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SYNCHRONIZATION OF FIREFLIES USING A MODEL OF LIGTH CONTROLLED OSCILLATORS

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Abstract: Natural systems often present spontaneous synchronicity; for example, fireflies flashing in unison or cardiac cells firing in synchrony. Those are distributed systems with decentralized control and fault-tolerance, the same features researchers seek in communication systems. Synchronicity can also be used to coordinate sampling across multiple nodes in a sensor network and is especially important in applications with high data rates. Basically, synchronization consists in an adjustment of rhythms among self-sustained oscillators due to a weak coupling that may act in different manners. This phenomenon has achieved great importance in the last years due to the fact that it is manifested in systems of very different nature such as physical, chemical, biological and electrical. Furthermore, phenomena involving synchronization in complex networks or synchronization in time-delayed systems have been intensively studied in recent years. We constructed an electronic model of fireflies using a light-controlled oscillator (LCO), whose free-run duty cycle can be modified and adjusted manually on the spot, and on which quantitative measurements of periods and phase differences may be performed with the required precision. Furthermore, this device allow us to connect an arbitrary number of LCOs in order to study the synchronization times for different kinds of links. Finally, we solved the model numerically finding that it reproduces our different experimental results with three interacting LCOs in different configuration.

keywords: synchronization, pulse-coupling oscillators, coupling configurations

Synchronization of nonlinear dynamical elements is observed in many natural systems, ranging from physical [1], chemical [2], biological [3] to electronic oscillators [4]. In particular, fireflies provide one of the most spectacular examples of synchronization in nature. Fireflies generate light from the lantern in the abdomen; it usually takes about 800 milliseconds to recharge the lantern and 200 milliseconds to produce a spark; the process may then repeat. Formal models of this behaviour describe a single firefly as a relaxation oscillator with a phase $0 < \Phi < 2\pi$ and period ω. Interaction occurs only when one oscillator sees the emission of the other and changes its rhythm in return. Mirollo and Strogatz [5] proved that for certain conditions, this type of coupled oscillators always synchronize. A simple electronical model for fireflies can be constructed using Light-Controlled Oscillators (LCOs) which constitute unidimensional relaxation oscillators described by two distinct time scales, with great parameter malleability and easy experimental implementation [5–7]. In this model the free-run duty cycle can be modified and adjusted manually on the spot.Furthermore, this device allow us to connect an arbitrary number of LCOs in order to study the synchronization states for different kinds of links. Further analysis of stable regions is carried for different configurations of three LCOs, where experimental and numerical results exhibit that depending on the adjacency matrix, phase bifurcations appear as a consequence of frequency detuning. In this work we present experimental and numerical results about the interaction of three quasi-identical LCOs in different configurations. Despite the differences between them, synchronization is reached and it exhibits robustness under environmental perturbations or intrinsic statistical variations.

The LCO used in this work is an open electronic version of an oscillator which mimics Gregarius fireflies. Basically, the LCO is composed of a LM555 chip to function in an astable oscillating mode [7]. It possesses an intrinsic period and pulse-like IR light emissions, both which can be manually modified on the spot enabling quantitative measurement of phase differences and period variations with the required precision. The characteristic frequencies, named λ and γ, corresponding to the charging and discharging states of the capacitor C respectively, are determined when no external perturbation is done. Timing components are set due to two variable resistors R_λ and R_γ, so the intrinsic longer charging period can be changed by acting on R_λ, and flashing can be widened by modifying the discharging state, thus, R_γ. Coupling is achieved by photo-sensor diodes connected in parallel, whom act as current sources when they are receiving IR light, shortening the charging time and making a longer discharging state. When all photo-sensors are masked, namely *in dark*, the periods only depend on the electronics. An LM555 constitutes the brain of the electronic firefly, managing these current deviations and setting the maximum charging and minimum discharging voltages at $2V_{cc}/3$ and $V_{cc}/3$ respectively, where V_{cc} is the source voltage value. Different configurations correspond to masking the sensors and varying coupling strength is managed by forcing electronically greater emissions.

The dynamical model that describes LCOs corresponds to the following set of differential equations [6, 7]:

$$\dot{V}_i(t) = \lambda_i \left[V_{cc} - V_i(t)\right] \epsilon_i(t) - \gamma_i V_i(t) \left[1 - \epsilon_i(t)\right]$$
$$+\beta \sum_{j=1,\, j\neq i}^{N} \delta_{ij} \left[1 - \epsilon_j(t)\right],\; i = 1, \ldots, N, \qquad (1)$$

where V_i is the i-th LCO voltage, β gives account of the coupling strength, δ_{ij} is the adjacency matrix element, and $\epsilon_i(t)$ is a variable created to represent the oscillator stage —takes the value 1 (charging stage) or 0 (discharging stage). The parameter λ_i (γ_i) is the inverse characteristic time scale for the charging (discharging) stage and is related to those of the LCO. The action of the coupling results in a raise of the asymptotic level of the capacitor stages (V_{cc} and 0 increase for charge and discharge, respectively).

For three LCOs, we analyzed 15 different configurations, corresponding to the six possible entries of the adjacency matrix. Stable states are analyzed taking constant β and detuning LCO periods for each configuration. Master-Slave (MS) and Mutual Interaction (MI) coupling are used between 2 LCOs in order to generate different configurations. Figure 1 shows two experimental configuration that we have used for the case of 3 LCOs.

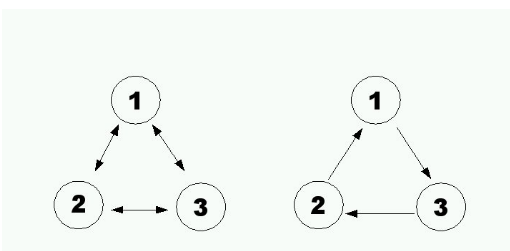

Figure 1 – Different configuration corresponding to 3 LCOs: MI configuration (left) and local coupling due to aring structure (right).

Dependig on the detuning between each LCOs, synchronization states can be achived for two or three LCOs. Furthermore, for certain values of detuning the system does not present full synchronization. Figure 2 shows the trajectory in phase space corresponding to different detunings and the three LCOs coupled in a ring. We can observed that for $\omega_1 < \omega_3 < \omega_2$, synchronization is achived between LCO_1 and LCO_2, (top pannel). For $\omega_2 < \omega_1 < \omega_3$ three LCOs are synchronized (bottom pannel).

Experimental and numerical results show that the synchronization range is similar in this situation compared to the case of two interacting LCOs.

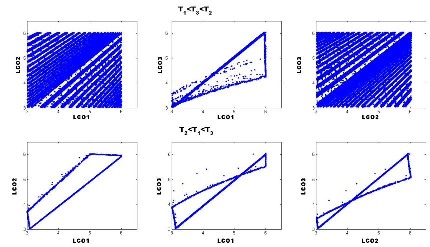

Figure 2 – Trajectories in phase space for 3 LCOs when a MI coupling is established between them and differents detuning are used.

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