

Column Generation Approach for the Uncapacitated Facility Location Problem

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A good strategy for the solution of a large-scale problem is its division into small ones. In this context, this work explores the lagrangean relaxation with clusters (LagClus) that can be applied to combinatorial problems modeled by conflict graphs and considers the graph partitioning to get clusters of vertices and edges. When removing the edges that connect the clusters, the conflict graph is divided in subgraphs with the same characteristics of the whole problem. Relaxing these edges in the lagrangean way, subproblems are solved, and an approximated solution is obtained with better bounds than the traditional lagrangean relaxation. Moreover, given that subproblems generate solutions to each cluster of independent form, a column generation approach for the UFLP is presented. This approach was applied to the Uncapacitated Facility Location Problem (UFLP) with new dual bounds for instances presenting large duality gaps.

The idea of the lagrangean relaxation with clusters (LagClus) was proposed by Ribeiro (2005) that considers the attainment of a conflict graph, the partitioning of this graph in clusters with the same characteristics of the original problem and the use of the lagrangean relaxation to incorporate the removed edges to the problem resolution. Figure 1 shows the phases of this process.

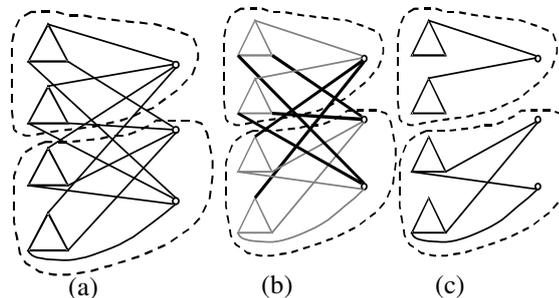


Figure 1 – Description of the LagClus. (a) Conflict graph. (b) Relaxation of the removed edges. (c) Subproblems' (clusters) resolution.

The Uncapacitated Facility Location Problem (UFLP) is a location problem extensively studied in the literature that involves fixed costs for locating the facilities, production costs and transportation costs for distributing the commodities between the facilities and the clients. The main objective of the UFLP is to choose the location of facilities to minimize the cost of satisfying the demand for some commodity.

Several solution approaches have been proposed for the UFLP in the last decades. There are exact algorithms like in (Körkel, 1989), but heuristics, metaheuristics and relaxations were natural choices for large-scale instances, due to the NP-hard characteristic of the UFLP (Cornuéjols et al, 1990). The greed heuristic developed by Kuehn & Hamburguer (1963) provided the base for the creation and application of several constructive algorithms, local search methods and sophisticated heuristics for this problem. Cornuéjols et al (1990) present a good historical summary, revision of applications, solution methods for the UFLP and an integer linear programming formulation for the UFLP.

To get the conflict graph (Atamtürk et al, 2000) for location problems, one has to work with the complement of the location variables ($\bar{y} = 1 - y$) (Cornuéjols & Thizy, 1982), which applied to the UFLP, becomes \overline{UFLP} . By partitioning the conflict graph $G = (V, E)$ in P parts V_1, V_2, \dots, V_P subgraphs $G_p = G[V_p]$ and a edge set $E_p = E(G_p)$ are defined. The edges on G that connect the subgraphs corresponding to vertices in different

clusters define the set $\hat{E} = E \setminus \bigcup_{p=1}^P E_p$ (edges in boldface in the Figure 1). Applying a graph partitioning heuristic to the conflict graph of \overline{UFLP} , a block-diagonal structure representation is obtained.

The classical implementation of column generation uses a coordinating problem, or restricted master problem (RMP), and column generator subproblems for the RMP. The last ones, by their dual variables, lead subproblems to the search of new columns that add new information to the RMP. Considering the problem defined by a block-diagonal structure representation above mentioned and if removed the set of restrictions defined by the edges on G that connect the subgraphs, the problem can be divided in P distinct subproblems. Thus, applying the Dantzig-Wolfe decomposition (DW) (Bazaara et al, 1990) for the linear relaxation (LP) of the \overline{UFLP} , the RMP is obtained.

The following steps are used for the column generation approach:

- a) Obtain the \overline{UFLP} .
- b) Obtain the RMP, by using the Dantzig-Wolfe decomposition.
- c) Generate the initial set of columns for the RMP.
- d) Solve the RMP and collect the dual variables.
- e) If there are not columns with negative reduced cost, stop.
- f) If the gap between the RMP value and the LagClus relaxation for the $\overline{UFLP} \leq$ a predefined value, stop.
- g) Insert columns
- h) If Number of columns \leq MaxColumn, go to step d).
- i) Remove columns and go to step d).

The concepts presented in this work were tested on instances with large duality gap that are computationally difficult for methods based on linear relaxation (Kochetov & Ivanenko, 2003), and are available at http://www.math.nsc.ru/AP/benchmarks/UFLP/Engl/uflp_dg_eng.html. The results show that the LagClus and the column generation approach present dual bounds with better quality than traditional lagrangean and linear relaxations for these difficult instances. The penalty is the computational time. It is intended to continue the efforts with the column generation approach to solve large-scale location problems, developing techniques that can reduce the computational time, like a more efficient treatment for the unproductive columns elimination, and the use of stabilization techniques. As a suggestion, the techniques shown in this work can be adapted and applied to other location problems, like the p-median.

REFERENCES

- Atamtürk A., Nemhauser G. L. and Savelsbergh M. W. P. (2000). *Conflict graphs in solving integer programming problems*, European Journal of Operational Research, 121, 40-55.
- Bazaara M. S., Jarvis J. J. e Sherali, H. D. (1990), *Linear Programming and Network Flows*, John Wiley & Sons, New York.
- Cornuéjols, G., Nemhauser, G. L. and Wolsey L. A. (1990), *The Uncapacitated Facility Location Problem*, in Mirchandani, P. B. and Francis, R. L., "Discrete Location Theory". Wiley-Interscience, New York, pp. 119-171.
- Cornuéjols G. and Thizy J. M. (1982) *Some facets of the simple plant location polytope*, Mathematical Programming, 23, 50-74.
- Körkel, M. (1989), *On the exact solution of large-scale simple plant location problem*, European Journal of Operational Research, 39, 157-173.
- Kochetov, Y. and Ivanenko, D. (2003) *Computationally difficult instances for the uncapacitated facility location problem*, in MIC2003: The Fifth Metaheuristics International Conference, Kyoto, Japan, 2003, pp. 41-1 – 41-5.
- Kuehn A. A. and Hamburguer M. J. (1963). *A heuristic program for location warehouses*. Management Science. 9, 643-66.
- Ribeiro, G. M. (2005) *Relaxação lagrangeana com divisão em clusters para alguns problemas de otimização modelados em grafos de conflitos*. 99 p. Proposta de Tese (Doutorado em Computação Aplicada) – Instituto Nacional de Pesquisas Espaciais, São José dos Campos, 2005.