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Comparison between linear and nonlinear control for the double pendulum using the minimum energy criterion

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1. INTRODUCTION

Control theory has been largely used in many different applications in engineering. This paper presents a comparison between a linear and a nonlinear control technique: LQR (Linear Quadratic Regulator) and nonlinear feedback or Computed Torque Control (CTC) [1] [2] respectively. The nonlinear system to be controlled is a robotic manipulator with two rigid links [3]. According to the numerical simulations, the application of linear control in a nonlinear plant like the one investigated here really limits the operation of the plant. The nonlinear law proposed applies better. The analysis of the control laws is done using a minimum energy criterion [4].

2. MODELING

The double pendulum system can be geometrically modeled as in Figure 1.

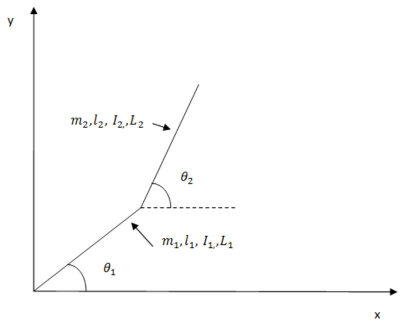

Figure 1 – Double Pendulum Model

In this figure, m_1 and m_2 are the masses of the bars, I_1 and I_2 are the moments of inertia of the bars, l_1 and l_2 are the positions of the center of mass of each bar and L_1 and L_2 are the total length of the bars. The angles θ_1 and θ_2 are represented in Figure 1. τ_1 and τ_2 are the torques in the joints

Using the Lagrangian [5] approach one determinates the governing equations of motion of this system [6]. These equations are given by:

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = \begin{bmatrix}\frac{D}{AD-BC} & \frac{-C}{AD-BC}\\ \frac{-B}{AD-BC} & \frac{A}{AD-BC}\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} - \begin{bmatrix}\frac{D}{AD-BC} & \frac{-C}{AD-BC}\\ \frac{-B}{AD-BC} & \frac{A}{AD-BC}\end{bmatrix}\begin{bmatrix}E\\ F\end{bmatrix} \qquad (1)$$

where,

$A = m_1 l_1^2 + I_1 + m_2 L_1^2$,
$B = m_2 L_1 l_2 \cos(\theta_2 - \theta_1)$,
$C = B$,
$D = m_2 l_2^2 + I_2$,
$E = -\dot{\theta}_2^2 m_2 L_1 l_2 \,\mathrm{sen}(\theta_2 - \theta_1) + \cos(\theta_1) g (m_1 l_1 + m_2 L_1)$,
$F = \dot{\theta}_1^2 m_2 L_1 l_2 \,\mathrm{sen}(\theta_2 - \theta_1) + \cos(\theta_2) g\, m_2 l_2$

The linearized governing equations are given by:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \dot{X}_3\\ \dot{X}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ W_1 & 0 & W_2 & 0\\ 0 & 0 & 0 & 1\\ W_3 & 0 & W_4 & 0\end{bmatrix}\begin{bmatrix}(X_1 - 90^\circ)\\ X_2\\ (X_3 - 90^\circ)\\ X_4\end{bmatrix} + \begin{bmatrix}0 & 0\\ W_5 & W_6\\ 0 & 0\\ W_7 & W_8\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \qquad (2)$$

where,

$$W_1 = -\frac{g(l1m1 + L1m2)(I2 + l2^2 m2)}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_2 = \frac{gL1 l2^2 m2^2}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_3 = \frac{gL1 l2 m2 (l1m1 + L1m2)}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_4 = -\frac{g l2 m2 (I1 + l1^2 m1 + L1^2 m2)}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_5 = \frac{I2 + l2^2 m2}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_6 = -\frac{L1 l2 m2}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_7 = -\frac{L1 l2 m2}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

$$W_8 = \frac{I1 + l1^2 m1 + L1^2 m2}{-L1^2 l2^2 m2^2 + (I1 + l1^2 m1 + L1^2 m2)(I2 + l2^2 m2)}$$

Equations 2 can be written in the form:

$$[\dot{X}] = [A][X] + [B][\tau] \qquad (3)$$

Using this linearization, the LQR control technique can be designed. The gains obtained in this way are used in the complete nonlinear system.

2.1 THE LINEAR CONTROL: LQR

According to the LQR approach[2], the torque vector in Equations 3 is replaced by the control law:

$$[\tau] = (-R^{-1}B^T K)(X - X_{ref}) \qquad (4)$$

where [R] is a 2x2 identity matrix and the gain matrix is obtained by the solution of the Riccati equation:

$$-A^{T}K - KA + KBR^{-1}B^{T}K = Q \tag{5}$$

[Q] is the identity 4x4 matrix in the general case.

2.2 THE NONLINEAR CONTROL: CTC

In this technique, the nonlinearities in the system are compensated via the torque. Using the general governing equations, Equations (1), the control torques are written as:

$$\begin{aligned} \tau_1 &= E + A\,U_1 + B\,U_2 \\ \tau_2 &= F + C\,U_1 + D\,U_2 \end{aligned} \tag{6}$$

In Equations (6), U_1 and U_2 are linear control laws used to complete the CTC control [2]. This linear law can be a PD control [5] or the LQR briefly discussed in 2.1. Both control laws are used here.

3. RESULTS AND CONCLUSIONS

The first analysis consists in verify if a linear control law is able to control the proposed nonlinear system. The numerical results indicate that it is possible to control this system for small angles and velocities, but for large angles and velocities it is necessary to use huge values for the gains in matrix Q. Figure 2 represents the torque necessary to control the joints considering large angles and velocities. Using only the linear LQR control in a nonlinear system.

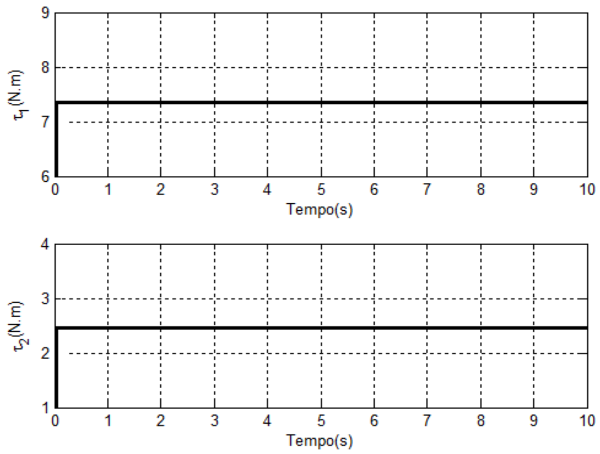

Figure 2 – Control torque for the nonlinear system investigated considering the LQR.

In the Figure is possible to see that is the control law is capable to control the system, but is necessary to implement a impulse response in the joint. In the real system is impossible to generate this type of torque, so this control law is not appropriate to control this system.

Considering the nonlinear feedback and two different control laws (PD and LQR) the necessary torque for the same task is shown Figure 3. Since the torque is proportional to the energy applied to the system, it is possible to observe that even if it is possible to control using a linear control law the energy necessary to do so is much larger than the one that needs to be applied using a nonlinear control law.

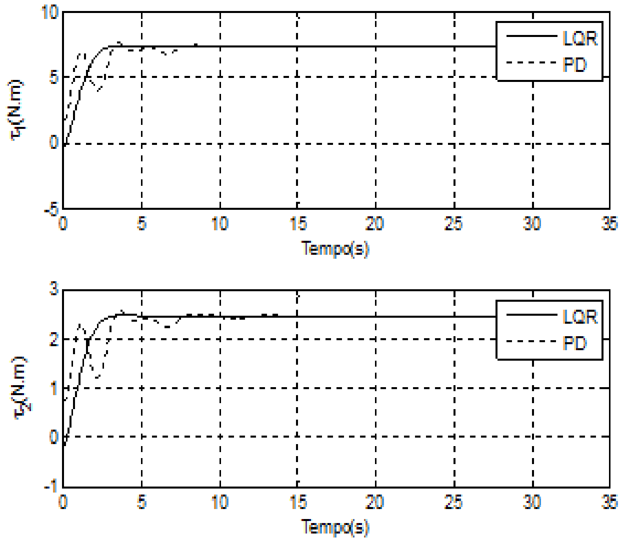

Figure 3 – Control torque for the nonlinear system investigated considering the CTC (PD and LQR included).

Through the graph in Figure 10 is possible to observe that the energy used the LQR control is smaller than that used by the DP to the torque applied to the first joint. This causes the energy used by the actuator is less to take them to the final state required. Regarded as the second joint torque in the energy used by control LQR is greater, but the LQR is softer which means that fewer variation in engine generating less wear. These results shows that in the real system the best control law, among which were analyzed, is the CTC with the LQR complement. Observing the data obtained by the simulations is possible to infer that the nonlinear control laws are most appropriate to control nonlinear systems. Moreover the CTC control Law is very effective in the nonlinear system however is necessary to chose the right law to complement that.

4. REFERENCES

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