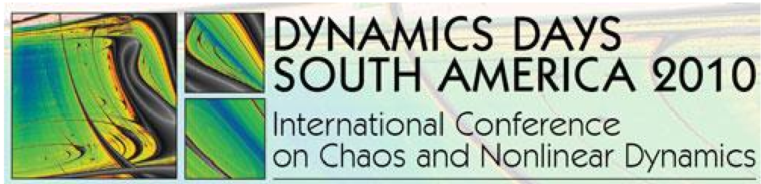

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CHAOS AND CRITICALITY IN CITY TRAFFIC UNDER RESONANT CONDITIONS

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1. ABSTRACT

The study of urban traffic from a physical viewpoint [1–6] has shown to be interesting not only due to its obvious social and economical [7] impact, but also due to its complexity [8–10] which is experienced daily by drivers. This complex behavior has been studied from many perspectives, ranging from statistical and cellular automaton models, to hydrodynamical and mean field approaches [11–14].

We first explore in detail the traffic model proposed in Toledo et. al. (Phys. Rev. E 70 016107, 2004) in which a single car travels through a sequence of traffic lights. A car in this sequence of traffic lights can have (a) an acceleration a_+ until its velocity reaches the cruising speed $v_{\max}$, (b) a constant speed $v_{\max}$ with zero acceleration, or (c) a negative acceleration $-a_-$ until it stops. Therefore, we can summarize the equations of motion for the vehicle, as

$$\frac{dv}{dt} = \begin{cases} a_+\,\theta(v_{\max} - v), & \text{accelerate} \\ -a_-\,\theta(v), & \text{brake} \end{cases}, \tag{1}$$

where $\theta(x)$ is the Heaviside step function. A normalized description includes the definitions $A_+ = a_+ L/v_{max}^2$ and $A_- = a_- L/v_{max}^2$.

As the car approaches the n^{th} traffic light with velocity $v_{\max}$ the driver must make a decision depending on the sign of $\sin(\omega_n t + \phi_n)$ at the distance $v_{\max}^2/2a_-$ (the last stopping point to arrive with null velocity at the traffic light). The frequency ω_n and the phase ϕ_n at the n^{th} traffic light are used to control the traffic.

If $\sin(\omega_n t + \phi_n) > 0$ (green light) the driver continues through the traffic light at speed $v_{\max}$. If $\sin(\omega_n t + \phi_n) \leq 0$ (red light) the driver starts braking with $-a_-$ until it reaches the traffic light with speed $v = 0$ and waits for the next green light, or until the light turns green again with $v \neq 0$, at which point it starts accelerating with a_+. The normalized frequency in this case is $\Omega/2\pi = L/Pv_{max}$, where P is the traffic light period.

The complex behavior that occurs when traffic lights are synchronized is studied. Two strategies are considered: all lights in phase, and a "green wave" with a propagating green signal at speed v_w. The normalized parameter in this case is $\alpha = v_{max}/v_w$.

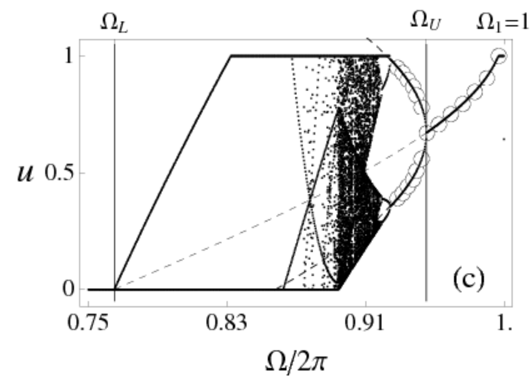

Figure 1 – Bifurcation diagrams for $u = v/v_{\max}$ as a function of $\Omega/2\pi$ for $A_+ = 2 \cdot 200/14^2$ and $A_- = 8 \cdot 200/14^2$, which correspond to realistic city traffic conditions.

The chaotic behavior shown for a given bound in the acceleration/braking ratio, as displayed in the bifurcation diagram of Fig. 1, is examined more carefully, and the region in parameter space for which we observe chaotic behavior is found. For example see Fig. 2. For $\Omega/2\pi < 1$ we have a period doubling bifurcation, and for $\Omega/2\pi > 1$ we have a period adding dynamics that can be described by supertracks. It it interesting that we can define aproximate scaling laws that allow us to describe the nontrivial dynamics of this model in a two dimensional parameter space.

It is also found that traffic variables such as traveling time, velocity, and fuel consumption, near resonance, follow critical scaling laws. For example, in the case of a greenwave, the analytical prediction for the average speed close to the resonance $\alpha = 1$, is

$$\frac{\langle v \rangle}{v_{\max}} = 1 - |1 - \alpha| \,. \tag{2}$$

This critical behavior is universal, in the sence that time and velocity scaling laws hold even for random separation between traffic lights.

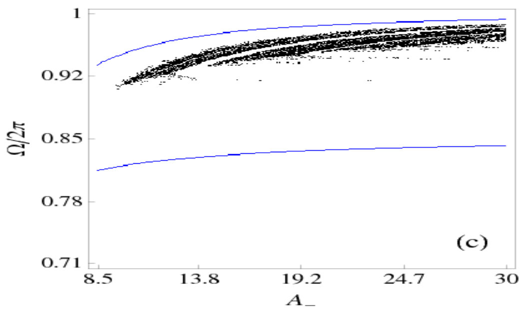

Figure 2 – Chaotic regions for $\Omega/2\pi$ *vs.* A_+. $A_- = 3 \cdot 200/14^2$.

We then analize the resiliece of this critical behavior as we include more cars into the systems and their interactions. We first notice that his critical behavior close to $\alpha \approx 1$ does not depend on the accelerations.Hence, it is natural to modeled this critical behavior with a cellular automata, and we will see that an analogous resonant behavior is found for the two strategies mentioned above. In this model, we divide the distance L_n between successive traffic lights by a number $N_n^c = L_n/\ell$ of cells that can be occupied by a vehicle or be empty, with ℓ the size of the cell, and n labeling the n^{th} traffic light. The car will move to the next cell in one time step τ if that cell is empty. Conversely, the vehicle will stay in its cell during the next time step if the next cell has a vehicle. Hence, the cars cannot pass each other, and the velocity takes two states 0 or 1. If the cell is at a traffic light, the car must stay in its cell while the traffic light is red. The resilience of the critical behavior is analyzed as we introduce velocity perturbations, which are defined by the parameter r. Cars in our model will have, at every time step, a probability r of not moving during one time step. Initially the street is empty. First, we inject cars at the left end at a rate $1/f = 1$, that is, we inject a car at every time step when the first cell is empty. The system is evolved during a time $10^3 P$ to eliminate transient behaviors, and then the dynamics is followed during a time $10^3 P$. The results is shown in Fig. 3 for $f = 5$.

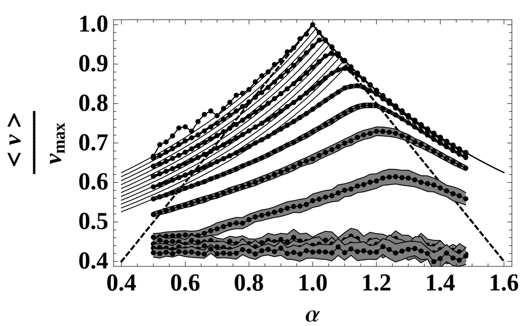

Figure 3 – Results for $f = 5$. r varies from 0 to 0.1, in increments of 0.01. Solid lines correspond to the analytical results, and dashed lines to Eq. 1. Line width represents the standard deviation of the car velocity over the simulation run.

The resonance is clearly visible in Fig. 3, and as we increase r it moves to the right as expected. This figure also brings the suggestion that as r becomes large enough, there is a coherent structure (a cluster of cars) propagating in the system, and on average the cars take about the same amount of time to reach the end of the simulation box, independently of v_{wave}, hence, of α. This emergent state, occurs before we reach a complete traffic jam, for a much higher value of r. This state must be analized in detail.

These results suggest the concept of transient resonances, which can be induced by adaptively changing the phase of traffic lights. This may be important to consider when designing strategies for traffic control in cities, where short trajectories, and thus transient solutions, are likely to be relevant.

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