Propagation of electromagnetic waves in the source region of the Auroral Kilometric Radiation in a cold plasma approximation.

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Abstract

In this work we use the magnetoionic theory to describe the propagation of electromagnetic waves inside a plasma cavity that is immersed in a weakly inhomogeneous, stationary magnetic field $B_{\oplus}$.

From the usual expressions for the dielectric tensor and dispersion relation, we derive a set of canonical equations that describe, within the geometrical optics approximation, the time evolution of the wave vector and the ray position inside the plasma cavity. The ray tracing equations thus derived are employed to study the wave propagation and amplification inside the source region of the Auroral Kilometric Radiation (AKR). To this end, we use a specific physical model, derived elsewhere (Gaelzer et al., 1992; Gaelzer et al., 1994), that models a two-dimensional cavity in the auroral region of the magnetosphere, inside of which the AKR is generated by the electron maser mechanism. With this information, we developed a computational code that describes the evolution of an electromagnetic wave spreading inside the source region of the AKR. A comparison with other works found in the literature is presented.

Introduction

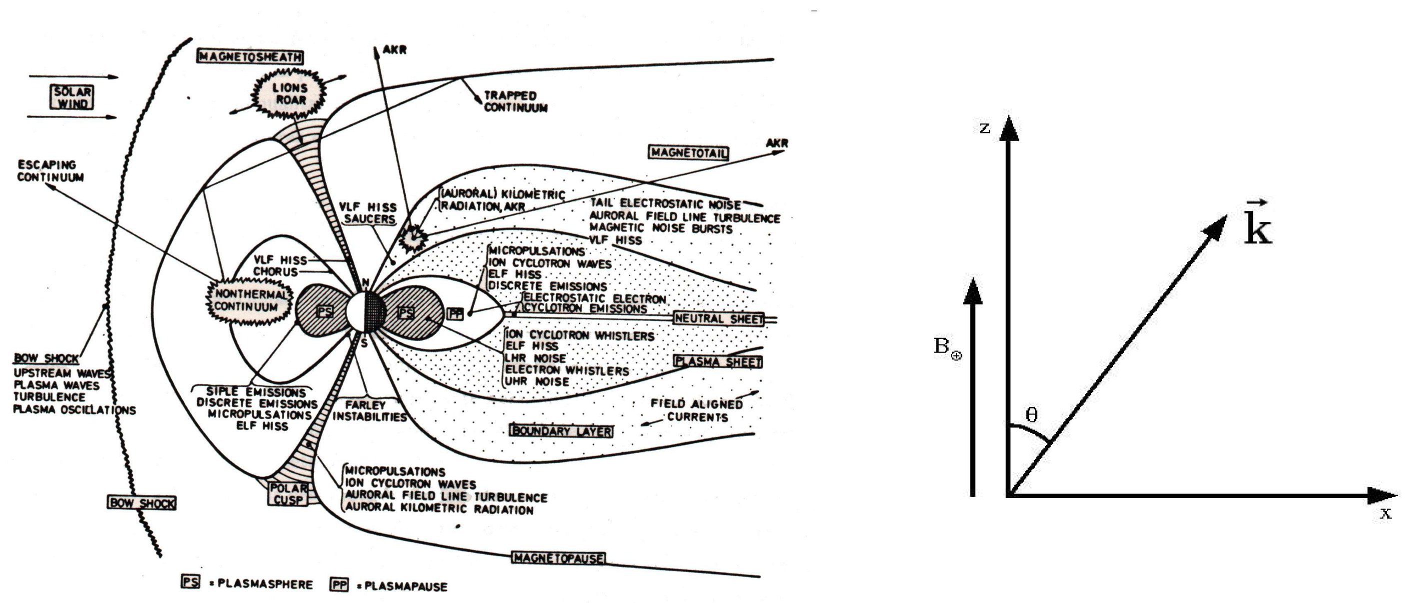

Figure 1: Physical model for the source region of the Auroral Kilometric Radiation (AKR), the picture on the left describe the magnetosphere regions on the Earth and the right is the model adopted in this work.

Dispersion Relation for Electromagnetic Waves

The auroral region is described by a two-dimensional slab, with the *z* axis pointing upward along the geomagnetic field and the *x* axis pointing in the magnetic latitudinal direction. The slab is considered to be homogeneous in the *y* direction. The dispersion relation may be written as

$$\Lambda=(\varepsilon_1\sin^2(\theta)+\varepsilon_3\cos^2(\theta))\mathrm{N}^4+(-\varepsilon_1\varepsilon_3(1+\cos^2(\theta))-(\varepsilon_1^2-\varepsilon_2^2)\sin^2(\theta))\mathrm{N}^2+\varepsilon_3(\varepsilon_1^2-\varepsilon_2^2)=0$$

where

$$\varepsilon_1=1-\frac{\mathrm{X}}{1-\mathrm{Y}^2},\quad \varepsilon_2=\frac{-\mathrm{XY}}{1-\mathrm{Y}^2},\quad \varepsilon_3=1-\mathrm{X}$$

and

$$\mathrm{X}=\frac{\omega_{pe}^2}{\omega^2},\quad \mathrm{Y}=\frac{|\Omega_c|}{\omega},\quad \vec{\mathrm{N}}=\frac{\vec{\mathrm{k}}\mathrm{c}}{\omega}$$

The parameters that are of interest in the source region of the AKR, namely the densities of the different plasma populations, the drift velocities, and the temperatures, are evaluated from a model based on the same methodology introduced by *Chiu and Shultz* [1978]. Essentially, profiles of density, temperature, and drift velocity are provided by the model, between two points along the geomagnetic field lines, called the source points. One of these points is at the top of the ionosphere, approximately at 100 km above the surface of the Earth, at the altitude where the frequency of collisions between the particles matches the electron cyclotron frequency. The other point is situated at the magnetic equator, where the hot plasma is injected from. The essential point to us is that the plasma in the source region is considered to be constituted by the following populations: (1) hot plasma of magnetospheric origin; (2) cold plasma of ionospheric origin; (3) backscattered electrons from the ionosphere; and (4) trapped electrons due to the electric and magnetic mirrors.

Another feature of the model that is worthwhile to mention here is that in our study we will consider only the radial variation of the geomagnetic field. The latitudinal dependence will be ignored because we are interested only in the portion of the ray path where occurs amplification, restricted here to a distance of a few hundreds of kilometers. The field is then given by $B_s = 0.6/z_s^3$ G, where z_s is the altitude of the point *s* in Earth's radii. Regarding the cavity with finite width, we simulated it by introducing the dependence of the total electronic density in the perpendicular direction, *x*, given by

$$\mathrm{n}_{\mathrm{Te}}(\mathrm{x},\mathrm{s})=\tilde{\mathrm{n}}_{\mathrm{Te}}(\mathrm{s})[\Delta-(\Delta-1)\mathrm{e}^{-(\mathrm{x}/\mathrm{L}_x)^2}]$$

Where $\tilde{n}_{Te}(s)$ is the total electronic density at the center, $\Delta\tilde{n}_{Te}(s)$ is the total density at the border, Δ is a real number greater than one, and L_x is a parameter that measures the width of the cavity. Temperature and drift velocity are described by exponentially decreasing profiles *Gaelzer et al.*, [1992].

The trajectory of the waves is obtained as a solution of the following equations:

$$\frac{\partial \mathrm{k}_\perp}{\partial \mathrm{t}}=-\frac{\partial\Lambda/\partial \mathrm{x}}{\partial\Lambda/\partial\omega}\qquad \frac{\partial \mathrm{x}}{\partial \mathrm{t}}=\frac{\partial\Lambda/\partial \mathrm{k}_\perp}{\partial\Lambda/\partial\omega}$$

$$\frac{\partial \mathrm{k}_\parallel}{\partial \mathrm{t}}=-\frac{\partial\Lambda/\partial \mathrm{z}}{\partial\Lambda/\partial\omega}\qquad \frac{\partial \mathrm{z}}{\partial \mathrm{t}}=\frac{\partial\Lambda/\partial \mathrm{k}_\parallel}{\partial\Lambda/\partial\omega}$$

The amplification factor *g* is obtained from

$$\mathrm{g}\ =\ -\int_{\vec{r}_0}^{\vec{r}}\vec{\mathrm{k}}''\cdot d\vec{\mathrm{r}}\ =\ -2\frac{\omega}{\mathrm{c}}\int_{\vec{r}_0}^{\vec{r}}\mathrm{N}_\perp''\,d\mathrm{r}'$$

Results

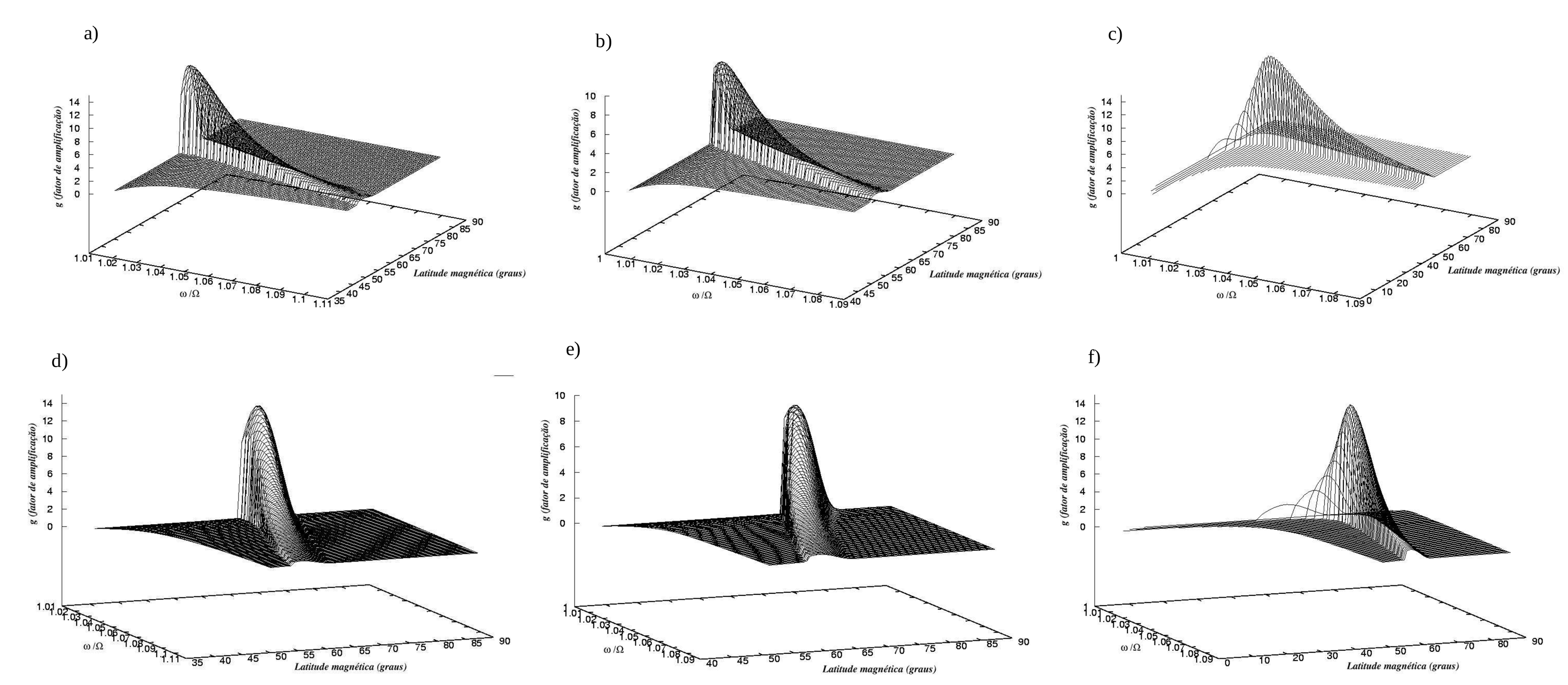

Figure 2: Amplification of the waves, with ω/Ω = 1.006 to 1.08 and $N_{\backslash\backslash}$ = 0.01 to 0.6 and x_0= 0 km. For a) and d) starting from z_0 = 2.5 R_E, with L_x = 500 km, for b) and e) starting from z_0 = 2 R_E, with L_x = 500 km, for c) and f) starting from z_0 = 2.5 R_E, with L_x = 800 km.

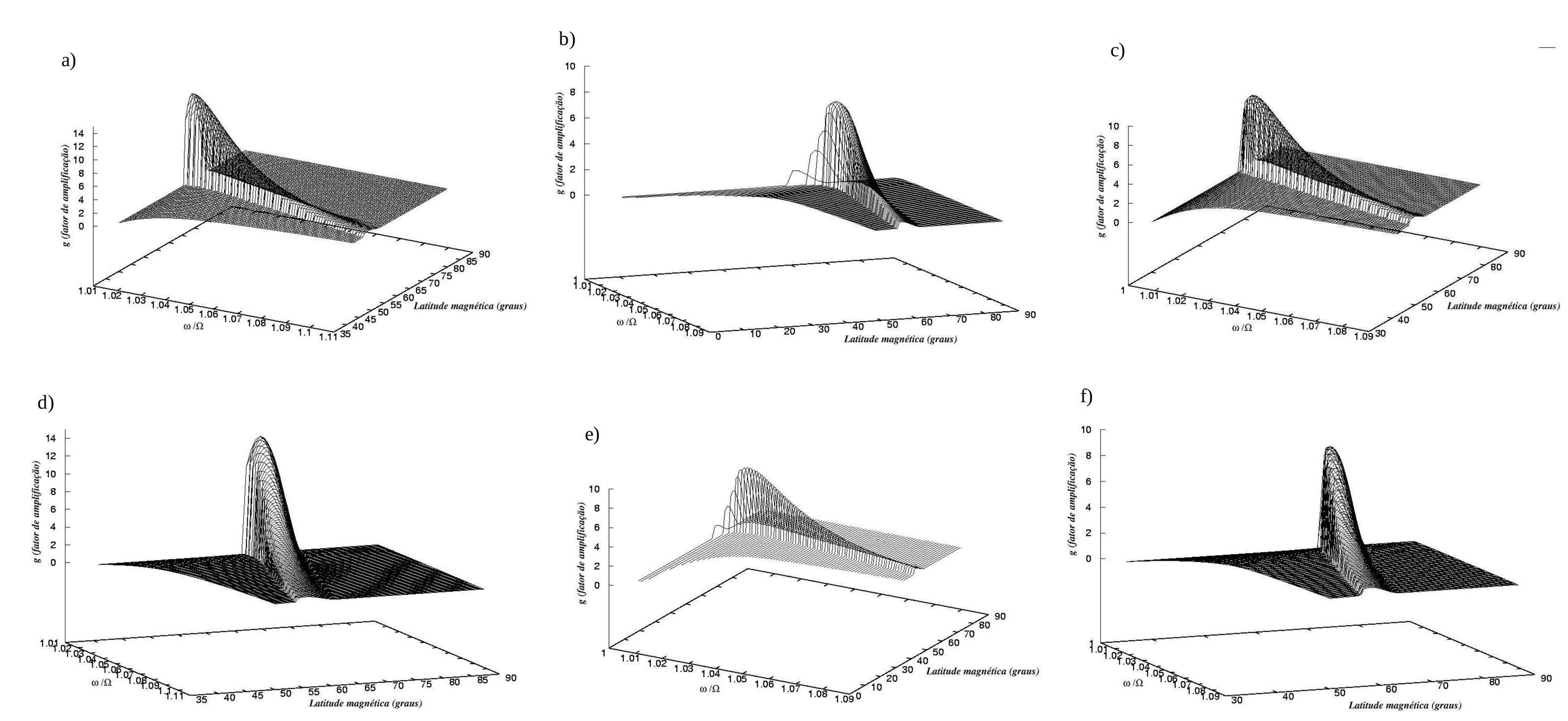

Figure 3: Amplification of tree waves, a) and d) starting from z_0 = 2.5 R_E, with L_x = 800 km and x_0 = 0 km, for ω/Ω = 1.016 to 1.1 and $N_{\backslash\backslash}$ = 0.01 to 0.6, for b) and e) starting from z_0 = 2.0 R_E, with L_x = 500 km and x_0 = -240 km, for ω/Ω = 1.006 to 1.08 and $N_{\backslash\backslash}$ = 0.01 to 0.6, for c) and f) starting from z_0 = 2.0 R_E, with L_x = 500 km and x_0 = -152 km, for ω/Ω = 1.006 to 1.08 and $N_{\backslash\backslash}$ = 0.01 to 0.6.

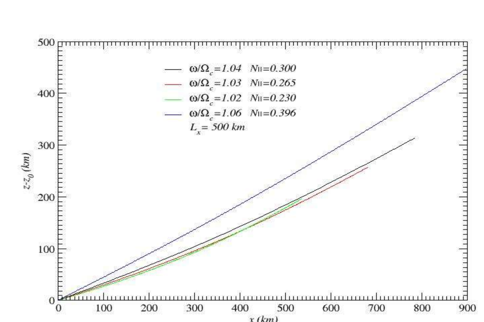

Figure 4: Values of g and z for four rays with different values of ω and $N_\parallel$, starting from z_0 = 2.0 R_E, for L_x = 500 km. The values of ω and $N_\parallel$ are the same as in figures 2 and 3.

Summary and conclusions

In the present work we derived the equations for the trajectory of the waves, using the equations of the geometrical optics, incorporating all relevant thermal and relativistic effects. The dispersion relation and the ray tracing equations were also particularized for the case of a generalized loss cone distribution function, with a drift velocity parallel to the ambient magnetic field, and a numerical code was developed to follow ray trajectories inside a two dimensional region.

The ray tracing code was utilized along with a model that simulates the conditions of density, temperature, and drift velocity of the plasma in the auroral region of the Earth, where the AKR is originated.

The conclusion of our studies is that there is an interval of frequencies not very close to the cyclotron frequency, with are significantly amplified through the cyclotron maser mechanism in the auroral regions, and for which the effects of perpendicular density gradients in the auroral cavities is such that the amplification of the waves is increased, as compared to infinite width cavities.

In a previous work, *Gaelzer et al. [1994]* assumed a thermal plasma to obtain the amplification factor for same parameters used in this work.

Here, assuming that all electron populations are cold, we have exactly the same spatial distribution for the different populations, but use the cold plasma dispersion relation to obtain the ray tracing equation. For frequencies far from Ω_e, the same trajectories are obtained, using either the cold plasma or the hot plasma dispersion relations, confirming that the cold plasma equations are the correct limit of the complete treatment.

Acknowledgments

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