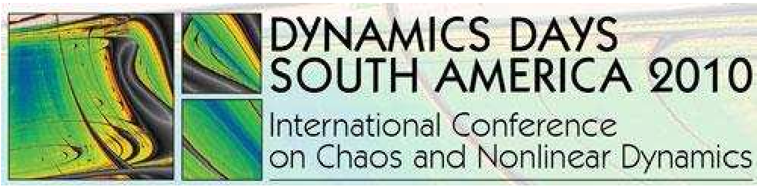

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A STUDY VIA PARAMETER SPACES OF THE TOKAMAP

Alan Celestino[1], *Holokx Abreu Albuquerque*[2], *Lúcio Minoru Tozawa*[3]

[1]UDESC, Joinville, Brasil, celestino.alan@gmail.com
[2]UDESC, Joinville, Brasil, dfi2haa@joinville.udesc.br
[3]UDESC, Joinville, Brasil, dfi2lmt@joinville.udesc.br

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The stability of the magnetic confinement of plasma inside tokamaks is an important problem that has been tackled using techniques from nonlinear dynamics. Formally, the magnetic field lines inside a tokamak are orbits of Hamiltonian systems of one and a half degrees of freedom that can be described by a two-dimensional return map [1]. A map like that can be obtained by evaluating the coordinates of the intersections between a Poincaré section perpendicular to the axis of the tokamak and the field lines.

The tokamap was proposed by Balescu *et al.* [2] to satisfy some conditions, considered minimal for tokamak: having an impenetrable axis and a realistic rotational transform of equilibrium profile that, in the model, can be freely chosen. The equations that define the map are,

$$\begin{aligned} y_{n+1} &= \tfrac{1}{2}\{P(x_n,y_n) + \sqrt{[P(x_n,y_n)]^2 + 4y_n}\}, \\ x_{n+1} &= x_n + \iota(y_{n+1}) - \tfrac{k}{(2\pi)^2}\tfrac{1}{(1+y_{n+1})^2}\cos 2\pi x_n, \quad (1) \\ P(x,y) &= y - 1 - \tfrac{k}{2\pi}\sin 2\pi x, \end{aligned}$$

where x is the poloidal angle, y is the radius, k is the parameter of disturbance and ι is the rotational transform of equilibrium. Here, we used the following profile of rotational transform,

$$\iota(y) = \frac{\iota_0}{4}(2-y)(2-2y+y^2), \quad (2)$$

where $\iota(0) = \iota_0$ is its value at the axis. It's common to use, for analysis via dynamic systems, the phase spaces of the map for typical values of the parameters k and ι_0. The characteristic phase space generated by the map can be seen in Fig. 1.

Although the analysis of phase spaces are satisfactory in some aspects, it is difficult to cover a large set of parameters, as each phase space is associated with only one pair (k, ι_0). However, many of the dynamical characteristics of a phase space can be determined by evaluation of the largest Lyapunov exponent (λ) [3], so that it is possible to study the system for a wide range of parameters. The computational tool that allows this approach is the parameter spaces, which can be considered as a calculation of the relationship between the largest Lyapunov exponent and the parameter pair (k, ι_0).

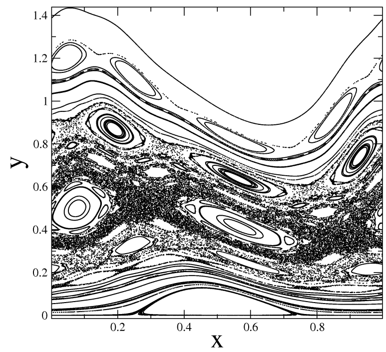

Figure 1 – Phase space genereted with typical values of the parameters $k = 4.0$ and $\iota_0 = 1.0$, with 10^2 initial conditions for 10^4 iterations.

In this work, we apply this tool for the study of tokamap, covering a region of parameters of interest to the physics of tokamaks. For the construction of parameter spaces, we used an initial condition of the confined sea of chaos, that can be seen in Fig. 1. We iterate this initial condition 10^6 times evaluating the largest Lyapunov exponent by the method of Wolf *et al.* [4]. The result is a point (k, ι_0, λ). Repeating this process for a set of 3.6×10^5 parameter pairs (k, ι_0), we obtain a surface in three-dimensional space, which we represented in 2D associating a color diagram to the exponent λ. The parameters space obtained for the system can be seen in Fig. 2, where black regions represent a quasi-periodic behavior, blue regions the confined sea of chaos, and yellow and red regions the divergence. In Fig. 3, we show an amplification of the green box present in Fig. 2. The dependence of the exponent λ with ι_0 for a given k is shown in Fig. 4, where we follow the red line that can be seen in Fig. 2.

The clear dividing line between the divergence and the other regions in Fig. 2 also sets the limit of interest to the physics of tokamaks, since there is no sense to study field lines diverging away from the tokamak.

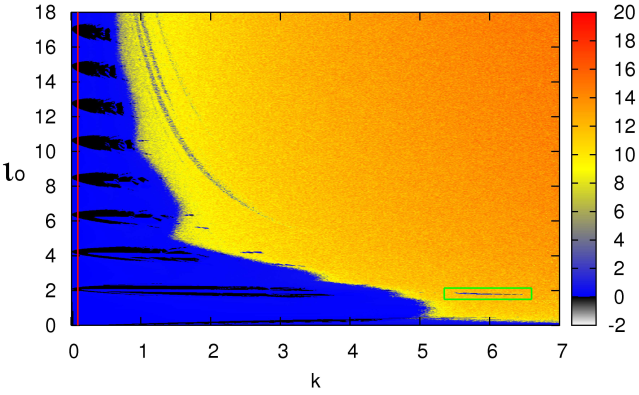

Figure 2 – Parameter space of Eqs. (1). The colors indicate the largest Lyapunov exponent in right side scale.

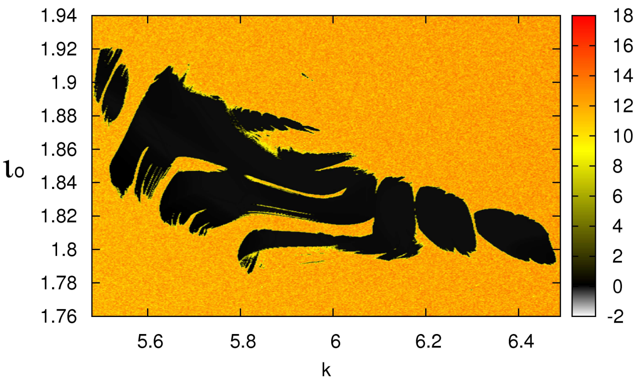

Figure 3 – Quasi-periodic island immersed in the divergence region, green box in Fig. 2.

The parameter space of Fig. 2 shows the existence of quasi-periodic islands immersed in the divergence region (one of this islands can be seen in Fig. 3), which, interpreting the divergence as a migration of the field lines out of tokamak, suggests a paradox. However, phase spaces in those regions indicate that the presence of islands is a characteristic of the initial condition and do not represents the general behavior of the system. Physically, we have a magnetic surface in the middle of the divergence.

In Fig. 4, we can identify the degeneration of the black structures that repeat periodically in the parameters space over the increment of ι_0. This degeneration probably indicates the intersection between the red line and the line of divergence in Fig. 2. However, even before this intersection, it is apparent that the behavior of λ becomes progressively more irregular as ι_0 increases, which indicates an instability of the system in relation to the variations of the parameters set in that region.

The results of this study are fully consistent with those obtained by other authors through the analysis of the phase spaces of the tokamap. Given the huge range of values of the parameters that can be studied, the technique of parameter spaces in tokamap proved be much more general, revealing

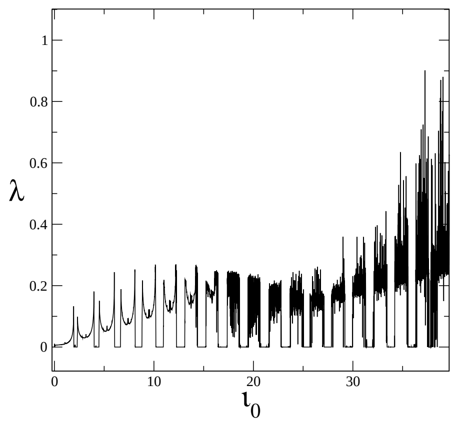

Figure 4 – Largest Lyapunov exponent behavior for different values of ι_0, where we fixed $k = 0.1$ and follow the red line shown in Fig. 2. We can observe the profile of the black structures immersed on the blue region.

some peculiarities of the system that could not be perceived through another technique. However, the analysis of phase space must be implemented together with the analysis of the parameter spaces, since the phase space is much more accurate to describe the system behavior for a given pair of parameters. The study also opens up questions about the evolution of Lyapunov exponent with the increase of ι_0, since the formation of periodic structures and their degeneration are not fully understood.

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