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A Discrete SIRS Model with Kicked Infection Probability

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The dynamics of epidemics has been based on the susceptible, exposed, infective and recovered continuous-time model (SEIR) [1], or on the simpler SIR model. Both models have endemic equilibria that are asymptotically stable or the disease dies out. Because oscillations are observed in the incidence of many infectious diseases (such as measles, mumps, rubella and influenza), it is of interest to determine how oscillating solutions can arise in epidemiological models. In the deterministic framework, to account for oscillations the transmission rate is allowed to vary seasonally or spatial heterogeneity is included [2, 3]. Periodic behaviour can be also observed if small world and finite recovery time effects on the infection probability are considered [4, 5]. Recently, we proposed a discrete-time version of a SIRS model which exhibits oscillations [6]. In fact, the inclusion in a SIR model of a positive feedback, from the removed class to a susceptible one, in a very narrow range of the corresponding control parameter, has the effect of enhancing periodically the spread of disease. On the whole, many more studies are needed in order to understand clearly the origin of possible oscillations in discrete-time models of disease propagation.

In this paper we add to the SIRS model of ref. [6] (*i*) a variable population size (assumed as logistic) and (*ii*) seasonal variability (introduced by means of a sequence of kicks, which change periodically the infection probability).

We consider a population consisting of susceptible, infected and recovered (usually, permanently immune) individuals *S*, *I*, *R*. As a consequence, the total population is

$$N = S + I + R. \qquad (1)$$

The basic discrete-time deterministic model (SIRS) has the form

$$\begin{aligned} S_{n+1} &= qS_n + cR_n \\ I_{n+1} &= (1-q)S_n + bI_n \\ R_{n+1} &= (1-c)R_n + (1-b)I_n \end{aligned} \qquad (2)$$

where, during each sampling interval *n*, *q* denote the probability that a susceptible avoids the infection, *b* is the proportion of individuals which remains infected ($0 \le b \le 1$), *c* ($0 \le c \le 1$) a fraction of recovered individuals, which lose immunity. The probability *q* is an arbitrary function $0 \le q(S, I) \le 1$ with the property $q(S, 0) = 1$. It depends on the particular form of propagation of the disease.

The probability *q* is modelled as follows:

(I) $q = 1 - pI/N$,

(II) $q = (1-p)^{I/N}$

(III) $q = \exp(-\alpha \cdot p \cdot I/N)$,

where α is a constant or can be modelled suitably.

The probability *p* can be perturbed by a sequence of kicks with amplitude *k* and period *t*

$$p = p_0 + k\delta_{n,t}, \qquad (3)$$

where (*a*) $\delta_{n,t} = 1$ if *n*/*t* is an integer and 0 if not, or (*b*) $\delta_{n,t} = 1$ for $n/t, n/t+1, \ldots, n/t+s$ (*n*/*t* and *s* are integer).

In the following figures time series of susceptible, infected and recovered individuals are shown, with constant population and *q* modelled by (I).

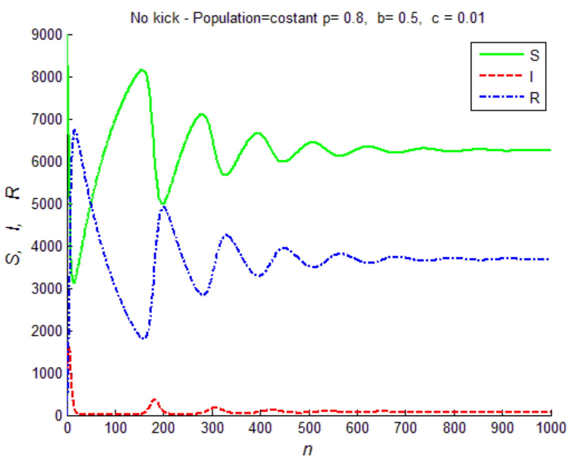

Fig 1

The kick parameter is $k = 0$ for the plot of figure 1, $k = p_0(I/S - 1)$ for the plot of figure 2 and $k = 0.2$ for the plot of figure 3.

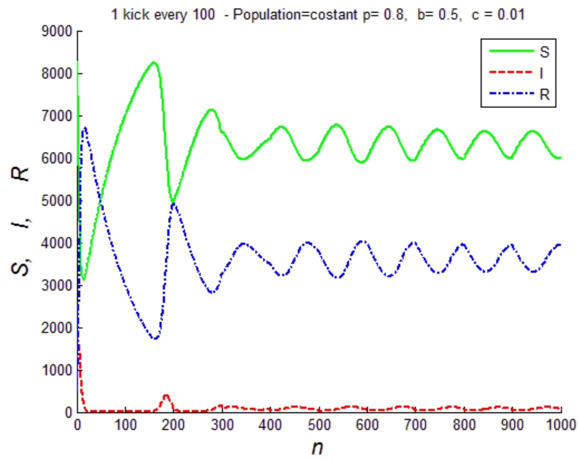

Fig. 2

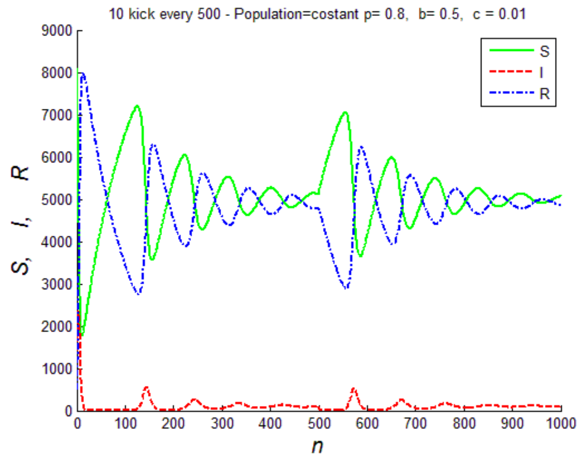

Fig. 3

Drawings show that in all these cases, the kicks force the system to oscillate around states of endemic equilibrium.

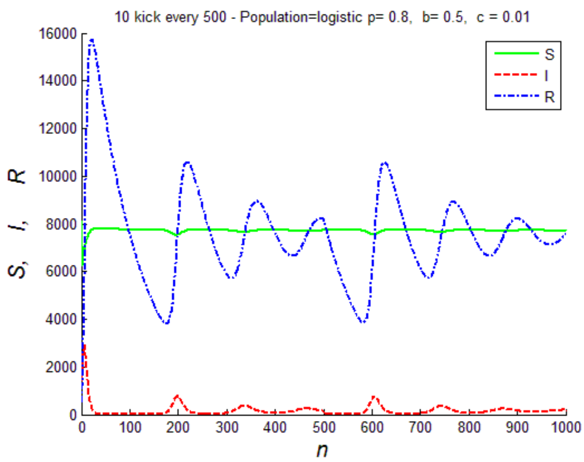

Fig. 4

In the figure 4 a variable population of susceptible is modelled as a logistic growth, with a factor of 2.8. By looking to the picture on sees easily that in this particular case susceptible population converges toward the fixed point of logistic equations.

In conclusion, we have shown that our model accounts for the periodicity observed in the real epidemic diseases. It is of further interest to analyse the combined effect of epidemic, logistic and kick parameters.

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