Breeding and predictability in chaotic model

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The use of ensemble forecasting and data assimilation shows the importance of local predictability properties of the atmosphere in space and in time (e.g., Toth and Kalnay, 1993). The local/regionall loss of predictability is an indication of the instability of the underlying flow computed from a numerical model, where small errors in the initial conditions (or imperfections in the model) grow to large amplitudes in finite times. The stability properties of evolving flows have been studied using Lyapunov vectors (e.g., Alligood et al, 1996), and, more recently, with bred vectors (Kalnay, 2001).
The "breeding method" is a well-established and computationally inexpensive method for generating perturbations for ensemble integrations. In examination of the local structure of the vectors indicates that there may be substantial redundancy when multiple independent breeding cycles are performed in parallel, and the vectors can be inefficient in spanning the range of locally growing perturbations. Breeding (Toth and Kalnay, 1997) was developed as a method to generate initial perturbations for ensemble forecasting in numerical weather prediction at the National Centers for Environmental Prediction (NCEP). The method involves simply running the nonlinear model used for the control a second time, periodically subtracting the control from the perturbed solution, and rescaling the difference so that it has the same size as the original perturbation. The rescaled difference (a bred vector) is added to the control run and the process repeated. Their growth rate is a measure of the local instability of the flow. Bred vectors (BV) are perturbations, related to Lyapunov vectors that capture fast growing dynamical instabilities of the solution of a numerical model. Bred Vectors (BVs) are computed as follows (Kalnay et al., 2003).
1) Start with an arbitrary initial perturbation $\delta f(x,t)$ of size A defined with an arbitrary norm. This initialization step is executed only once. The size of A is essentially the only tunable parameter of breeding.
2) Add the perturbation to the basic solution, integrate the perturbed initial condition with the nonlinear model, and subtract the original unperturbed solution from the perturbed nonlinear integration

$$\delta f(x,t+\Delta t) = M[f(x,t)+\delta f(x,t)] - M[f(x,t)] \qquad (1)$$

3) Measure the size $A+\delta A$ of the evolved perturbation $\overline{\delta f(x,t+\Delta t)}$, and divide the perturbation by the measured amplification factor so that its size remains equal to A:

$$\delta f(x,t+\Delta t) = \overline{\delta f(x,t+\Delta t)}A/(A+\delta A) \qquad (2)$$

Steps 2) and 3) are repeated for the next time interval and so on. It has been found that after a short transient time of the order of the time scale of the dominant instabilities. In practical applications, bred vectors are are intrinsically local in space and time, and they are finite amplitude, finite time vectors. (Figure 1).
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Like an Example to study bred vectors, we reproduce the Research Internships in Science and Engineering (RISE) experiment with the 3-variable Lorenz model that indicate that orthogonalizing the bred vectors can result in significantly improved performance. This experiment showed that the regime changes in Lorenz's model are predictable. The purpose of this paper is to describe the breeding method that explores chaotic model predictability and its results.
The Lorenz model provides a practical test case with qualitatively realistic properties. Atmospheric behavior involving barotropic and baroclinic instabilities is considered somewhat analogous to Lorenz model behavior because of the exponential instability of the model's trajectories and its abrupt regime changes. Classic Lorenz (1963) three-variable model with standard parameter values: σ=10, b = 8/3, and r = 28 result in chaotic solutions (Figure 2.). The model was integrated with a 4th order Runge-Kutta numerical scheme. We used two sets of the Lorenz equations starting with different initial condition. We first perform breeding on the Lorenz model integrated with time steps Δt=0.01, and a second run started from an initial perturbation δx_0 added to the control at time t_0. Every 8 times steps we take the difference δx between the perturbed and the control run,

rescale it to the initial amplitude and add it to the control. We also measure the growth rate of the perturbation per time step as $1/8*(ln\,|\delta x\,|/|\,\delta x_0\,|)$, following Evans et al. (2004). Figure 3a shows the use of this simple procedure it allows us to estimate the stability of the attractor. Moreover, the growth rate measured by breeding provides remarkably precise "forecasting rules", illustrate in Figure 3b, that could be used by a forecaster living in the Lorenz attractor to make "extended range forecasts" about when will the present regime end, and how long will the next regime last. The presence of a red star shows bred vector growth in the previous 8 steps was greater than 0.064, it can be used to forecast that the next orbit will be the last one in the current regime. The blue stars indicate a negative growth rate, meaning that the perturbations are actually decaying. The results shown in figure 3a suggested that the bred vector growth would allow estimating of high and low predictability.
This is only an illustration how breeding scheme can be employed to get a predictability rules for a chaotic system. Our objective is to extend such methodology to investigate the predictability for other relevant chaotic regimes as the Chua's system associated to the dynamics in the electric circuits (Alligood et al., 1996), and the three coupling waves for solar activities connected to the space weather process (Chian et al.,1999)..

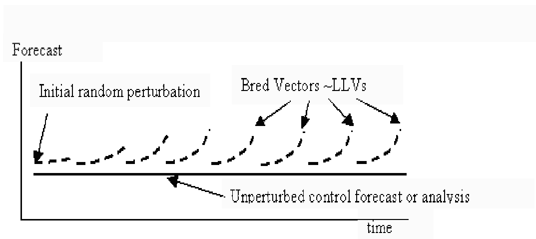


Figure 1-Schematic of the method to generate bred vectors (Evans et al, 2004).

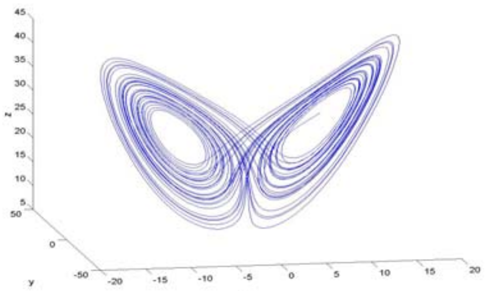


Figure 2- Solutions of the Lorenz model equations showing two chaotic regimes

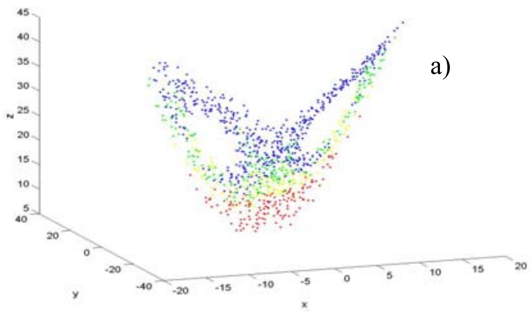
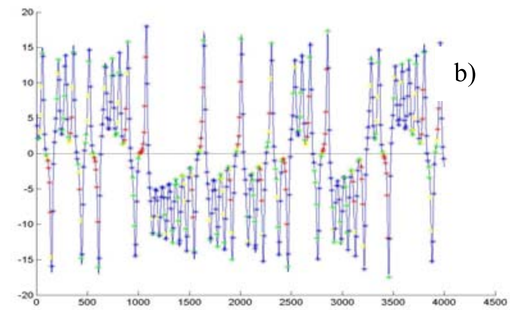


Figure 3 – a: The Lorenz classic attractor colored with the bred vector growth



b: X(t) for the classic Lorenz model with red stars providing "forecasting rules".

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