field E. In the calculations, the integral is replaced by a sum of successive Monte Carlo sweeps through the lattice with the unit of time $dt = 1$. We use typically 4×10^4 Monte Carlo steps to compute averages and 20 different realizations of disorder.

To analyze the numerical results we need a scaling theory of the nonlinear response near a second-order phase transition. A detailed scaling theory has been described in the context of the current-voltage characteristics of vortex-glass models[13] but it can be directly applied to the present case. Since the glass transition occurs at $T = 0$ with a power-law divergent correlation length $\xi \propto T^{-\nu}$ and the external field introduces an additional length scale $l \propto kT/E$, the dimensionless ratio E/JG_L can be cast into a simple scaling form[13] in terms of the dimensionless argument ξ/l,

$$J/EG_L = F(E/T^{1+\nu}), \tag{6}$$

where F is a scaling function with $F(0) = 1$. This scaling form indicates that a crossover from linear behavior, when $F(x) \sim 1$, to nonlinear behavior, when $F(x) \gg 1$, is expected to occur when $x \sim 1$ which leads to a characteristic field $E_c \propto T^{1+\nu}$ at which nonlinear behavior sets in.

The nonlinear response J/E and an Arrhenius plot of the linear conductance G_L are shown in Fig. 1. The data show the expected behavior for a $T = 0$ transition. The ratio J/E in Fig. 1(a) tends to a finite value for small E, corresponding to the linear conductance G_L in Fig. 1(b) with an activated behavior. This activated behavior is consistent with a zero-temperature transition and finite correlation length at nonzero temperatures which leads to a finite energy barrier U for vortex motion. In general, an energy barrier exponent[16] ψ can also be defined from $U \sim \xi^{\psi}$ for a temperature-dependent energy barrier. The pure Arrhenius activated behavior in Fig. 1(b) is consistent with an exponent $\psi \sim 0$. As can be seen from Fig. 1(a), there is a smooth crossover from linear behavior, when J/E is roughly a constant, to nonlinear behavior for increasing E at each temperature which appears at smaller E for decreasing temperatures in agreement with the expected crossover behavior at a characteristic field $E_c \propto T^{1+\nu}$.

We now verify the scaling hypothesis of Eq. (6) and obtain a numerical estimate of the thermal correlation length exponent ν using two different methods. Figure 2(a) shows the temperature dependence of E_c defined as the value of E where E/JG_L starts to deviate from a fixed value of 2. From the expected power-law behavior for the crossover field $E_c \propto T^{1+\nu}$ we obtain a direct estimate of $\nu = 1.4(2)$ in a log-log plot. From a scaling plot of the nonlinear response according to Eq. (6), ν can also be obtained by adjusting its value so that the best data collapse is obtained as shown in Fig. 2(b). The data collapse supports the scaling behavior and provides an independent estimate of $\nu = 1.3$. From the two independent estimates we finally obtain $\nu = 1.35 \pm 0.2$.

Our estimate of $\nu = 1.35 \pm 0.2$ from the scaling analysis of the vortex response is consistent with previous estimates of the spin-glass correlation length exponent ν_s obtained in the phase representation of the XY spin glass.[2,3,5] These calculations give numerical estimates with comparable uncertainties ranging from $\nu_s=1$ to 1.2. It also agrees with a recent calculation in the phase representation of Eq. (1), $\nu=1.1\pm0.2$, using the same scaling analysis.[17] This suggests that the relevant length scale for vortex dynamics is set by the spin-glass correlation length which determines short-range phase coherence. Since the chiral-glass correlation length exponent has been estimated to be significantly larger, in the range[2–4,12] $\nu_c=1.8$ to 2.6, it also supports the scenario in which the phase and chiral variables in the *XY* spin glass are decoupled on large length scales and order with different correlation length exponents. However, as the errorbars of these estimates are significant large, a single critical exponent[6,12] may not be completely ruled out on pure numerical grounds and further work will be necessary to completely settle this issue.

It should be noted that the decoupled scenario for spin and chiral variables near the same transition temperature, suggested by our numerical results, does not contradict general arguments of renormalization-group theory[22] which allows for the possibility of nontrivially decoupled fixed points. In fact, since the model has continuous and Ising-like symmetries, one would expect that the effective Hamiltonian describing the critical behavior could be written in terms of a disordered ferromagnetic *XY*-spin model, with a zero-temperature transition, coupled to an Ising spin-glass model, representing phase coherence and chiral degrees of freedom, respectively. Ferromagnetic *XY* spin models with a zero-temperature transition do exist as, for example, in diluted *XY* models at percolation threshold.[20,21] Although the exact form of the effective *XY* and Ising Hamiltonians and the coupling term are not known, the continuous and discrete symmetries of the model are consistent with an energy density coupling of the form $\Sigma_r E_s(r)E_c(r)$, where E_s and E_c are the local energy densities for the *XY* spins and chirality, respectively. Such a coupling term is known to occur in the effective Hamiltonian of frustrated *XY* models with weak disorder.[7] For a stable decoupled fixed point, the coupling term should be an irrelevant perturbation, corresponding to an eigenvalue $\lambda=2-x<0$ evaluated at the unperturbed fixed point, where $2x$ is the correlation function exponent. Using the scaling relation[22] $x=2-1/\nu$ for the energy density correlations, and the proposed numerical values for the correlation length exponents $\nu_s=1.3$ and $\nu_c=2$, we find indeed that $\lambda=2-x_s-x_c<0$ as required for a stable decoupled fixed point. If the transition at $T=0$ corresponds to a decoupled fixed point then phase and chiral variables can order with different correlation length exponents. These arguments, by no means, show that a decoupled transition is actually realized in the *XY* spin glass but it makes plausible the assumption of distinct divergent correlation lengths at the same transition temperature used in our analysis of the numerical data.

Finally, our calculation for the two-dimensional *XY* spin glass, which indicates that vortex dynamics probe mainly the spin-glass correlation length rather than the chiral-glass correlation length, also suggests that the finite-temperature resistive transition observed recently by Wengel and Young[10] in numerical simulations in the vortex representation of the three-dimensional *XY* spin-glass model should be attributed to spin-glass ordering. This is in fact consistent with more recent calculation[11] indicating that the lower critical dimension for spin-glass ordering may be just above 3.

This work was supported by Fundação de Amparo à Pesquisa do Estado de São Paulo, FAPESP (Proc. 99/02532-0).

[1] J. Villain, J. Phys. C **10**, 4793 (1977).
[2] H. Kawamura and M. Tanemura, Phys. Rev. B **36**, 7177 (1987); J. Phys. Soc. Jpn. **60**, 608 (1991).
[3] P. Ray and M.A. Moore, Phys. Rev. B **45**, 5361 (1992).
[4] H.S. Bokil and A.P. Young, J. Phys. A **26**, L89 (1996).
[5] S. Jain and A.P. Young, J. Phys. C **19**, 3913 (1986).
[6] M. Ney-Nifle and H.J. Hilhorst, Phys. Rev. B **51**, 8357 (1995).
[7] E. Granato and J.M. Kosterlitz, Phys. Rev. Lett. **62**, 823 (1989).
[8] M. Benakli, E. Granato, S.R. Shenoy, and M. Gabay, Phys. Rev. B **57**, 10 314 (1998).
[9] H. Kawamura, J. Phys. Soc. Jpn. **64**, 711 (1995); H. Kawamura and M.S. Li, Phys. Rev. Lett. **78**, 1556 (1997).
[10] C. Wengel and A.P. Young, Phys. Rev. B **56**, 5918 (1997).
[11] J. Maucourt and D.R. Grempel, Phys. Rev. Lett. **80**, 770 (1998).
[12] J.M. Kosterlitz and N. Akino, Phys. Rev. Lett. **82**, 4094 (1999).
[13] R.A. Hyman, M. Wallin, M.P.A. Fisher, S.M. Girvin, and A.P. Young, Phys. Rev. B **51**, 15 304 (1995).
[14] F.V. Kusmartsev, Phys. Rev. Lett. **69**, 2268 (1992).
[15] D. Doninguez, E.A. Jagla, and C.A. Balseiro, Phys. Rev. Lett. **72**, 2773 (1994).
[16] M.P.A. Fisher, T.A. Tokuyasu, and A.P. Young, Phys. Rev. Lett. **66**, 2931 (1991).
[17] E. Granato, Phys. Rev. B **58**, 11 161 (1998).
[18] J.V. José, L.P. Kadanoff, S. Kirpatrick, and D.R. Nelson, Phys. Rev. B **16**, 1217 (1977).
[19] E. Granato and J.M. Kosterlitz, Phys. Rev. Lett. **81**, 3888 (1998).
[20] D. Stauffer and A. Aharony, *Introduction to Percolation Theory* (Taylor & Francis, London, 1992).
[21] E. Granato and D. Domínguez, Phys. Rev. B **56**, 14 671 (1997).
[22] J. Cardy, *Scaling and Renormalization in Statistical Physics* (Cambridge University Press, Cambridge, 1996).