DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

**INPE – National Institute for Space Research**
**São José dos Campos – SP – Brazil – July 26-30, 2010**

# Influence of energy changes in breathers

*Gabriel Gouvêa Slade[1], José Roberto Ruggiero[2], Elso Drigo Filho[3]*

1Unesp, São José do Rio Preto, Brazil, ggslade@hotmail.com
2Unesp, São José do Rio Preto, Brazil, zerug@ibilce.unesp.br
3Unesp, São José do Rio Preto, Brazil, edrigof@gmail.com

**Abstract**: This work studies the dynamical behavior of breathers under the influence of energy changes. To create the breather we used the anti-continuous limit and studied its stability through the Floquet theory. Using the information entropy we calculated the effective number of oscillators with significant energy and determined if there is or not the formation of a spatially localized structure.

**keywords**: Applications of Nonlinear Sciences, Breathers, Energy Localization.

## 1. INTRODUCTION

Breathers are time periodic and spatially localized solutions [1]. This kind of solution is generated by the combination of two factors: the discreteness and the nonlinearity [2]. Mackay and Aubry [3] proved the existence of breathers in the one-dimensional lattice when the system is composed by weakly coupled nonlinear solutions. Their proof was based on the anti-continuous limit and showed that the solution persists in a space of periodic solutions with a fixed period and decays exponentially in space.

Here, we follow the numeric proceedings suggested by Marin and Aubry [4] to create the breather and use the Floquet theory to analyze its stability. This study gives to us solutions with different coupling parameter that can be stable or not. However, the energies involved are fixed and dependent to the breather frequency and coupling.

In this work, we verify the influence of energy changes in the dynamic behavior of breathers. In order to do this, we use a stable solution and vary its energy. Based on the information entropy [5] we calculate the number of oscillators with effective energy and use it like a criteria to energy localization.

## 2. PRESCRIPTION TO CREATE THE BREATHER

The Lagrangian of the system is:

$$L_{\mathrm{y}} = \sum_{j=1}^{N}\left\{\frac{m}{2}\dot{y}_j^2 - \frac{k}{2}(y_{j+1}-y_j)^2 - \frac{1}{2}V(y_j)\right\}, \qquad (1)$$

and represents a chain of harmonic oscillators with an additional nonlinear on site potential $V(y_j)$. Here $V(y_j)$ is the Morse potential written, in a simplified way, as:

$$V(y) = (\mathrm{e}^{-y}-1)^2. \qquad (2)$$

The breather can be created using the principle of the anti-continuous limit suggested by MacKay and Aubry [3] and implemented by Marin and Aubry [4] and Cuevas [6]. So, at the beginning we considered the uncoupled system, i.e. $k = 0$. With this limit the system is composed of isolated oscillators under the influence of the nonlinear on site potential.

Because we are looking for periodic solutions, they can be written in terms of a Fourier cosine series:

$$y_j(t) = z_j^0 + 2\sum_{l=1}^{lm} z_j^l \cos(l\omega_{\mathrm{b}}t), \qquad (3)$$

where $\omega_{\mathrm{b}}$ is the breather frequency. Thus, the equations of motion can be changed through substitutions of a set of algebraic equations to obtain the $lm+1$ coefficients of $z_j^l$ parameters (3), these equations can be solved by the Newton method.

Once we found the breather profile for the uncoupled system, $k = 0$, we introduced the coupling into the system by small increments on $k$, $\delta k$. The next step is to find the numerical solution for $k=\delta k$ by using the Newton method and the original profile obtained for $k=0$. This new solution is used like a seed for the case $k = 2\delta k$ and so on until we reach the desired value of $k$.

## 3. RESULTS AND DISCUSSION

The results reported below were obtained using chains with 21 oscillators (N=21) and the number of coefficients in the Fourier series was $lm = 17$. For numerical simulations we used periodic boundary conditions. The stability of the system was studied using the Floquet theory [7]. The system is stable if none of the Floquet multipliers have a module greater than one.

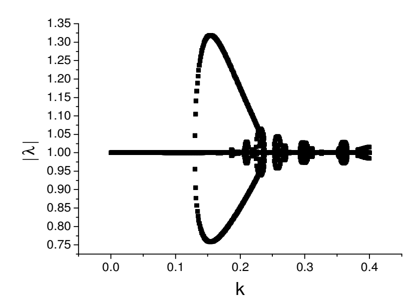


**Figure 1: The absolute value of Floquet multipliers versus coupling parameter *k*.**

In figure 1 we present the results for the absolute value of the Floquet multipliers as function of the coupling parameter $k$ with breather frequency $\omega_b = 0.8$. For small values of $k$ all the Floquet multipliers have module equal to one, so the stability is guarantee. After a certain value of $k$ some bifurcations starts to occur and the stability is lost.

The dynamical behavior of a stable solution is the formation of a spatially localized structure in the chain, and it persists for all time. This analysis allows knowing the behavior of the system as function of the coupling parameter, but does not give directly any information about the influence of the energy changes in the breather. To test this we start from a stable solution, $k = 0.1$, and change its energy decreasing or increasing all the initial positions of this solution. The criterion to analyze the behavior of the system relative to the energy localization is based on the information entropy and measure the number of oscillators with significant energy of the system, $n_{osc}$.

The Figure 2 shows the $n_{osc}$ values in function of the energy. We note the existence of a minimum value of $n_{osc}$ that is equivalent to the energy of the created breather ($E = 0.645$). When the energy diverges from this value the $n_{osc}$ starts to grow and the energy starts to spread out through the chain, losing the localization. In these situations the system presents the behavior of harmonic chains.

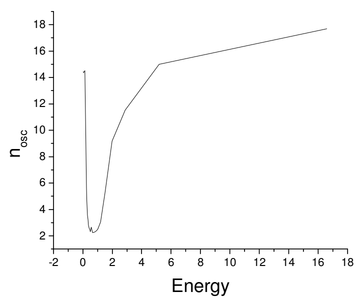


**Figure 2: The number of oscillators with significant value in function of the energy.**

The Fourier analyses allow us to rationalize an explanation for this fact. For energies below $E = 0.645$, the increase of the $n_{osc}$ value is related to the fact that the excited modes beneath to the optical branch, $1 \leq \omega \leq \sqrt{1 + 4k}$ . When we deal with energies above $E = 0.645$, the highest values of $n_{osc}$ occur because the excited modes are inside the acoustic branch, $0 \leq \omega \leq \sqrt{4k}$. These situations are shown in Figure 3.

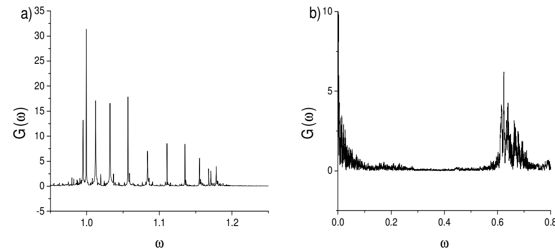


**Figure 3: Modulus G(ω) of the complex Fourier transform of the eleventh oscillator. a) $E = 0.0556$ and $n_{osc} = 14.366$, b) $E = 1.5$ and $n_{osc} = 5.265$.**

## 4. CONCLUSION

The energy changes in a stable solution of breathers make variations in the dynamic behavior of the system. When the energy of the system is below the energy of the breather, the localization is not found because the nonlinear modes are not excited and the system oscillates in the optical branch. For energies above the energy of the created breather, the lost of localization occurs because the system acquire energy sufficient to overcame the on site potential of some oscillators and it shows a behavior similar to purely harmonic network. This result shows the importance of the energy in localized phenomena that involves nonlinearity and discreteness.

## ACKNOWLEDGMENTS

Thanks for CNpq by support.

References

[1] Aubry S 1997 *Physica D* **103** 201-50
[2] Flach S and Willis C R 1998 *Physics Reports* **295** 181-264.
[3] MacKay R and Aubry S 1994 *Nonlinearity* **7** 1623-43
[4] Marin J L and Aubry S 1996 *Nonlinearity* **9** 1501-28.
[5] Luca J, Lichtenberg A J and Liebermon 1995 *Chaos* **5** 283-97.
[6] Cuevas J 2003 *Localización y Transferencia de Energia em Redes Anarmónicas No Homogéneas* Phd thesis Universidad de Sevilla.
[7] Hoppensteadt F C 2000 *Analysis and Simulation of Chaotic Systems* (New York: Springer) chapter 1 pp 1-24.