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EDGE OF CHAOS IN THE GOY SHELL MODEL OF FULLY-DEVELOPED TURBULENCE

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The dynamical systems approach to turbulence explains numerous complex phenomena such as the transition from laminar to turbulent flows, amplitude-phase synchronization, broadband power spectrum and intermittency [1–3]. Complex motions arise in a dynamical system due to chaotic attractors and chaotic saddles. The tendency for turbulence in pipe flows not to persist but decay to a laminar state if the observation time is long enough is explained by the assumption that the turbulent state corresponds to a chaotic saddle; the boundary between the chaotic saddle (turbulence) and the laminar state is determined by the edge of chaos [4].

Our aim is to investigate the phenomenon of edge-of-chaos in a three-dimensional Gledzer-Ohkitani-Yamada (GOY)-shell model of fully-developed fluid turbulence [5]. In this model, the wave number is one-dimensional and discretized by octaves as $k_n = 2^{n-4}(n = 1, 2, \cdots, N)$. The dynamical equation for the complex shell velocity u_n is given by

$$\left(\frac{d}{dt} + \nu k_n^2\right) u_n = i[a_n u_{n+1} u_{n+2} + b_n u_{n-1} u_{n+1} + c_n u_{n-1} u_{n-2}]^* + f\delta_{n,1}, \quad (1)$$

where $*$ denotes the complex conjugate, and the coupling constants are takes as $a_n = k_n, b_n = -\delta k_{n-1}, c_n = (1-\delta)k_{n-2}$, $b_1 = c_1 = c_2 = a_{N-1} = a_N = b_N = 0$ to conserve the energy $E = \sum_n |u_n|^2$ when the viscosity ν and the external forcing f vanish. We consider the case $\delta = 1/2$, where the helicity $H = \sum_n (-1)^n k_n |u_n|^2$ is also conserved as in Navier-Stokes turbulence. The external forcing is imposed on the first shell to avoid energy transfer to lower wave numbers. Many numerical evidence have been found for the K41 scaling of the energy spectrum in the inertial range of the shell model. For the higher order structure functions, a deviation from the K41 scaling, i.e., intermittency, has been found similar to the Navier-Stokes turbulence. In addition, the probability density function of the velocity u_n has been found to depart further from Gaussian distribution function at higher wave numbers.

We construct a bifurcation diagram for Eq. (1) by varying the control parameter ν, and focus our attention in the region of a p-7 periodic window. We study the role played by chaotic saddle in transient chaos inside the periodic window and permanent chaos outside the periodic window. Inside the periodic window, a surrounding chaotic saddle coexists with a banded attractor (periodic or chaotic), and is responsible for chaotic transients that mimic the dynamics of the chaotic attractor outside the periodic window. Figure 1 shows the Poincaré map $\mathrm{Im}\{u_1\} = 0.4$ of the surrounding chaotic saddle at $\nu = 0.001799$ (green dot), in the middle of the periodic window, jointly with p-7 periodic attractor (black cross). The surrounding chaotic saddle is robust and persists beyond the saddle-node bifurcation and crisis. At the onset of saddle-node bifurcation, a type-1 Pomeau-Manneville intermittency appears. At the onset of crisis, the banded chaotic attractor loses its stability and is converted to a banded chaotic saddle. The resulting crisis-induced intermittency displays random switching between laminar and bursty regimes, due to the coupling between banded and surrounding chaotic saddles.

We apply the method of bisection to study edge-of-chaos in the GOY-shell model of fully-developed fluid turbulence. When a periodic attractor and a chaotic saddle coexist, as in a periodic window, there exist two possible trajectories for a given initial condition in the phase space: i) the trajectory can converge directly to the attractor, or ii) the trajectory can visit the vicinity of chaotic saddle before it converges to the attractor. Under this circumstance, we can define two regions or pseudo-basins in the phase space: the laminar basin, related to i), and the chaotic basin related to ii). The boundary between these pseudo-basins is called edge of chaos [4]. We show that in a periodic window, the edge state is the p-7 unstable periodic orbit (UPO) that emerges from a saddle-node bifurcation at the start of the periodic window. At the end of the periodic window this UPO and its stable manifold collide with the banded chaotic attractor, leading to an interior crisis. An efficient method to find the edge state is the bisection method. First, we select two initial conditions, u_L and u_C, in the laminar and chaotic regions respectively. Any path that connects them must intersect the edge. Then we

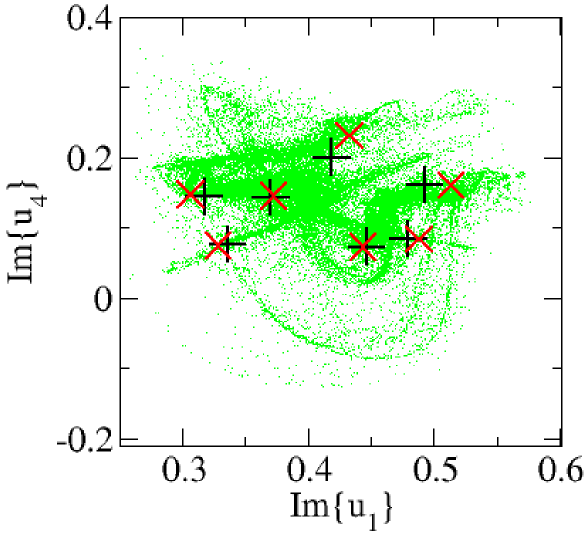

Figure 1 – Poincaré map of p-7 periodic attractor (black cross), chaotic saddle (green dot) and the edge of caos given by a p-7 UPO (red cross), at $\nu = 0.001799$.

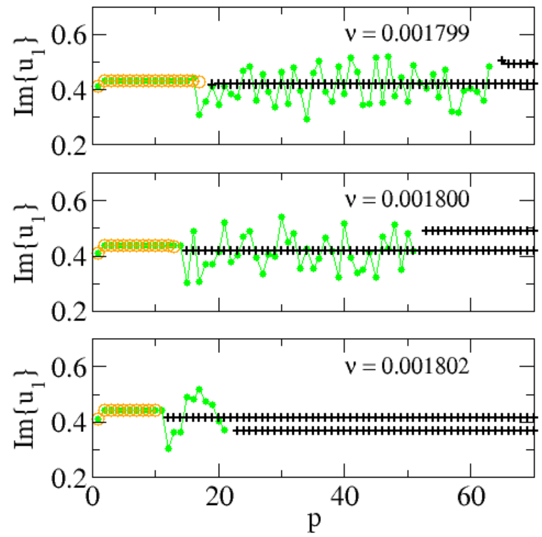

Figure 2 – Poincaré time series of trajectories on the laminar (orange circle) and chaotic side (green dot) of the edge of caos before converging to the p-7 periodic attractor (black cross).

integrate the initial condition u_M=$(u_L + u_C)/2$ and decide which side it belongs to. By sucessive bisections we reduce the distance $d = ||u_L - u_C||$, and bringu_L and u_C close to the stable manifold of the edge UPO. Integrating Eq. (1) using u_L and u_C as initial conditions we generate trajectories that follow the stable manifold of the edge, spending sometime near the UPO, and then diverging either to the attractor or to the chaotic saddle, respectively. Red cross in Fig. 1 represents the Poincaré point of the edge UPO obtained for $\nu = 0.001799$. Figure 2 shows the time series of trajectories at the laminar side (orange circle) and the chaotic side (green dot), for three different values of ν within the periodic window. The initial separation of u_L and u_C is 10^{-12}. In the three cases both trajectories have the same periodic behaviour in the beginning, and then they separate. The laminar trajectories converge quickly and smoothly to the attractor (black cross), and the chaotic ones spend sometime near the chaotic saddle before converging to the attractor. The time that laminar and chaotic trajectories remain close to the edge state decreases as ν increases. This is because the unstable eigenvalue λ_U of the edge state increases as the control parameter increases. The variation with the control parameter ν of the unstable eigenvalue (λ_U) of the edge of chaos (p-7 UPO), the average lifetime of the chaotic saddle (τ), the maximum Lyapunov exponent ($\lambda_{\max}$), the fractal dimension of the stable (D_S) and unstble manifold (D_U) of the chaotic saddle are shown in Table 1. We note that τ and D_S decrease, and D_U increases with ν. This is in agreement with Fig. 2, where trajectories at the chaotic side (green dot) take less time to converge to the attractor, as ν increases.

Table 1 – The dynamical variation with ν of the edge of chaos and chaotic saddle.

ν	λ_U	τ	$\lambda_{\max}$	D_S	D_U
0.001799	4.128	746	0.0059	22.77	2.637
0.001800	5.596	525	0.0062	22.69	2.661
0.001802	8.708	263	0.0061	22.36	2.979

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