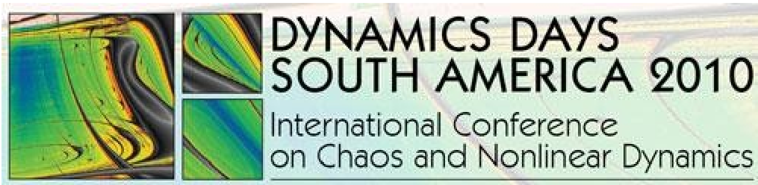

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PATTERN FORMATION AND NON-LOCAL INTERACTION: CONTINUOUS VERSUS DISCRETE MODELS

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Recently, the topic of pattern formation due to non-local interaction, in the context of population dynamic, has been widely discussed along the literature [1-4]. Taking a non-local generalization of the Fisher equation [1] which can be deduced from a microscopic model for the individual's interaction [2], the appearance of cellular pattern was elucidated. By using bi-stable models, the appearance of localized structures was studied [3]. This type of localized states could exhibits a wide range of complex behaviors, as self-replication [4].

Moreover, nonlocal interaction seems to be relevant in many other fields, as stripe formation in the visual cortex [5] or magnetic systems [6].

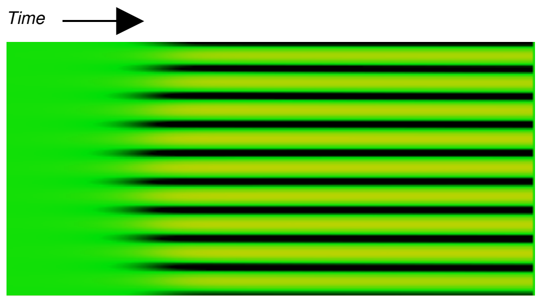

Figure 1 – Typical pattern formation of a continuous model, from a uniform state to a static pattern. The specific model appears in Ref. [4].

On the other hand, the use of discrete system as cellular automata to model such processes is also a well reputed approach. From the pioneers works on self-reproducing machines [7,8], to the studied of cellular automata as a special case of dynamical system [9,10].

The aim of this poster is to present a discrete model of population dynamics, that gives account of the same effect that some of above continuous models [3,4]. To wit, a one dimensional cellular automaton that takes into account cooperative and competitive interaction between the individuals. But, in this case no-motion of the individuals will be considerate.

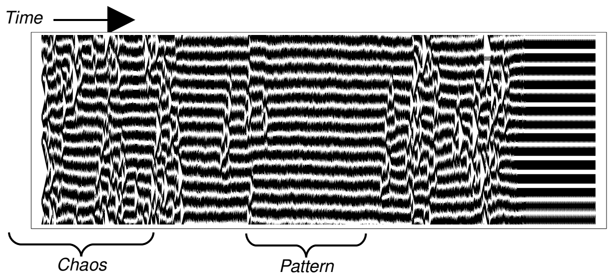

Figure 2 – Pattern formation of our discrete model, which exhibits intermittence between a chaotic state and the pattern.

As for the case of continuous models, this discrete model exhibits pattern formation. Namely, a spatially coherent structure, with a well defined wave number. However, the mechanism involved in the formation is quite different. While in the case of continuous models the pattern emerges from a uniform state, and becomes in a stationary structure, as it is shown in Figure 1. For the case of the cellular automaton the pattern emerges form a chaotic state, through intermittence. To wit, moving some parameter related with the non-local interaction, the chaotic state exhibits bursts of a coherent pattern, and returns to the chaotic state. As the parameter (related with the non-locality) is increased the bursts of pattern are longer and more frequent, and the episodes of chaos are reduced. Figure 2 shows an example of this kind of dynamics. Finally, when the parameter related with the non-locality is large enough, the system exhibit a completely regular pattern (with some phase oscillation).

The transition from chaos to the pattern is characterized using an order parameter related with the discrete Fourier transform. It is also commented the relation with other well known characterization of cellular automata, like the Langton's parameter [11]. Moreover, introducing our own parameter, the relation between this transition and the edge of chaos is explored. It is important to emphasize that the discussion is mostly focused in the language of dynamical systems.

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