

Scaling of Bootstrap Current in the Framework of a Self-Consistent Equilibrium Calculation

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1. Introduction A detailed study of the bootstrap current dependence upon plasma profile parameters and the effects on plasma equilibrium quantities such as internal plasma inductance, normalized and poloidal beta values, loop voltage and central safety factor, is performed in the framework of a self-consistent equilibrium calculation in tokamak plasmas. In this model, the total plasma current is composed by the diamagnetic, Pfirsch-Schlüter and the neoclassical ohmic and bootstrap currents [1]. The bootstrap current here is generated by thermal particles and its profile is calculated according to the model described by Sauter et al [2]. Variations of Z_{eff} , plasma elongation, magnetic field and plasma current are also carried out. This study is performed for the spherical tokamak ETE (Experimento Tokamak Esférico) at INPE, in Brazil, and our results are compared to a general empirical scaling law proposed by Hoang in [3]. Due to some deviations observed between Hoang's scaling and our results, a new scaling law is being tested and the results of this study are discussed in the present work.

2. Procedure for the study We have considered a fixed reference case of plasma profiles and varied each of the profile parameters under study keeping all the others constant. Doing this, we can follow the trend of the bootstrap current for variations of a single parameter in the self-consistent calculation, which is very interesting from the experimental point of view. The pressure, electron and ion temperature profiles were taken as Gaussian shaped functions as in $f(\rho) = f(0) \exp[-\alpha_f (\rho/(w_f - \rho))^2]$. The parameter $f(0)$ determines the central value, whereas α_f and w_f control the gradient and the width of the profile determining the value at the boundary. The pressure profile is a fixed input in the equilibrium calculation so the density profiles were derived taking into account the total pressure in the plasma and the quasi-neutrality condition. Stability analyses were not included in the code even though we have been careful in order not to consider normalized beta values above 6. Our reference case was taken with the following parameters: $p(0)=15\text{kPa}$, $T_{e,i}(0)=1\text{keV}$, $T_{e,i}(a)=0.1\text{ keV}$, $\alpha_p = 3$, $\alpha_{T_e} = 0.02$, $\alpha_{T_i} = 2$, $\kappa(a) = 2$, $\delta = 0.3$, $I_p=200\text{kA}$, $B_0=0.4\text{T}$ and $Z_{\text{eff}}=1$ and, the bootstrap current was calculated according to the model described by Sauter et al [2]. Separate studies on $T_e(0)$, $T_i(0)$, α_{T_e} and α_{T_i} , keeping all the other parameters fixed, required pre-calculated pressure profiles in order to fix the density profile among all cases analysed. Variation ranges were set from 0.4 to

1.6 keV for $T_{e,i}(0)$ and from 0.02 to 10 for $\alpha_{Te,Ti}$. Figure 1 shows the bootstrap current fraction when these parameters are varied.

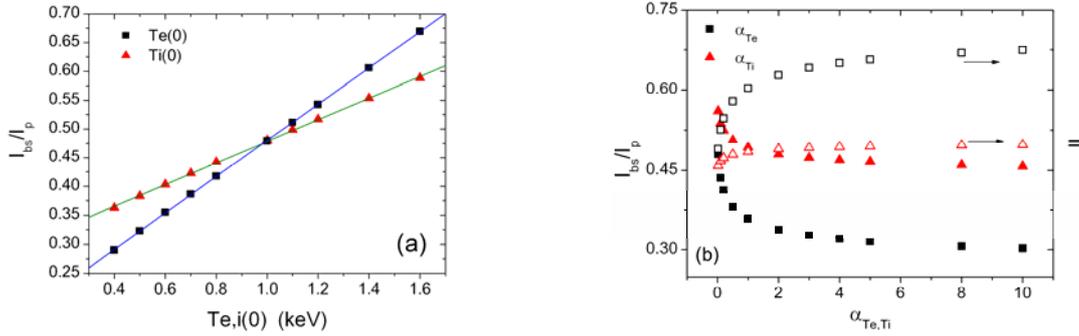


Figure 1: Behaviour of I_{bs}/I_p as $T_{e,i}(0)$ and $\alpha_{Te,Ti}$ are varied

The increase of I_{bs}/I_p , observed as $T_{e,i}(0)$ increase, is related with higher temperature and pressure gradients achieved and lower collisionality. Regarding the temperature peaking factors, it is clearly observed in Fig. 1b that the increase of the bootstrap current fraction is closely related with the broadening of the current density profile $j(\rho)$, as $\alpha_{Te,Ti}$ decrease. Broader temperature profiles, (specially in the electron case), when all other profiles are kept constant, cause the broadening of $j(\rho)$, noticed from the decrease of the internal plasma inductance l_i . Scans of $p(0)$ and α_p in relation to the reference case and for fixed temperature profiles are translated by variations of the density profile. Variation ranges for these parameters were considered from 6.0 to 18.0 kPa for $p(0)$ and from 0.5 to 10.0 for α_p . The increase of I_{bs}/I_p for higher values of $p(0)$ and broader pressure (or density) profiles is basically related to the increase of the density and pressure gradient terms in both cases. The maximum of bootstrap current profile will coincide with the maximum of the pressure gradient profile, peaking closer to the centre as α_p increases. The bootstrap current fraction was also analysed in relation to some equilibrium parameters. A wide variation of the internal plasma inductance l_i is observed for variations of $\alpha_{Te,Ti}$ and α_p . It decreases as the peaking factors decrease and this is associated with the broadening of the current density profile that generates higher fractions of bootstrap current. The central safety factor increases for broader pressure and temperature profiles and higher values of $T_{e,i}(0)$ and $n_e(0)$. Its increase is also mainly related to the broadening of the current density profile. Wide variation ranges for V_{loop} are obtained for variations of the electron temperature profile parameters due to the connection of this profile with the plasma conductivity. In relation to β_{pol} and the normalized beta β_N , we see for the former that the normal linear trend for the I_{bs}/I_p in relation to this parameter is not followed for α_p variations whereas for β_N , the linear trend is not observed for α_{Te} and κ .

3. Scaling Law A comparison of I_{bs}/I_p provided by our self-consistent equilibrium code for ETE with an empirical scaling law established by Hoang et al [3], based on experimental data of machines of large aspect ratio and circular cross-sections, is shown in Fig. 2. There is a reasonable agreement between our calculations and Hoang's scaling except for plasmas with high β_N , possibly because this scaling was empirically obtained for high aspect ratio plasmas where high β_N values are not easily achieved. Hoang's scaling is written as $I_{bs}/I_p = 0.45 \epsilon^{0.5} \beta_{pol}^{0.92} c_p^{0.47} c_j^{-0.45}$, with $c_p = p(0)V/\int p dV$ and $c_j = j(0)/(I_p/S_T(a))$. The parameters c_p and c_j are related to the peaking of the pressure and current density profiles respectively, and $S_T(a)$ to the area of the plasma cross-section. We tested a new scaling in order to try to diminish deviations observed between our equilibrium calculations and Hoang's scaling. Following a suggestion by Pomphrey [4], this new scaling is written in terms of the normalized beta value since in this way we can perform future studies of profiles effects on the bootstrap current at constant achievable beta values. This is justified by the fact that the achievement of high beta values and high fractions of bootstrap current, both desired in tokamak devices, compete with each other (see ref.[4] and references therein). It is also interesting to express the bootstrap current fraction in terms of the edge q value, considered here as q_{cyl} , since it is this parameter that has to be kept constant when making comparisons at different aspect ratios [5]. Moreover, from the analyses described in the previous section, we conclude that the shape of the current density profile and the magnitude of the density gradient have an important effect on I_{bs}/I_p . For this reason, an extra dependence on the internal plasma inductance parameter, stronger than that brought inside β_{pol} , was considered in this scaling. Finally, a dependence on the pressure peaking factor was also emphasized. Our new scaling was then expressed as:

$$\frac{I_{bs}}{I_p} = 5 \epsilon^{1/2} C_{bs} c_p^{1.012} \frac{\beta_N q_{cyl}}{li^{1.243}} \left(\frac{R_0(a)}{a} - \frac{\delta}{4} \right) \left(1 - \frac{\delta^2}{8} \right) \quad (2)$$

$$\text{with } \beta_{pol} = 5 \beta_N q_{cyl} / \left[li \left(R_0(a)/a - \delta/4 \right) \left(1 - \delta^2/8 \right) \right] \quad (3) \text{ and } q_{cyl} = 5a^2 \kappa B_0 / \left[I_{p(MA)} R_0(a) \right] \quad (4)$$

Variations of several plasma parameters as described in the previous section were carried out generating 360 points for the ETE tokamak. Note that in our fitting (Eq. 2), the dependence on li is about 25% stronger than the dependence brought inside β_{pol} (Eq. 3). The terms in parentheses are related to plasma geometry. Figure 2a shows a comparison between our scaling (black points) and Hoang's scaling (green) against the results obtained from the self-consistent

equilibrium calculation. There is a better agreement between our scaling with the equilibrium calculations. The blue and pink points represent an equilibrium with $\alpha_p = 0.5$ for Hoang's and the present work scalings respectively, with all other parameters given as in the reference case. The biggest discrepancy between Hoang's scaling and the equilibrium calculation is observed in this case. For this particular set of parameters, $\beta_N = 4.8$, relatively high for conventional aspect ratio tokamaks. Figure 2b shows our scaling when the dependence on c_p is not taken into account. The colored points illustrate scans of α_p in relation to a given reference case showing a linear behaviour that deviates from the normal trend. In Fig. 2c the dependence on c_p is accounted as described in Eq. 2, showing a much better agreement with the equilibrium calculation results. Finally, Fig.2d shows a histogram with the errors obtained and their frequency of occurrence when our scaling is compared with the equilibrium calculation result.

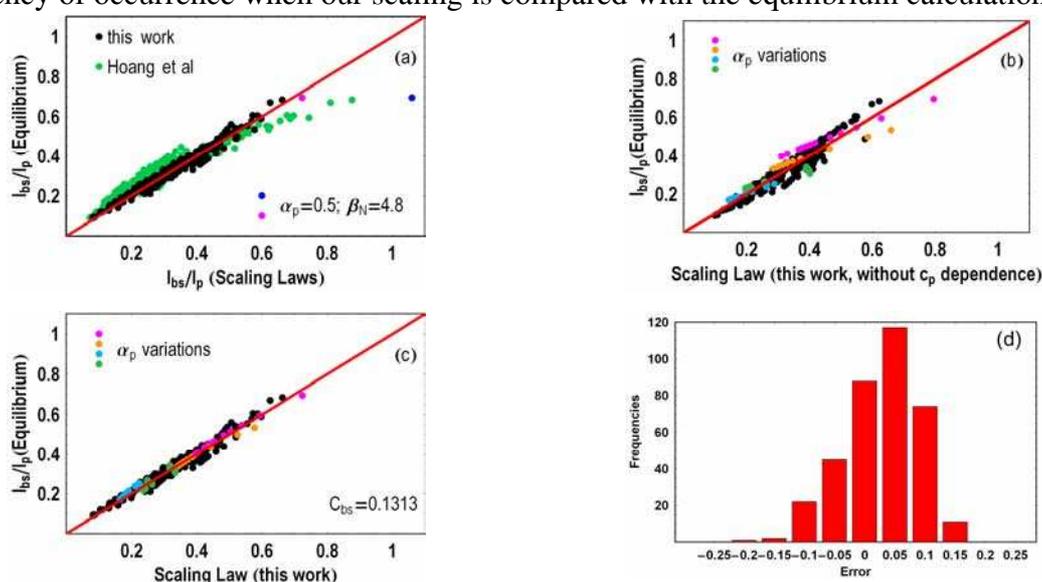


Figure 2: Comparison of scaling laws with equilibrium calculations: this work (black), Hoang (green) (a); this work scaling without c_p dependence (b); this work scaling with c_p dependence (eq.(2)) (c); histogram of errors (d).

Conclusions A study on the bootstrap current dependence upon plasma profiles was carried out and a new scaling law for I_{bs}/I_p , based on a self-consistent equilibrium calculation for ETE, is proposed. This scaling provides errors mostly up to 10% for ETE but should still be tested in other machines, preferably with experimental data.

References

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