Potential vorticity representation of the atmospheric climate processes.

Michael V. Kurgansky: Departmento de Física de la Atmósfera y del Océano, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Concepción, Chile.
email: kurgansk@udec.cl

Igor A. Pisnichenko: Instituto Tecnológico SIMEPAR (Sistema Meteorológico do Paraná), Curitiba, Brasil.
e-mail: igor@simepar.br

Resumo

A aplicação do conceito de vorticidade potencial de Ertel (VPE) às investigações dos processos climáticos na atmosfera da terra dirigidos pelas fontes de calor e atrito está sendo discutida. Uma liberdade fundamental disponível na definição geral de VPE, que consiste na escolha arbitrária da função da temperatura potencial, pode ser usada para chegar na modificação de VPE com taxa do tempo de mudança mínima na quantidade total de substância de vorticidade potencial (a "carga de vorticidade") acumulada no hemisfério da atmosfera. Essa VPE "modificada da melhor maneira" (q daqui por diante) é representada como "uma variável lenta do clima". Dentro deste ponto de vista, baseado no uso da característica meteorológica q, são considerados dois regimes ideais atmosféricos. O primeiro regime é associado com FDP (a Funçào de Densidade da Probabilidade) de q de forma exponencial. O segundo regime é caracterizado pela repartição igual da massa de ar entre q-valores. Os estados reais do sistema atmosférico de clima estão compreendidos entre estes dois regimes limitados (geralmente mais perto do primeiro) e sua dinâmica pode ser expressada nos termos da proximidade relativa a cada um destes regimes. A abordagem proposta tem um potencial de grande redução do número dos graus de liberdade, fisicamente necessários, nos modelos de clima. Uma dificuldade conceitual central sobre a maneira de reconstrução de q-distribuição geográfica, baseada no conhecimento de sua FDP, é discutida e os exemplos de modelo de tal reconstrução são dados. Baseando-se na análise global da série dos dados, os exemplos de como esta abordagem estatística na base de vorticidade potencial q pode ser aplicada para descrição da variabilidade climática da atmosfera nas escalas intra- e inter-anuais, são dados.

INTRODUCTION

The atmosphere-ocean climate system can be described by equations which result from fundamental principles of mass, momentum and energy conservation. In addition to these principles, the atmospheric and oceanic flows obey a general Ertel's potential vorticity (EPV) theorem (Ertel 1942).

In the atmospheric context, the broad area of EPV application covers theoretical investigations, diagnosis, analysis and modelling of tropospheric and stratospheric dynamical processes (e.g., Hoskins et al. 1985; Kurgansky and Tatarskaya 1987; Holton et al. 1995). Far less frequently, EPV is considered an indicator of atmospheric long-term, climate processes, although a few appealing attempts to apply 'PV-thinking' in the climate theory have to be mentioned (e.g., Koshyk and McFarlane 1996; Kirk-Davidoff and Lindzen 2000.).

The well-known freedom available in the general definition of EPV, namely that the potential temperature (PT) entering Ertel's potential vorticity may be replaced by any monotonic function χ of PT without loss of material conservation property, means that under adiabatic and frictionless approximation no particular PV modification has greater fundamental physical significance than any other. Although they can have some advantages for visualisation of large-scale processes in the troposphere and stratosphere (Kurgansky and Tatarskaya 1987; Lait 1994).

The situation basically changes when both diabatic heating and frictional forcing are important, as it is common in the climate problem. Here, the existing arbitrariness in EPV definition can be used to arrive at an 'optimal' modification of potential vorticity that minimises the time rate of change in the total amount of potential vorticity substance (the 'vorticity charge') accumulated in the hemispheric atmosphere. In such circumstances, there is some sort of balance, or nearly cancellation (in an integrated, average sense) between frictional and diabatic PV forcing. This 'optimally modified' EPV (MPV, or alternatively q, hereafter) examples a 'slow climate variable' owned regular statistical properties, namely its probability density function (PDF) is fair close to a reference exponential PDF, as shown empirically by Kurgansky and Prikazchikov (1994), who used 1979-1980 First Global GARP Experiment (FGGE) data, and by Kurgansky and Pisnichenko (2000), who calculated monthly-mean PDFs of q for 1980-89 European Centre for Medium Weather Forecasts (ECMWF) data.

METHODS AND DATA

Principal equations.

Following Obukhov (1964) we take $\chi(\Theta)=-p^*(\Theta)/g$, where p^* is a reference pressure depended upon potential temperature Θ, and g is the gravity acceleration, and apply the term "potential vorticity" to the quantity

$$q = \rho^{-1}\mathbf{Z}\cdot\nabla\left(\chi(\Theta)\right) = \rho^{-1}\mathbf{Z}\cdot\nabla\left(-\,p^*(\Theta)/g\right), \qquad (1)$$

having an order of magnitude of the Coriolis parameter ($10^{-4}\ \mathrm{s}^{-1}$). Here, ρ is the density and $\mathbf{Z}$ is the absolute vorticity. For the p^* we use the function

$$p^* = A - B\cdot\tan^{-1}\left[C\left(\Theta-\Theta_0\right)\right]. \qquad (2)$$

Here, A, B, C and Θ_0 are some constants.
From (1) it follows that the integral

$$Z_A = \iiint\limits_V q\rho\, dV \equiv \iiint\limits_V [\mathbf{Z}\cdot\nabla(-\,p^*(\Theta)/g)]dV\ ,$$

taken over the volume V of the atmosphere becomes the well-defined finite quantity and is named the "atmospheric vortex charge", which stresses an analogy with the electric charge conservation law in electrodynamics. When V contains the hemispheric portion of the atmosphere one arrives at the atmospheric vortex charge either over the Northern Hemisphere Z_{NH} or over the Southern Hemisphere Z_{SH}, such that $Z_A= Z_{NH}+ Z_{SH}$.

Dynamical fundamentals

Under the influence of diabatic heating $\dot{\Theta}$ and frictional forces $\mathbf{F}$ the potential vorticity q transforms according to the equation (e.g. Haynes and McIntyre 1990)

$$\dot{q} = \rho^{-1}\left[\mathbf{Z}\cdot\nabla\dot{\chi} + \nabla\chi\cdot\nabla\times\mathbf{F}\right], \qquad (3)$$

where a dot above variables denotes the material time-derivative. Equation (3) keeps its form under the transformation: $\chi\Rightarrow\chi^*=\Phi(\chi)$, $q\Rightarrow q^*=\Phi' q$, Φ being an arbitrary function. Isoscalar surfaces $q=const$ and $\chi=const$ divide the atmosphere into (q, χ) solenoids, along which air masses flow during adiabatic and frictionless processes. It was shown in Kurgansky and Pisnichenko (2000) that a unique choice of χ exists, when air mass contained in an infinitely thin and quasi-zonally oriented (q, χ)-solenoid is nearly preserved in diabatically and frictionally driven atmospheric flows, that is equation $\left(\partial\dot{q}/\partial q\right)+\left(\partial\dot{\chi}/\partial\chi\right)=0$ is fulfilled, and this choice corresponds to the case of an "optimal" potential vorticity modification.

Basic statistical arguments

Consider the quantity $\mu(q,\chi)\ dqd\chi$, which equals the part of the total atmospheric mass, enclosed in the solenoid formed by intersection of the surfaces q, $q+dq=const$ and $\chi,\chi+d\chi=const$. If $\iint \mu(q,\chi)\ dqd\chi=1$, where an integration is taken over all q and χ values, then $\mu(q,\chi)$ function may be regarded as the probability density of q and χ values for a randomly chosen air particle (Obukhov 1964). The atmospheric vortex charge equals $Z_A=m_A Q_A$, where m_A is the total atmospheric mass, and $Q_A=\int q\mu(q,\chi)dqd\chi$ is the first momentum of $\mu(q,\chi)$ distribution. The $\mu(q,\chi)$ distributions were calculated separately for the Northern Hemisphere (NH) and the Southern Hemisphere (SH). They are $\mu_{NH}(q,\chi)$ and $\mu_{SH}(q,\chi)$, respectively.

Reference distribution

We seek a stationary reference air mass distribution μ between infinitely thin, quasi-zonally oriented (q,χ)-solenoids. Following classical arguments by Gibbs (1902) and taking into account the principle of "potential vorticity substance" conservation, the most simple and fundamental choice is to set

$$\mu=\mu_B(q,\chi)=\mu_0 \exp\{-q/Q\},$$

with μ_0 as a constant.
Integration over all q values gives $\int \mu_0 Q\, d\chi=1$, and, as a consequence

$$\mu_B(q)=\int\mu_B(q,\chi)d\chi=\exp\{-q/Q\}\int\mu_0 d\chi=Q^{-1}\exp\{-q/Q\}.$$

Further on, $\mu(q,\chi)$ is also reduced to the 1-D probability density $\mu(q)=\int\mu(q,\chi)\, d\chi$.

A statistical approach to the problem of searching for the optimal PV modification is suggested. The informational entropy defined according to Shannon and von Neumann (e.g. Katz 1967) is introduced for description of the air mass statistical distribution on PV and χ values. It can be interpreted as a general measure of the degree of uncertainty in PV and χ values for a randomly chosen air parcel. Reference distribution supplies the conditional maximum of informational entropy provided that the total amount of modified potential vorticity substance, i.e. 'atmospheric vortex charge', is kept constant. The informational entropy deficit, taken as a difference between the maximum possible and actual information entropy values, tends to vanish when canonical (PV, χ) coordinates are used.

For both hemispheres separately the distribution

$$\mu_B(q)=|Q|^{-1}\exp(-q/Q), \qquad (4)$$

supplies the maximum of the informational entropy $H=-\int\mu\log\mu dq$, provided

$$Q=\int q\mu_B(q)dq=\int q\mu(q)dq, \qquad (5)$$

holds. The informational entropy maximum value is equal to

$$H_B=-\int\mu_B\log\mu_B dq=\log|Q|+const, \qquad (6)$$

and the informational entropy deficit

$$\Delta H=H_B-H \qquad (7)$$

characterises the degree of closeness of $\mu(q)$ and $\mu_B(q)$ distributions.

"***Optimal" potential vorticity modification***

When seeking the "optimal" function p_0^* we gave variations in C and Θ_0 in (2). It was assumed that A=681 hPa and $B=(2/\pi)A$=433.5 hPa. ECMWF data for 1980-89 were used and for every year the January and July monthly-mean distributions $\mu(q)$ were calculated for 64 different pairs of C and Θ_0 values for NH and SH, separately. The values of Q, H, H_B and $\Delta H=H_B-H$ have been calculated and then averaged over the entire 10-year period. As it is seen in Fig. 1a, $\Delta H=\Delta H(C,\Theta_0)$ attains minimum values in the vicinity of a line given by the linear regression equation $\Theta_0-292.55=321.4\times(C-0.04614)$. Here, Θ_0 is expressed in degrees Kelvin (K) and C is given in K^{-1}. Figure 1(b) shows that the position of minimum for the corresponding standard deviations of ΔH coincides with that of the mean ΔH values. This confirms that that we have arrived at a statistically stable state of minimum. For a further analysis, the concrete values of Θ_0=293 K and C=0.04614 K^{-1}, lying close to the regression line, have been chosen. Figure 2 demonstrates evident closeness between $\mu(q)$ and $\mu_B(q)$ distributions for such an optimal PV modification. Here, monthly-mean statistics for January 1980 in the NH have been used. Some systematic deviations occur only at high PV values.

RESULTS AND DISCUSSION

Informational entropy interannual variations

Interannual evolution of the informational entropy H for Θ_0=293 K and C=0.04614 K^{-1} within the 1980-89 period is presented in Fig. 3. Correlation matrix $r_{ij}=r(Y_i,Y_j)$, i,j=1,2,3,4 ($Y_1 = H_{JAN}^{NH}$, $Y_2 = H_{JAN}^{SH}$, $Y_3 = H_{JUL}^{NH}$, $Y_4 = H_{JUL}^{SH}$) between JAN, JUL and SH, NH informational entropy values after the subtraction of the linear trends in Y_i looks like as

$$\mathbf{r}' = \begin{pmatrix} 1 & 0.83^* & 0.04 & 0.68^* \\ 0.83^* & 1 & -0.20 & 0.81^* \\ 0.04 & -0.20 & 1 & -0.14 \\ 0.68^* & 0.81^* & -0.14 & 1 \end{pmatrix},$$

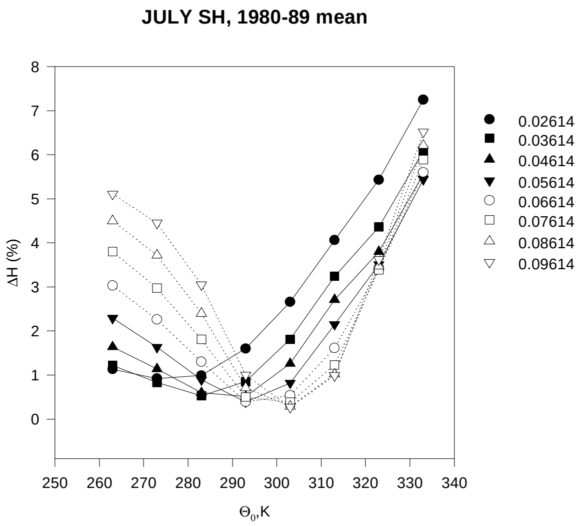
where the only values marked by an asterisk are significant. We observe that the hemispheres in July are discoupled but in January they are strongly enough linked in the interannual time-scale. Possible linkage might originate from either phenomena like El Niño events occurring around January or standing planetary waves propagation across the equator. Due to huge thermal inertia of oceans dominating over the Southern Hemisphere, Y_2 and Y_4 values are well-correlated too. Linkage between Y_1 and Y_4 occurs via Y_2, and $r'_{14} \approx r'_{12} r'_{24}$.

The above mentioned linear trend can be described by linear regression equations $Y_i=a_iX+b_i$, i=1,2,3,4, where X denotes years and varies in the range of 80-89. The a_i and b_i coefficients are given in Table 1.

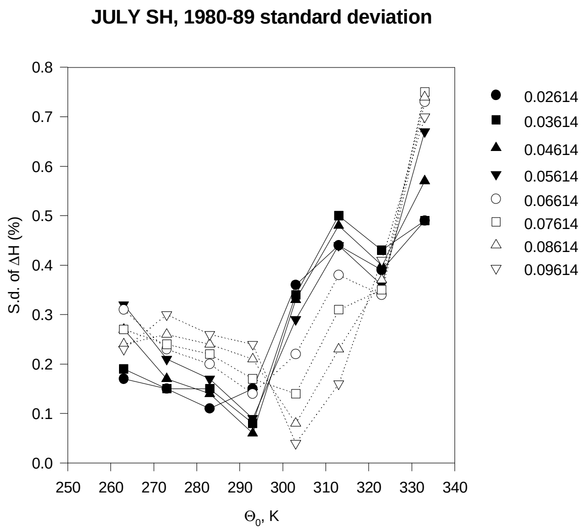
<u>TABLE 1</u>

i	1, NH (January)	2, SH (January)	3,NH (July)	4,SH (July)
a_i	-0.000685	0.00176	0.0000242	0.00147
b_i	0.725	0.501	0.609	0.544

Approximating actual H values by $H_B \approx (\log 19)^{-1}\log|Q|+const$ and using the linear regression equations for describing the interannual evolution of H during years we can obtain the formula: $\eta_i = \left|Q_i^{1989}\right| / \left|Q_i^{1980}\right| = 19^{10 a_i}$, which express decadal increase or decrease of vorticity charge. The coefficients η_i have the following values: η_1=0.980 (NH, Jan), η_2=1.053 (SH, Jan), η_3=1.001 (NH, Jul), and η_4=1.044 (SH, Jul).



(*a*)



(*b*)

<u>Figure 1</u>. (*a*) Dependence of informational entropy deficit ΔH on the potential temperature $\Theta=\Theta_0$ at the maximum of $\left|dp^*(\Theta)/d\Theta\right|$ in (2) for 8 different values of the parameter C (K^{-1}) that

determines the vertical temperature lapse rate. The case of the Southern Hemisphere, July is considered; ΔH are calculated using monthly-mean statistics averaged over 1980-89 period, computed ΔH values are expressed in percents of log19; (*b*) Standard deviations of ΔH from their mean values for 1980-89. Other notations are as in (*a*); SH, July.

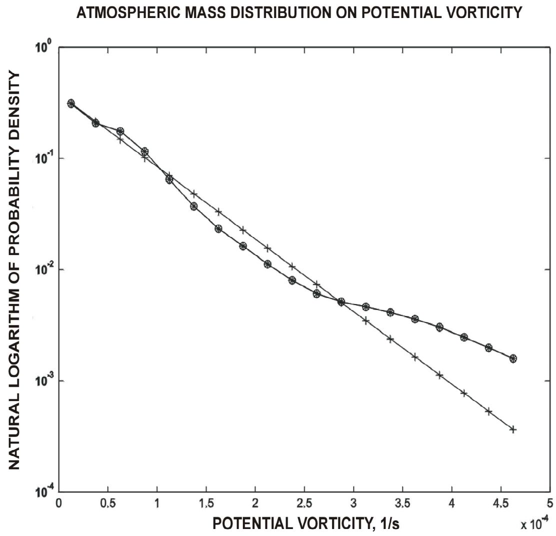


Figure 2. Atmospheric mass distribution on optimally modified PV (Θ_0=293 K, C=0.04614 K^{-1}) for SH, January 1980; (–⊗–⊗–⊗–) is the actual distribution and (–+–+–+ –) is the corresponding reference distribution.

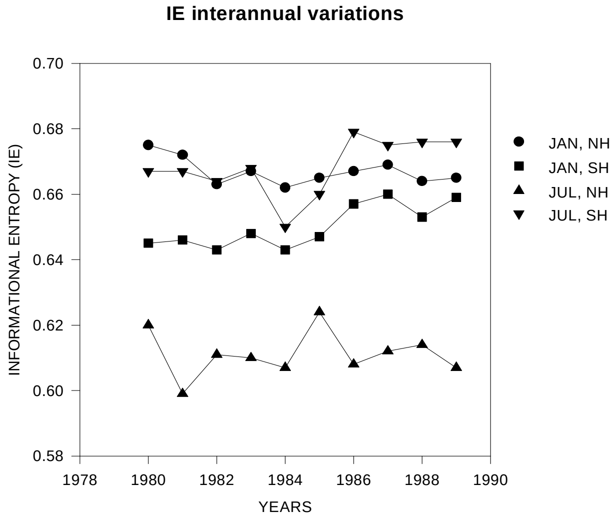


Figure 3 Interannual variability of the informational entropy H of monthly-mean air mass distribution on optimally modified PV for the 1980-89 period.

Statistically significant values of η>1 for SH, indicating an increase of vorticity charge in this hemisphere during 1980s, may be either the consequence of actually occurring Antarctic region cooling (Kelly and Jones 1996) or an artefact of changing the ECMWF operational analysis model in 1983 from a grid point to a spectral one (see also Simmons and Keay 2000). We plan to separate these effects more clearly in the nearest future, with the use of NCEP data set. In any case, the obtained results confirm how MPV statistics may serve a sensitive tool to diagnose data archives and AGCM outputs.

Dynamical-statistical model of zonal atmospheric circulation.

One of the most important and hard issue of modern climatology is the study of the spatial-temporal distribution of rainfall to foresee very strong downpour. Here we try to relate the evolution of precipitation patterns over Central and Southern Chile with variations in MPV statistics of large-scale atmospheric circulation.

Let us suppose the explicit existence of two different circulation regimes in the atmosphere. The first regime, which we conditionally call the Hadley regime, occupies the low-latitudinal equatorial belt till latitude φ_0 and includes both the direct Hadley meridional circulation cell and permanent subtropical high-pressure anticyclones. The second regime with high-magnitude value and horizontally well-mixed potential vorticity (horizontal mixing has to be understood as resulting in horizontally uniform field of MPV-expectation values) situated above the latitude φ_0 will be named the Rossby regime.

The exponential distribution (4), introduced above, serves as a reference probability density function of encountering an air particle with MPV value equal to q. Considering the Southern Hemisphere, we will treat pseudo-scalar q as positive and growing in the equator-pole direction, and latitudinal coordinate x=sinφ is also assumed positive.

It is assumed that the value q=q^* exists, which separates air masses belonging to the region and corresponding to the above mentioned regimes of circulation. All air masses with q<q^*, which we call anticyclones, fall into the Hadley regime zone, and air masses with q>q^*, which we called cyclones, fall into the Rossby regime zone. The probability of a randomly selected air mass belonging to the Rossby regime can be calculated using the formula

$$\int_{q^*}^{\infty} \mu(q)\,dq = 1 - x_0 . \tag{8}$$

Using (4) to estimate the integral on the left-hand-side of (8), one arrives at an explicit formula

$$\exp\left\{-\frac{q^*}{Q}\right\} = 1 - x_0 , \tag{9}$$

that links three parameters: q^*, Q and x_0. According to the ergodicity assumption, the ensemble averages within the Rossby zone are replaced with local time-means and, in addition, MPV-expectation values are considered spatially uniform and equal to

$$q_R = \int_{q^*}^{\infty} q\,\mu(q)\,dq \Bigg/ \int_{q^*}^{\infty} \mu(q)\,dq = q^* + Q > Q , \tag{10}$$

if equation (9) is used.

When considering two terrestrial hemispheric atmospheres as uncoupled, which is a reasonable approximation when one is interested in processes in the extra tropics of intra-annual duration, then it can be assumed that within Hadley zone the local time-mean values of q are spatially uniform and equal to q_H. In this case one has, similar to (10),

$$q_H = \int_0^{q^*} q\mu(q)\,dq \Bigg/ \int_0^{q^*} \mu(q)\,dq = -\frac{q^*(1-x_0)}{x_0} + Q < Q , \tag{11}$$

and, by definition, $Q = x_0 q_H + (1-x_0) q_R$.

Suppose that q^* is a quasi-constant, and its value is insensitive to seasonal changes. In the case considered, one has a step (discontinuity) in expected q-values across x=x_0, equal to

$$q_R - q_H = \frac{q^*}{x_0} > q^* , \tag{12}$$

which is seasonally dependent, via $x_0(t)$, and increases in magnitude during the winter season because $x_0(t)$ decreases (see below).

Here, we note that a second equilibrium regime of the atmospheric general circulation exists, which corresponds to air mass equipartitioning on q-values, provided q-s are given within the finite interval (0, q_{max}). A theoretical support for this may be the follows. If $\mu(q)$ is the PDF of q, such that $\int_0^{q_{\max}} \mu(q)\,dq = 1$, then the informational entropy $H = -\int_0^{q_{\max}} \log\mu(q)\,\mu(q)\,dq$ attains maximum value $\hat{H} = \log q_{\max}$ when $\mu = q_{\max}^{-1} = const$. This PDF corresponds exactly to the atmospheric zonal circulation with meridional MPV profile $q = x q_{\max}$, e.g. when the atmosphere is observed in the state of solid-body-like super-rotation. This uniform distribution is apparently distinct from that of the actual atmospheric climate state. In order to judge on the degree of closeness between the uniform and exponential distributions, we construct a reference exponential distribution

$$\mu^*(q) = (2/q_{\max})\exp\{-2q/q_{\max}\},$$

which has the same vorticity charge $Q^* = q_{\max}/2$ as the uniform distribution. This μ^*-distribution possesses the entropy $H^* = \log(e q_{\max}/2)$. The difference

$$H^* - \hat{H} = \log(e q_{\max}/2) - \log q_{\max} = \log(e/2) \approx 0.307$$

serves as a natural unit of informational entropy differences between two opposite limiting circulation regimes. To compare the results based on continuous and practically used discrete μ-distributions (see more details in Kurgansky and Pisnichenko 2000) we took the value Q=–0.63×10^{-4}s^{-1} (Southern Hemisphere, January, 1983). Here, with good accuracy, 2|Q|=5Δq and hence $\hat{H} = \log 5 \approx 1.609$, where Δq=0.25×10^{-4}s^{-1} is the unit of q-discretisation, applied in practical computations, with the total number N=19 of these Δq-cells, covering the q-axis from 0 to q_{max}=4.75×10^{-4}s^{-1}. From data of Kurgansky and Pisnichenko (2000) it follows for this month that $H^* = \log 19 \cdot 0.651 \approx 1.917$ and $H^* - \hat{H} \approx 0.308$, which practically coincides with the estimate for the continuous distribution. The entropy deficit (difference between H^* and informational entropy of the actual atmospheric state) equals to log19·0.0029≈0.0085 (Kurgansky and Pisnichenko 2000), which is less than 3% of $H^* - \hat{H}$. Thus we can conclude that the actual atmospheric circulation regimes might be considered as confined (by their informational entropy H values) between these two limiting reference circulation regimes (by their informational entropy values), having (i) exponential and (ii) uniform PDF of MPV, correspondingly, but positioned much closer to the former.

If $q^*=const$ (in reality we suppose that it alters more slowly than Q), then it follows from (9) that any long-term time-variation of δx_0, including the seasonal one, which correspond to the variations δQ, is described by the formula

$$\delta x_0 = -\frac{\delta Q}{Q}\left(1 - x_0\right)\log\frac{1}{1 - x_0}.$$

The maximum response in δx_0 to relative changes $\delta Q/Q$ is achieved for $x_0=1-e^{-1}\approx 0.632$ ($\varphi_0\approx 39.2°$), with a plateau in the vicinity of this latitude, and for a rather broad latitudinal range one can use a working formula

$$\delta x_0 \cong -0.368\left(\delta Q/Q\right). \qquad (13)$$

Available climatological data on surface air pressure distribution over Chile (Saavedra and Foppiano 1992 and references therein) enable one to judge on the latitudinal position of the location of maximum surface pressure (LMP), $x_{MP}=\sin\varphi_{MP}$ in our notations, with φ_{MP}~35° during the Southern Hemisphere winter and φ_{MP}~42° in summer, the annual-mean position being about φ_{MP}=38.5°. Saavedra et al. (2000) showed how well the LMP index quantifies the seasonal course of precipitation frequency in the Central Chile for $x \le x_{MP}$. It is not so straightforward to relate x_0 and x_{MP} without making some further model assumptions, but nevertheless, a good correlation between seasonal changes in x_0 and x_{MP} should be expected, or at least, reasonably hypothesized.

One can arrive at the same estimate for x_0, basing on quite general arguments. From one side, in the reference state (4) the informational entropy equals to $H = \log Q + \mathrm{const}$ (cf. Kurgansky and Pisnichenko 2000). From the other side, according to general statistical principles, a properly defined entropy of the Rossby's regime, S, is related to the probability of that regime (9) by the logarithmic formula

$$S = \log\exp\left\{-q^*/Q\right\} + \mathrm{const} \equiv \left(-q^*/Q\right) + \mathrm{const}.$$

Equalizing the characteristic 'temperatures' ϑ and ϑ_*, defined by $\vartheta^{-1} = dH/dQ$ and $\vartheta_*^{-1} = \left(\partial S/\partial Q\right)\big|_{q^*=\mathrm{const}}$, respectively, we get $q^*/Q = 1$ and hence $1 - x_0 = 1/e$, the latter because of (4). Therefore, we have succeeded to determine the 'statistically equilibrium' (climate-mean) position of the Rossby's regime border, which roughly coincides with 40° latitude. Small variations in Q-values, i.e., due to an annual course in insolation, would not violate the entropy difference H–S, if $\vartheta=\vartheta_*$ and q^* is kept constant. Therefore, this two-zone picture of general atmospheric circulation is well-consistent with a seasonal dependent forcing, under a sole assumption that the seasonal cycle amplitude is sufficiently small.

The representation of the edge of the Rossby's regime zone as an abrupt step in q-expectation values is, certainly, a crude idealisation, which moreover contradicts the very demands of atmospheric general circulation functioning, because such an edge would tend to act as a barrier to eddy meridional transport of MPV due to Rossby wave restoring mechanism. The eddy meridional transport of MPV is a necessary consequence of poleward eddy heat transport. Therefore, it is taken that a transitional zone of finite, synoptic-scale width Λ between Hadley and Rossby regime zones exists. It is also assumed that within the a transitional zone the expectation values of q are given by the expression

$$\bar{q}(x) = q_H + (q_R - q_H)w(x) \equiv q_H[1 - w(x)] + q_R w(x), \qquad (14)$$

where $w(x)$ is taken in a highly idealised analytical form

$$w(x) = \begin{cases} 0, & 0 \le x < x_0 - \Delta x \\ \frac{1}{2}\sin\left(\frac{\pi}{2}\cdot\frac{x - x_0}{\Delta x}\right) + \frac{1}{2}, & x_0 - \Delta x \le x \le x_0 + \Delta x \\ 1, & x_0 + \Delta x < x \le 1 \end{cases}, \qquad (15)$$

Notice how $w(x)$ can be interpreted statistically as the probability for a cyclonic air mass to be found at any latitude, labelled by x. The parameter Δx is related to the characteristic half-width $\Lambda/2$ of the transitional zone by the formula

$$\Delta x = \left(1 - x_0^2\right)^{1/2}(\Lambda/2a) \approx 0.775(\Lambda/2a), \qquad (16)$$

the latter has taken for x_0=0.632.

One can determine the characteristic width Λ of transitional zone in terms of Δq-step between Rossby and Hadley regimes. Assuming of maximum intensity of meridional MPV flux across the transitional zone one can arrive to the expression

$$\Lambda = 2\pi(K/\Delta q)^{1/2}, \qquad (17)$$

where K is the eddy diffusivity, which can be put equal to the heat diffusivity ≈$2.10^6 m^2 s^{-1}$, used for example in the energy balance models, as the meridional MPV transport are necessary to transfer heat from equator to polewards. Substituting (17) to (16) and using for Δq expression (12) we obtain

$$\Delta x = 0.382\cdot 10^{-6}\left(\frac{K\ x_0}{q^*}\right)^{1/2} \qquad (18)$$

Let us also assume that this transitional zone, as a whole, exposes seasonal meridional displacements according to a harmonic law

$$x_0(t) = A + B\sin\varpi t \qquad (19)$$

with $2\pi/\varpi = 1\,\mathrm{yr}$. To gain some insight into the problem, assume for a while that $B \ll \Delta x$, though in reality $B \sim \Delta x$, as we will see further on. Substituting (19) into (15) and using Taylor series expansion, one obtains within the interval $(A-\Delta x,\ A+\Delta x)$

$$w(x,t) \cong \frac{1}{2}\sin\left(\frac{\pi}{2}\cdot\frac{x-A}{\Delta x}\right) + \frac{\pi B\sin\varpi t}{4\Delta x}\cos\left(\frac{\pi}{2}\cdot\frac{x-A}{\Delta x}\right) + \frac{1}{2},$$

which clearly demonstrates a pronounced tendency towards increase of the amplitude of the seasonal cycle in w-values in the vicinity of the latitude x=A, corresponding to annual-mean position of the centre of the transitional zone. If to take into account that Δx also changes with time, according to (18), then we can specify slightly more the above written formula

$$w(x,t) \cong \frac{1}{2}\sin\left(\frac{\pi}{2}\frac{x-A}{(\Delta x)_0}\right) + \frac{\pi[B + (x-A)B/(2A)]\sin\varpi t}{4(\Delta x)_0}\cos\left(\frac{\pi}{2}\frac{x-A}{(\Delta x)_0}\right) + \frac{1}{2},$$

where $(\Delta x)_0 = 0.382\cdot 10^{-6}(K A/q^*)^{1/2}$.

Rainfall frequency

If it is assumed that every air mass of polar origin brings with it a certain probability of precipitation, equal to P_R, then the probability of rainfall $P(x, t)$ at a given latitude equals

$$P(x,t) = p(x,t)\cdot P_R \equiv \left[\frac{1}{2}\sin\left(\frac{\pi}{2}\cdot\frac{x - x_0(t)}{\Delta x}\right) + \frac{1}{2}\right]\cdot P_R, \qquad (20)$$

with $x_0(t)$ given by (19).

To prove these hypotheses, we took available monthly data on the distribution of precipitation (in days) for 15 meteorological stations in Central and Southern Chile covering the period 1950-1969 (Table 2, column 3-5) and fitted these data, using formula (20) with Δx=0.160, A=0.616 and B=0.047. The computed values for winter solstice (maximum rainfall), summer solstice (minimum rainfall) and equinox conditions (that are nearly coincide with annual-mean values), also given in Table 2 (column 6-8), depict reasonable agreement with empirical data, if some inconsistencies are disregarded for the most northern and southern stations, probably due to a discontinuity of the second x-derivative of $w(x)$ at $x=x_0(t)\pm\Delta x$. Notice, how A=0.616 is close to the theoretical value $x_0\cong0.632$ (see above) and B=0.047 corresponds to the amplitude of 3.47°×2°≈6.94° of annual displacements of the transitional zone, which agrees with the reported in Saavedra and Foppiano (1992) total magnitude of meridional shifts of MPL close to 7°.

TABLE 2.

Distribution of precipitation (in days) over Chile for period 1950-1969 (Department of Geophysics, University of Concepción, Chile, June 1971), columns ## 3 – 5, and its simple parameterisation, columns ## 6 - 8.

LOCATION	LATITUDE	MIN N° OF DAYS	MAX N° OF DAYS	MEAN N° OF DAYS	MIN N° OF DAYS	MAX N° OF DAYS	MEAN N° OF DAYS
La Serena	29.9°	0	2	1	0	4	1
Ovalle	30.6°	0	2.5	1.25	0	5	1.5
Valparaíso	33.0°	0	7	3.5	1	8	4
Santiago	33.5°	0	7.7	3.75	1	9	4
Constitución	35.4°	1	11.5	6.25	3	12	7
Linares	35.9°	1	10.5	5.75	3	13.5	8
Concepción	36.8°	2	16	9	4.5	14	9
Los Ángeles	37.5°	2	14	8	5	15	10
Padre Las Cases	38.8°	6	18	12	7	16.5	12
Valdivia	39.4°	7.5	22	14.75	8	17.5	13
Puerto Montt	41.5°	10	21	15.5	11	19.5	16
Castro	42.5°	12	23	17.5	12.5	20	17
Isla Guafo	43.7°	13	22	17.5	14	21	18
Puerto Aysen	45.4°	16	22	19	16	21	20
Cabo Raper	46.9°	21.5	21.5	21.5	17.5	22	21

In order to explain this B-value within our MPV-based framework we apply formula (13), which tells us that it is sufficient to assume seasonal variations $\delta Q/Q\approx0.13$ to explain the observed variations in x_0. The obtained value is approximately twice as large as calculated in (Kurgansky 1991), based on FGGE data set for the entire Southern Hemisphere; notice how for the Northern Hemisphere $\delta Q/Q\approx0.20$, for the same FGGE year. These quantitative discrepancies may be explained by an inhomogeneous distribution of variability of this transitional zone over the Hemisphere, and possibly, by the regional effect of Andes. The likely influential factors are also imperfections in the procedure of the 'optimal' PV modification and related deviations between the actual and reference PDF of MPV, if not to mention all other extreme simplifications in our model.

Finally, some MPV-related comments concerning the assumption of a 72% probability of rainfall (as empirical data suggest) inside the Rossby's zone have to be added. When expressed in MPV-terms, this rainfall probability can be given by the formula

$$P_R = \int_{\omega}^{\infty} \mu(q)dq \Big/ \int_{q^*}^{\infty} \mu(q)dq = \frac{\exp\{-\omega/Q\}}{1-x_0},$$

where ω is a lower threshold value of MPV above which rainfall starts, for a randomly chosen air mass. Equally, ω may be interpreted as a minimal value of MPV to explain the front formation, necessary for precipitation, within the taken air mass. If to adopt P_R=0.72, then $\omega = -Q\log(0.72 \cdot 0.368) \cong 1.33Q$. In this case formula (4) gives $q^* = Q$ and, therefore, within the interval $Q<q<1.33Q$, with 28%-probability for q to belong it (in a randomly chosen location), the precipitation within the Rossby's zone are locally absent, just because of q-values are non-sufficient to provide meteorological pre-conditions, necessary for precipitation.

Climate variations

Formula (20) can be used to estimate the sensitivity of rainfall frequency patterns, with respect to climate changes over the Southern Hemisphere. These changes can be quantified in terms of a shift of an annual-mean position of the transitional zone.

$$\delta P = \frac{dP}{dA}\delta A = -\frac{\pi}{4\Delta x}\cos\left(\frac{\pi}{2}\cdot\frac{x-A}{\Delta x}\right)P_R\,\delta A$$

The response δP reaches maximum value at $x=A$:

$$\delta P_{\max} = -\frac{\pi}{4\Delta x}P_R\,\delta A,$$

where δA is related with slow time-variations of Q by (13). When (i) using a linear regression equation

$$Q = \alpha + \beta \cdot \Delta T,$$

with ΔT as the equator-to-pole surface temperature difference and α, β as the empirical constants which fit both calculated Q-values and ΔT data, and (ii) assuming that mean-hemispheric surface temperature $\langle T \rangle$ is related to ΔT and equatorial temperature T_E by the formula: $\langle T\rangle = T_E - (1/3)\Delta T$, with T_E =*const*, one gets that $\delta\langle T\rangle$ =1 K leads to δA =0.019 and δP_{max}=–0.065, e.g. global hemispheric warming of 1K results in a decrease of precipitation frequency at $x=A$ by about 20% of its present value in this location. The obtained δA value corresponds to the equatorward shift of 1° of the transitional zone. In this connection it is necessary to mention how N. Saavedra, E. Müller and A. Foppiano in "Monthly-mean rainfall frequency model for Central Chile Coast: some climate inferences"(Submitted to International Journal of Climatology: currently under revision) have arrived at qualitatively similar conclusions by estimating the changes in precipitation patterns over Central Chile with the help of a linear regression equation to relate rainfall frequency with the latitudinal position of the location of monthly-mean surface air pressure maximum(LMP) and by considering scenarios when LMP=±1, ±2, ±3°.

For assessment of a possible ENSO effect we in a highly idealized manner assume that $\langle T\rangle$=*const* and therefore $\delta(\Delta T) = 3\delta T_E$. If to assume that during El Niño years δT_E =0.5 K, then δP_{max}=0.015 and the probability of precipitation increases by a few percent at $x=A$.

If one is interested in zonal mean atmospheric features, then this indeed very simple and crude scheme seems to predict qualitatively right tendencies towards increase precipitation in the subtropics during warm El Niño events and the corresponding decrease during cool La Niña ones (cf. Kiladis and Mo 1998). As far as it concerns Chile, significant quantitative underestimation occurs, nearly by the order of magnitude. For instance, in Aceituno (1988) the 1941-1983 rainfall data in Santiago for austral winter semester give the composite rainfall values 40% higher for positive SO phase, if compared to negative ones. We explain this discrepancy by our zonal-average approach and full neglect of regional, longitude-dependent peculiarities, which can strongly amplify the tendencies in hemisphere-scale processes.

CONCLUDING REMARKS

(1) Using the existing arbitrariness in a general Ertel's PV definition it is possible to construct a PV modification q, which corresponds to the highest possible degree of closeness between actual and reference, exponential, PV distributions. The exponential distribution has the same amount (per unit mass) of vortex charge Q over the Hemisphere as the real one, and possesses the maximum value of the informational entropy. In the latter sense, it is considered as a steady equilibrium climate distribution.

(2) Using ECMWF data and inspecting the January and July PV statistics temporal behaviour, we have detected a progressive growth of the vorticity charge in the Southern Hemisphere during the 1980s. This trend is accompanied by a decrease of the vortex charge in January for the Northern Hemisphere. The challenge is either to attribute these PV trends in the Southern Hemisphere to the real atmospheric warming over the Antarctic region or to explain it in the terms of a change in ECWMF operational analysis system in 1983. In the latter case, the proposed PV statistical computations may serve as a highly sensitive tool to diagnose the operational analysis systems quality.

(3) In the informational entropy related framework, two idealised steady equilibrium atmospheric regimes can be considered for a hemispheric atmosphere. The first is associated with the exponential distribution of q and the second is characterized by equipartitioning of air mass between q-values. The actual state of atmospheric climate system is confined between these two limiting regimes (generally, much nearer to the former) and their dynamics can be quantified in terms of relative closeness to each of these regimes.

(4) Using closeness between real and reference MPV distribution and assumption on the critical dependence on q of extra-tropical circulation systems, a simple zonal atmospheric model has been proposed to explain the seasonal climate variations in rainfall frequency over Central and Southern Chile in terms of meridional displacements of the critical latitude x_0 separating between cyclonic (high-latitude) and anticyclonic (low-latitude) air masses, marked by their q-values. The annual mean position φ_0=39.2° has been theoretically specified, which agrees fairy well with precipitation data over Chile. Changes in precipitation patterns over Chile, which accompany possible climate variations over the Southern Hemisphere are also estimated, using the proposed zonal model as a diagnostic tool.

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