EVOLUTION OF THE MAGNETOSPHERIC STORM-RING CURRENT WITH A CONSTANT TIME DELAY

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Abstract

Using the energy balance equation for the ring current during magnetic storms, a theoretical study of the response of this current is done, for the case of a constant time decay τ. The input energy function for the balance equation is assumed to be described by a simple time variation during the injection time, such that an analytical response can be obtained. The model is used for 5 of the 10 intense storms in the interval August 1978–December 1979, for which the ISEE-3 interplanetary data are available. The energy input function for these 5 events (those with less data gaps) is assumed to be one of both, the azimuthal interplanetary electric field or the Akasofu's coupling function. These input functions are approximated by one of the simple mentioned input functions and the solution obtained from the energy balance equation, for different values of τ, is compared to the actual evolution of the ring current (derived from the geomagnetic index Dst). The sets of input functions and τ values that better reproduce the observed storm evolution are adopted as the best approximation. As a conclusion, it is found that the more appropriate values of τ are longer than those determined in previous studies, especially for the case of more intense storms.

The energization of the ring current by the interplanetary plasma is usually described by a first order differential equation, namely the balance energy equation. Let D be the pressure corrected Dst index, $D = Dst - b\sqrt{p} + c$, where p is the disturbed-day ram pressure of the solar wind, ρV^2 (with ρ and V being the solar wind density and velocity, respectively), b is a constant of proporcionality and c gives the quiet-day contribution to D. Then, the balance equation can be written as

$$\frac{dD}{dt} + \frac{D}{\tau} = Q \qquad (1)$$

where τ is the ring current decay time and Q represents the energy input for the energization of the ring current. [e.g. see 1–6]. In the present note, we solve analytically equation (1) for the case for which the function Q has simple functional forms, and assuming that τ is a constant. The considered time dependence for Q are the rectangular and the triangular input function cases.

For the case of a rectangular input function, with a negative constant value, Q_o, during a given time interval and is equal to zero outside it, a very simple analytical result can be obtained for $D(t)$, namely, $D(t) = D(0)e^{-t/\tau} + Q_o \tau g(t)$, where $g(t)$ is given by $1 - e^{-t/\tau}$, for $t < T$, and $e^{-(t-T)/\tau} - e^{-t/\tau}$, for $t \leq T$. The peak value of D occurs at the instant $t = T$ and is given by $|D_p| = |D(T)| = |e^{-T/\tau} D(0) + Q_o \tau g(T)|$.

The other simple case for the input energy that we consider here is the triangular function, defined as, $Q(t) = \frac{Q_o}{T_1} t$, if $0 \leq t < T_1$; $\frac{Q_o}{T_2}(T_1 + T_2 - t)$, if $T_1 \leq t < T_1 + T_2$; and 0 otherwise, where here Q_o is again a negative constant. With this expression for $Q(t)$, the solution for equation (1) also becomes, $D(t) = D(0)e^{-t/\tau} + Q_o \tau g(t)$, where $g(t)$ is now given by $\frac{t-\tau}{T_1} + \frac{\tau}{T_1} e^{-t/\tau}$, for $0 \leq t < T_1$, $\frac{T_1+T_2+\tau-t}{T_2} + \frac{\tau}{T_1} e^{-t/\tau} - (\frac{\tau}{T_1} + \frac{\tau}{T_2})\, e^{(T_1-t)/\tau}$, for $T_1 \leq t < T_1 + T_2$, and $\frac{\tau}{T_1} e^{-t/\tau} - (\frac{\tau}{T_1} + \frac{\tau}{T_2})\, e^{(T_1-t)/\tau} + \frac{\tau}{T_2} e^{(T_1+T_2-t)/\tau}$, for $t > T_1 + T_2$. Also for this case the absolute value of $D(t)$ reaches a peak at a given time t_p which is a function of τ for given values of T_1 and T_2. This relatioship can be numerically found for given values of T_1 and T_2.

In order to predict the variation of D as derived from the energy balance equation, some assumption about the actual energy input to the ring current, and consequently on the value of Q, must be done. A simple relationship between the theoretical input functions and the actual power injected to the ring current can be established, in the case that the input can be described for one of both, E_y or the Akasofu's ϵ function. For these cases that relationship can be respectively written as, $Q = 5\,[E_y]$, and $\alpha\, 10^{-17}\,[\epsilon]$, where Q is given in nT/hr, $[E_y]$ is the numeric value of E_y given in mV/m, $[\epsilon]$ is the numeric value of the Akasofu's ϵ function given in erg/sec, and α is a factor of efficiency that seems to be in the range 0.1–0.3.

We consider five of the ten intense storms that occurred during the interval August, 1978, through December, 1979, as shown in the first column of Table I. These events were selected due to the relatively low data gaps presented. Since for most of the considered storms the level of D previous and posterior to the storm has an approximatly constant value, this is considered as the zero level for the disturbance. Therefore, to compute the peak of D, D_p, we substract that constant value from the absolute observed peak. On the second column of Table I the main features of each of the events are summarized. This features are the peak value of D, D_p, and the time scale t_p, which gives the duration of the observed main phase, as well as the value of τ for the recovery phase, τ_r, discussed below. For each of the events, the input functions E_y and ϵ are computed. Then the actual functions are approximated by one of the simple inputs given in the previous sections, namely, the rectangular or triangular shape. The characteristics of these approximate functions are summarized in the third column of Table I. As it can be seen, three of the events (# 1, 2 and 4) are best fitted by a triangular input function, one event (# 3) is best fitted by a rectangular input function, and one (# 5) by a double rectangular input with durations indicated in Table I.

Once the input signal is identified with one of the simple models, we infer the value of τ from the observed peak D_p, in assotiation with the characteristic time scales found in the respective approximations. The value of Q_o for each case is computed as mentioned above, for the parameter α equal to 0.1 and 0.3, and is shown on columns

4 and 5 of Table I. Therefore, having the ordinate of the curve $|D_p/Q_o| = f(\tau)$, it is possible to determine the corresponding τ for each case.. The resulting value for τ is given in column 6 of Table I. The estimated error (number in parenthesis) is assumed to be mainly due to the uncertainty in the value of Q_o, since this source of error is larger than the others at least for one order of magnitude. It should be noticed that this error propagates more strongly for large values of τ. The values of τ, as well as the corresponding input functions, that shown the best results for the final fitting between the theoretical curve and the observed storm evolution, are given in bold characters in Table I. After the best fit curve has been selected and the corresponding τ determined, the theoretical response curve can be computed. When comparing the results of this analytical curve to the observed evolution of the storm, we notice that the large scale variations of $D(t)$ at the main phase are well reproduced. On the other hand, the fitting is not too good for the recovery phase. We have contourned this discrepance considering that after the energy input stops, the ring current relaxes in a simple exponential way and determined the values of τ for the recovery phase, τ_r, by adjusting this exponential decay to the observed function, as shown in column 2 of Table I.

TABLE I

Event	Storm Features	Test Function	Approximate Input Features	Q_O (nT/h)	Computed τ $(\delta\tau)$
August 27–28, 1978	$\|D_p\| = 245$ nT,	$\epsilon(\alpha = 0.1)$	Triangular shape,	53	9.6 (1.5)
	$t_p = 10$ h,	$\epsilon(\alpha = 0.3)$	$T_1 = 11.7$ h,	159	1.8 (0.5)
(1)	$\tau_r = 14$ h.	**E_y**	$T_2 = 4.0$ h.	53	**8.7 (1.5)**
September 29–30, 1978	$\|D_p\| = 207$ nT,	**$\epsilon(\alpha = 0.1)$**	Triangular shape,	107	**2.8 (0.7)**
	$t_p = 6.7$ h,	$\epsilon(\alpha = 0.3)$	$T_1 = 3.7$ h,	322	0.8 (0.2)
(2)	$\tau_r = 10.7$ h.	**E_y**	$T_2 = 8.4$ h.	105	**2.9 (0.7)**
November 25–26, 1978	$\|D_p\| = 123.5$ nT,	$\epsilon(\alpha = 0.1)$	Rectangular shape,	22	> 20
	$t_p = 5.7$ h,	$\epsilon(\alpha = 0.3)$	$T = 5.4$ h.	66	2.0 (0.3)
(3)	$\tau_r = 9$ h.	**E_y**		35	**5.5 (1.4)**
March 10–11, 1979	$\|D_p\| = 118$ nT,	$\epsilon(\alpha = 0.1)$	Triangular shape,	26	> 10
	$t_p = 7$ h,	$\epsilon(\alpha = 0.3)$	$T_1 = 6.2$ h,	78	2.0 (0.3)
(4)	$\tau_r = 8$ h.	**E_y**	$T_2 = 3.4$ h.	35	**9.5 (3.5)**
September 18–19, 1979	$\|D_{p1}\| = 118$ nT,	**$\epsilon(\alpha = 0.1)$**	Double square,	25	**13 (5)**
	$t_{p1} = 5$ h,	$\epsilon(\alpha = 0.3)$	$T_1 = 5.8$ h,	75	1.6 (0.3)
(5)	$\tau_r = 10.7$ h.		$T_2 = 3.0$ h,		
			$\Delta T = 0.8$ h.		
		E_y	Double square,	35	4.7 (1.0)
			$T_1 = 6.0$ h,		
			$T_2 = 2.7$ h,		
			$\Delta T = 0.8$ h.		

Starting from the energy balance equation(1), the values of τ for the main phase of the considered events, approximated by either the simple rectangular or triangular functions, (Table I, column 6). The energy input to the ring current is considered to be given either by the azimuthal electric field, E_y, or the Akasofu's, ϵ, function with the coefficients of energy transformation given by above. The value of τ_r, for the recovery phase, is estimated from a simple exponential decay, starting at the instant for which the input energy of the model has ceased (Table I, column 2). The combination of input function and τ that best fit the resulting $D(t)$ to the observed behavior of the ring current are shown by bold-characters figures on Table I.

From the present analysis is not possible to obtain a unique value of τ for the analized storms, since in each case the assumed form for the energy input function has to be taken into account. However, the estimation of τ in the way above described allows one to establish a highly probably threshold for its physical value, such that the energy balance equation is satisfied. For the intense storms here considered, the results shown on Table I for the estimated values of τ are between about 3 hours (event #2) and 13 hours (event #5). These values seem to be larger than those determined in a previous study [5] but within the expected order of magnitude, under the constrain assumed here, namely constant τ [i.e., 7, 8]. On the other hand, Detman et al. [8], using the adaptive filtering numeric technique for the same ISEE-3 events, have found larger values of τ.

The conclusion from the present study is that actual value of the ring-current decay time for the main phase, τ, seems to be a function of energy input, in agreement with previous studies [5, 8]. This fact imposes an intrinsec threshold for the resulting ring current that avoids that a determinated natural energization limit could be exceeded. Therefore, if the solar wind input is highly energetic, like for instance event #2 (September 29–30, 1978), a small value of τ (around 3 hrs.) will be sufficient to allow the development of an intense storm ($|Dst_p| = 207$ nT). On the other hand, for cases for which the solar wind energy input is not so energetic, like event #4 (March 10–11, 1979), the existence of relatively larger values of τ (around 10 hrs.), allows the development of an intense storm (($|Dst_p| = 118$ nT) as well. It is not clear as yet how the whole mechanism of the ring current formation works and can be regulated in order to get self adjusted to determined threshold. However, the aim of this note is to call the attention to the physical limitation imposed to the values of the ring current decay time, τ, as a result of the balance energy equation.

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