SELF-CONSISTENT THEORY FOR COAXIAL GYROTRON CAVITY

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1 Introduction

Electron cyclotron resonance heating (ECRH) for future controlled thermonuclear fusion research requires radiation sources in the 100 – 300 GHz range for continuous operation at the megawatt power level. Rapid advances in the development of high power gyrotron oscillators have greatly improved prospects of using such sources for ECRH. Gyrotrons are particularly attractive sources for ECRH because they can operate continuously at low voltages ($< 100\,\mathrm{kV}$). A severe constraint on high power, high frequency gyrotron design is posed by ohmic losses in the cavity walls. On account of this technical restriction, the ohmic heating of the resonator walls should be limited to about $5\,\mathrm{kW/cm^2}$ for presently available cooling techniques. To circumvent this restriction, large, overmoded cavities are thus required. However, as the resonator cross section increases, the number eigenfrequency in the range of interest increases also, giving rise to multimode oscillation. In this regard, an important issue in gyrotron research is the investigation of mode selection techniques aiming at single-mode operation of overmoded cavities.

Concerning the transverse electromagnetic structure of the parasitic mode, two methods of mode selection can be considered. The first, referred to as electronic selection, involves placing the electron beam in a radial position inside the cavity so that a maximum coupling with the desired mode might be achieved. The second method makes use of the electrodynamical properties of the coaxial resonator. In fact, it has been demonstrated that a lossy coaxial insert can greatly reduce the mode spectrum of a highly overmoded, 1 MW gyrotron cavity.[1] The inner structure of the coaxial cavity, designed to operate in the $TE_{12,7}$ mode at 280 Ghz, consisted of a conical rod made from silicon carbide with a geometry specially devised to suppress high-order TE parasitic modes while keeping the working mode almost unperturbed.

Nevertheless, it remains an important question to verify whether gyrotron coaxial cavities can provide high efficiencies as those associated with empty cavities. The present paper draws attention to this problem.

2 Basic equations

The self-consistent nonlinear theory is well established for conventional gyrotrons with cylindrical, hollow cavities.[2] In the following, such a theory is extended to coaxial gyrotrons to calculate the efficiency for the cavity under consideration (Fig. 1). From the equation of motion for electrons and from the wave equation describing the electric field axial distribution, we obtain the system of coupled equations

$$\frac{du}{d\zeta} = -\frac{d\mathcal{H}}{d\Lambda} \tag{1}$$

$$\frac{d\Lambda}{d\zeta} - \Delta + u = \frac{d\mathcal{H}}{du} \tag{2}$$

$$\left(\frac{d^2}{d\zeta^2} + \kappa_{\|}^2\right) F(\zeta) = I_0 \left\langle (1-u)^{s/2} e^{-i\Lambda} \right\rangle_{\omega t_0} \tag{3}$$

where

$$\mathcal{H} = Re[(1-u)^{s/2} F(\zeta) e^{i\Lambda}],$$

$$F = 4\frac{k_{\perp mp}}{k_0}\frac{G_{m\pm s}(R_f k_{\perp mp})}{\gamma_0}\frac{s^s \beta_{\perp 0}^{s-1}}{s! 2^s}\frac{eV(\zeta)}{mc^2},$$

$$I_0 = 2,36 \times 10^{-2} G_{m\pm 1}(R_f k_{\perp mp}) I_A \times \frac{\beta}{\gamma_0} \frac{k_\perp}{k_0} \left(\frac{s^s \beta_{\perp 0}^{s-4}}{s! 2^s} \right),$$

$$\Delta = \frac{2}{\beta_{\perp 0}^2} \left(1 - \frac{s\omega_{c0}}{\omega} \right),$$

and

$$\kappa_{\parallel} = \frac{2\beta_{\parallel 0}}{\beta_{\perp 0}^2} \frac{ck_{\parallel}}{\omega}$$

where u is the transverse efficiency, defined as the efficiency of energy extraction from the perpendicular component of the beam velocity, Λ is the slowly varying part of the electrons gyrophase, s denotes the harmonic number, and $\langle ... \rangle_{\omega t_0}$ represents average over initial gyrophase. Indices $\perp$ and 0 denote, respectively, direction perpendicular to the resonator axis and initial values; k is the wavenumber, $G_{m\pm s}^2$ is the electron beam-field coupling factor given by

$$G_{m\pm s}(R_f k_{\perp mp}) = A_{mp} \mathcal{C}_{m\pm s}(R_f k_{\perp mp}) \times \left\{ \begin{array}{l} (-1)^s \\ 1 \end{array} \right.$$

where the minus (plus) sign corresponds to the wave rotation in the same (opposite) direction as the electron cyclotron motion,

$$|A_m|^2 = \frac{1}{\pi \chi_{mp}^2} \left\{ \left(1 - \frac{m^2}{\chi_{mp}^2} + \bar{Z}_{R_e}^2 \right) \mathcal{C}_m^2(\chi_{mp}) - \frac{1}{C^2} \left(1 - \frac{m^2 C^2}{\chi_{mp}^2} - \bar{Z}_{R_i}^2 \right) \mathcal{C}_m^2(\chi_{mp}/C) \right\}^{-1}$$

and

$$\mathcal{C}_m(x) = [N_m'(\chi_{mp}/C) + \bar{Z}_{R_i} N_m(\chi_{mp}/C)] J_m(x) - [J_m'(\chi_{mp}/C) + \bar{Z}_{R_i} J_m(\chi_{mp}/C)] N_m(x).$$

I_A is the beam current expressed in amperes, $\bar{Z}_{R_g}$ is surface impedance of the coaxial cavity walls ($g = e$ or i) normalized to the vacuum impedance, β is the electron velocity normalized to the speed of light, $k_{\parallel}$ is the axial wavenumber and ω is the wave frequency.

In this model, the dynamic of electrons is described by a system of nonlinear ordinary differential equations coupled to the wave equation. The coupling term is provided by the beam current which appears in the wave equation as a source term. Solution to these equations should satisfy radiation boundary conditions. The input boundary condition for the RF field is an evanescent wave, while the output radiation boundary condition is an outgoing travelling wave, as giving by

$$\left(\frac{dF}{d\zeta} \mp i\kappa_{\parallel} F \right) \Big|_{\zeta_i, \zeta_0} = 0. \tag{4}$$

The boundary conditions for the beam are represented by the initial velocities and phase for electrons entering the interaction region. For a monoenergetic, unbunched beam the initial phases are uniformly distributed over the initial gyration ring, such that

$$\Lambda_j = 2\pi j / N \qquad j = 1, 2, ..., N \tag{5}$$

3 Results and Conclusion

To find the solution to the set of self-consistent Eqs. 1-3, two computer programs were implemented. The first generates roots of the equation $\mathcal{C}_m'(\chi) = 0$ along the cavity axis. Such values are then read by the second program which solves simultaneously a system of $2(N+2)$ coupled first-order differential equations. Such a system consists of two dynamical equations for each electron and four equations for the RF field.

To satisfy the input boundary condition, the following solution to Eq. 4 is used

$$F(\zeta) = f e^{i\kappa_{\parallel}\zeta} \tag{6}$$

where the plus and minus signs correspond, respectively, to $\Im(\kappa_{\parallel}) < 0$ and $\Im(\kappa_{\parallel}) > 0$. The boundary condition for the electromagnetic field is fully specificied by the initial amplitude F and by the wave frequency ω, since

$$\kappa_{\parallel}^2 = \frac{\omega^2}{c^2} - \frac{\chi_{mp}^2}{R_e^2} \tag{7}$$

The solution of Eqs. 1-3 subject to boundary conditions Eq. 4 constitutes an eingenvalue problem. For constant values of beam current, magnetic field and for the given cavity geometric parameters, the eingenfunctions correspond to discrete pairs of F and ω. Such values are not known a priori and must be found by an iterative procedure. On initial entry, estimate values for F and ω are used, and, on exit, the reflection coefficient

$$|R| = \left| \left(\frac{df}{d\zeta} + i\kappa_{\parallel} f \right) \Big/ \left(\frac{df}{d\zeta} - i\kappa_{\parallel} f \right) \right| \simeq 0$$

is calculated. If $|R| > 5 \times 10^{-3}$, new values of F and ω are supplied to the program and the process is repeated for a narrower range of values to minimize $|R|$.

As shown in Fig. 2, considerable disagreement was found between the efficiencies of the cold and the self-consistent models. In the cold-cavity model the equations are uncoupled since the right-hand side of the wave equation is made equal to zero. Setting $B_0 = 111.6\,\mathrm{kG}$ and an electron velocity ratio $\alpha_0 = 1.5$ gives the highest efficiency (20.8 %) for 1 MW output power. This translates into a beam current of 60 A for a design beam voltage of 80 kV. Using the above parameters, in the self-consistent model, the axial distribution of the electric field and the efficiency are then calculated for both the coaxial and empty cavities. As expected, Fig. 2 shows that $V(\zeta)$ and η corresponding to the coaxial and empty cavities are almost the same, since the coaxial insert keeps the operating $TE_{42,7}$ mode nearly unperturbed. To explain the disagreement between the two models, it should be noted that the field amplitude F associated with the self-consistent model is higher than in the cold model for a given beam. Accordingly, from the energy balance equation, $U = QP/\omega$, where U is the cavity stored energy, both the Q factor and output power P in the self-consistent model ($Q = 860$, $P = 1.7\,\mathrm{MW}$) are larger than in the cold model ($Q = 509$, $P = 1.0\,\mathrm{MW}$), thus giving a higher field amplitude.

On the other hand, the fixed-field approximation, whereby the axial distribution of the RF field $V(\zeta)$ is described by a gaussian function, allows the transverse efficiency $\eta_\perp$ to be given in terms of only three variables, namely: ζ_L, the normalized cavity interaction length, the field amplitude F, and the mismatch parameter Δ which gives the detuning between the cyclotron frequency ω_c and the wave frequency ω. Fig. 3 shows isoefficiency curves on the plane $F - \zeta_L$ for optimum Δ, that is, for each value of F and ζ_L, the parameter Δ has been chosen so that the maximum efficiency is achieved. It can be noted that for a fixed $\zeta_L = 6.25$ (the effective cavity length), the larger F is, the higher $\eta_\perp$ will be, thus explaining the higher efficiency obtained in the self-consistent calculation.

Fig. 4 shows output power as function of beam current for optimum magnetic field. As can be seen, the highest efficiencies are associated with output powers larger than 1 MW. However, as the cavity has been designed to operate around 1 MW output power, the corresponding efficiency is only 28 % for a 45 A electron beam current. For the cavity in question, the efficiency does not peak at the design output power, due to the fact that the cavity length is unfavourably short.

A consideration that would circumvent this limitation consists in increasing the cavity length. In that way, higher efficiency are obtained for lower field amplitudes. Using a cavity mid-section length twice as great, the maximum efficiency increases from 21 % to 39 % for 1 MW output power as shown in Fig. 5. However, the resulting Q factor (1351) becomes higher than that for the short cavity (509). Such a higher Q factor increases the cavity wall loading, and this should be consistent with continuous operation megawatt gyrotron design as the ohmic wall losses in the cavity must be kept below $5\,\mathrm{kW/cm^2}$.

In conclusion, the results present here constitute an application of the self-consistent gyrotron theory extended to coaxial cavities. For the cavity examined, a strong self-consistent effect is observed in the efficiency calculation with respect to the cold-cavity approximation. Nevertheless, as has been demonstrated in this study, coaxial cavities can provide efficiencies as high as those associated with hollow cavities. Presently, optimized gyrotron coaxial cavities are being investigated whose designs are consistent with continuous operation for ECRH of fusion plasmas.

References

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