Generalized Watershed and PDEs for Geometric-Textural Segmentation

Anastasia Sofou and Petros Maragos

National Technical University of Athens

School of Electrical and Computer Engineering

Computer Vision, Speech Communication and Signal Processing Group (http://cvsp.cs.ntua.gr)

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ABSTRACT

In this paper we approach the segmentation problem by attempting to incorporate cues such as intensity contrast, region size and texture in the segmentation procedure and derive improved results compared to using individual cues separately. We propose efficient simplification operators and feature extraction schemes, capable of quantifying important characteristics like geometrical complexity, rate of change in local contrast variations and orientation, that eventually favor the final segmentation result. Based on the morphological paradigm of watershed transform we investigate and extend its PDE formulation in order to satisfy various flooding criteria, and couple them with texture information thus making it applicable to a wider range of images.

OVERVIEW

- ✓ Image Preprocessing and Simplification
- ✓ Image Decomposition into Constituent Components
- ✓ Feature Extraction
- ✓ Generalized Watershed and PDEs
- ✓ Coupled Contrast-Texture Segmentation
- ✓ Experimental Results
- ✓ Comparisons and Evaluations
- ✓ Conclusions

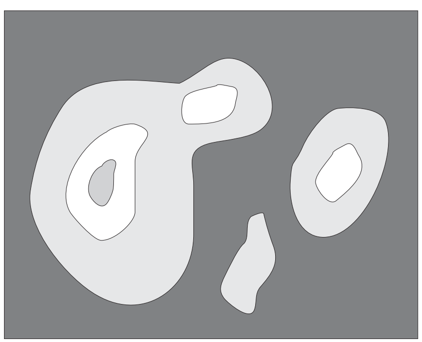
IMAGE SIMPLIFICATION

- Noise Reduction
- Structure Simplification
- Redundant Information Removal
- Preservation of Geometrical Structure and Objects' Contours

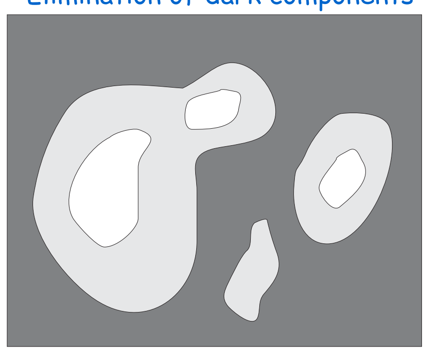
Tool: Connected Operators

Properties:

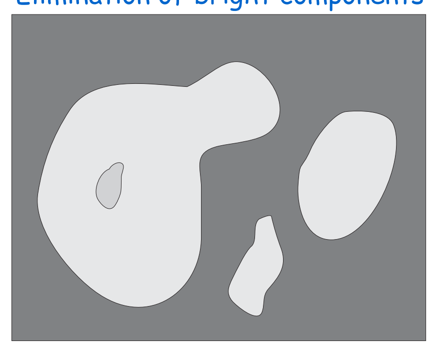
- Merging connected components and flat zones
- Preservation of geometrical structure and objects' contours
- No introduction of new contours



Elimination of dark components



Elimination of bright components

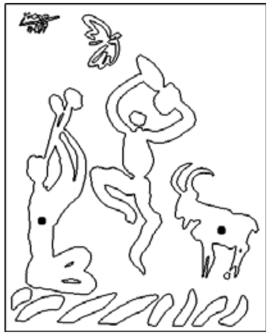


CONTRAST FILTERING – Connected Operators Based on Reconstruction

Set Reconstruction (opening)

$\rho^-(M\,|\,X) =$ Connected component of X that includes M $= \lim\limits_{n\to\infty}\left(\delta_B(\ldots\delta_B(\delta_B(M\,|\,X)\,|\,X)\,|\,X)\right)$


Binary Image


Markers


60 iterations


120 iterations


Final Result

Reconstruction Closing

$$\rho^+(m\,|\,f) = \lim_{n\to\infty} \varepsilon_B^n(m\,|\,f)$$

$$\varepsilon_B(m|f) = (m \ominus B) \vee f$$


Greyscale image f

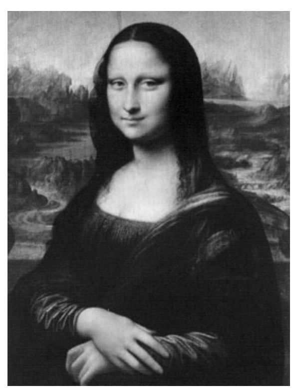

Reconstruction Opening
(m= f - 40)

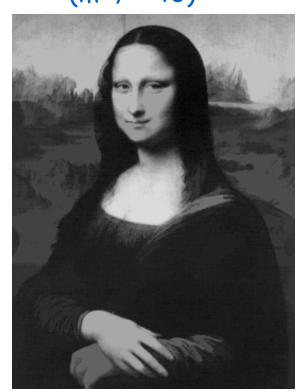

Reconstruction Closing
(m= f + 40)

Reconstruction Opening

$$\rho^-(m\,|\,f) = \lim_{n\to\infty} \delta_B^n(m\,|\,f)$$

$$\delta_B(m|f) = (m \oplus B) \wedge f$$





AREA FILTERING - Connected Operators based on Area

Binary Area Opening

$$\alpha_n^- = \bigcup_i \{X_i : \text{Area}(X_i) \geq n\}$$

Binary Area Closing

$$\alpha_n^+(X) = [\alpha_n^-(X^c)]^c$$

Upper Level Sets

$$X_\vartheta(f) = \{(x,y) : f(x,y) \geq \vartheta\}$$

Binary Image

Area Opening, n=200

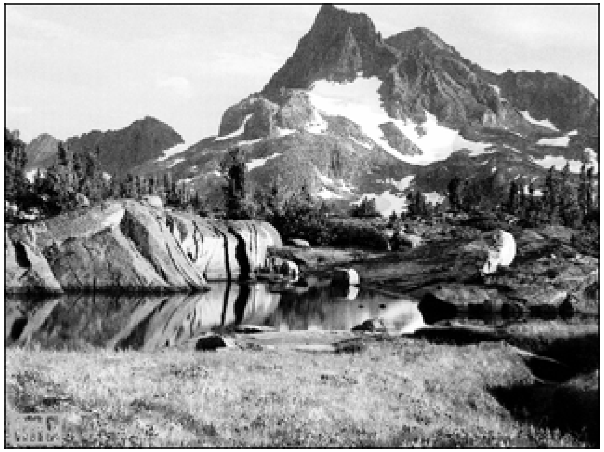
Area Opening n=1200

Greyscale Area Opening

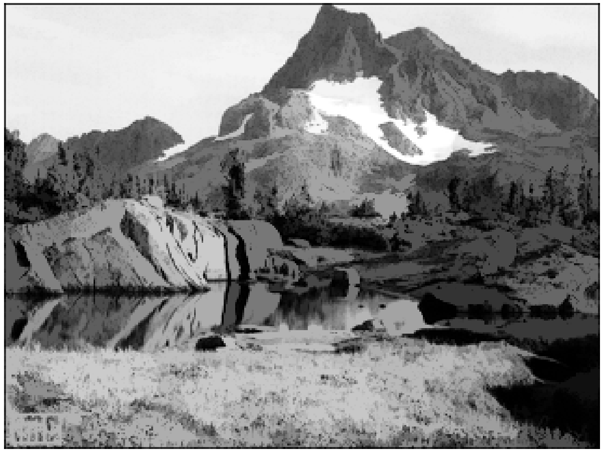
$$\alpha_n^-(f)(x,y) = \sup\{\vartheta : (\text{x,y}) \in \alpha_n^-(X_\vartheta(f))\}$$

Greyscale Area Closing

$$\alpha_n^+(f) = \sup\{\vartheta : (x,y) \in \alpha_n^+(X_\vartheta(f))\}$$



Greyscale Image



Area Opening

Area Closing

VOLUME FILTERING - Connected Operators Based On Volume

$$X_\vartheta(f) = \bigcup_i X_i \text{ και } Y = (X_\vartheta(f))^c = \bigcup_j Y_j$$

Upper Level Set Volume Opening

$$\beta_n^-(X) = \{X_i : \text{Area}(X_i) \cdot \vartheta \geq n\}$$

Upper Level Set Volume Closing

$$\beta_n^+(Y) = \{Y_j : \text{Area}(Y_j) \cdot \vartheta \geq n\}$$

Grayscale Volume Opening

$$\beta_n^-(f)(x,y) = \sup\{\vartheta : (x,y) \in \beta_n^-(X_\vartheta(f))\}$$

Grayscale Volume Closing

$$\beta_n^+(f)(x,y) = \sup\{\vartheta : (x,y) \in \beta_n^+(X_\vartheta(f))\}$$

Grayscale Image

Area Opening

Volume Opening

Area Closing

Volume Closing

LEVELINGS - Self Dual Filtering

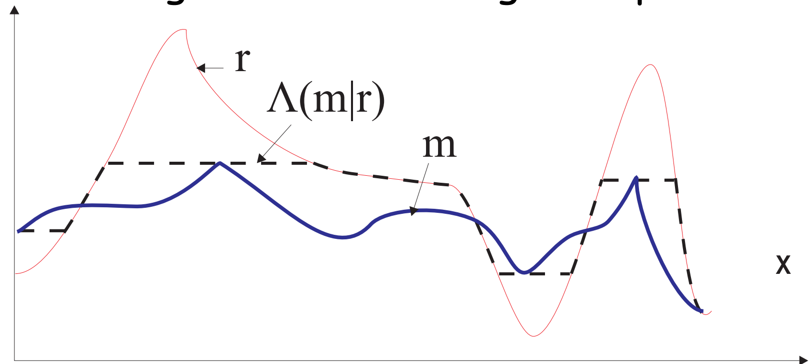
Self Dual Filtering: Symmetrical treatment of bright and dark image components

Leveling

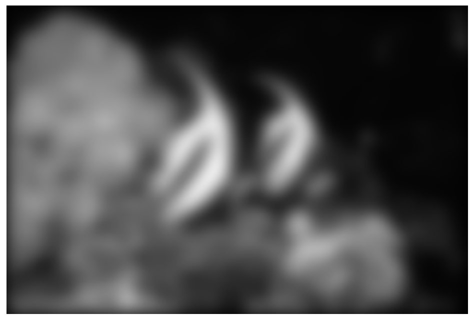
$$\Lambda(m \mid r) = \lim_{k \to \infty} f_k, \;\; f_k = \lambda(f_{k-1} \mid r), \;\; f_0 = m$$

$$\lambda(f \mid r) = (\delta(f) \wedge r) \vee \varepsilon(f)$$

δ dilation, ε erosion,with disk B



Image



marker m

Leveling

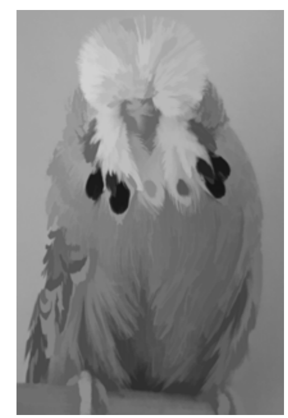
Alternating Sequential Filtering

$$\Psi_{\mathrm{ASF}}(f) = \varphi_n(\gamma_n(\ldots(\varphi_2(\gamma_2(\varphi_1(\gamma_1(f)))))\ldots))$$

φ closing, γ opening

Image f

$\Psi_{ASF}(f)$, n=6



$\Psi_{ASF}(f)$, n=10

IMAGE DECOMPOSITION INTO CONSTITUENT COMPONENTS

$f = u + v + w$

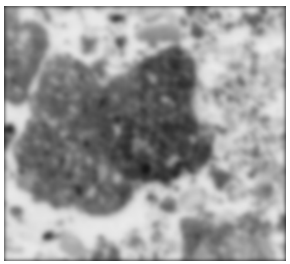
Image = geometrical structure + texture + noise

u : cartoon, v : texture, w : noise

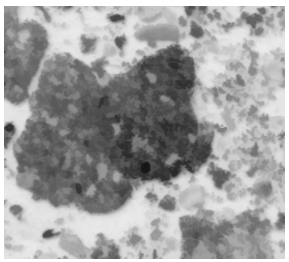
$$u_1 = \Lambda(m_1 \mid f), \ldots, u_n = \Lambda(m_n \mid u_{n-1})$$

$$\boxed{u = \Lambda(m \mid f), \quad v = f - \Lambda}$$

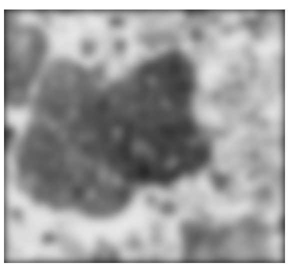
Levelings Pyramid



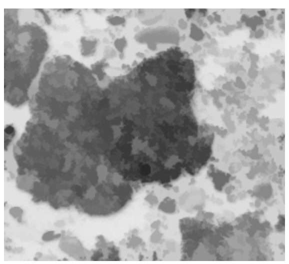
m_1



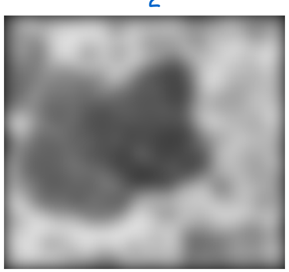
Λ_1



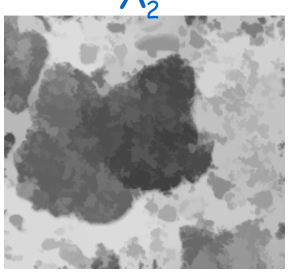
m_2



Λ_2



m_3

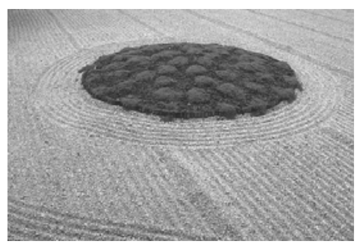


Λ_3

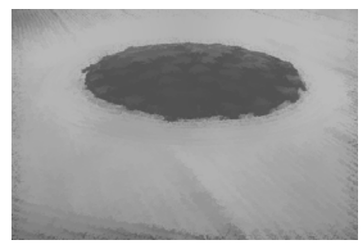
Cartoon u: geometrical structure information, partly smooth with flat plateaus

Texture v: texture information, texture oscillations (quick variation of intensity)

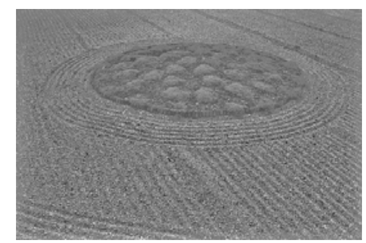
u+v Decomposition



Image



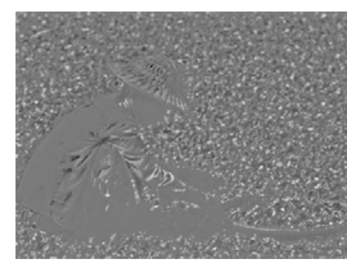
Leveling cartoon



Texture

Image

Leveling cartoon



Texture

FEATURE EXTRACTION

Image f

Morphological Gradient

Edge Features

- Morphological Gradient (edges)

$$M_{\nabla}(f) = [(f \oplus B) - (f \ominus B)] / 2r$$

Information about object contours and regions edges

Region Features - Markers

- Generalized Top-Hat Transform (peaks)

$$WTH(f) = f - \gamma(f)$$

γ : opening

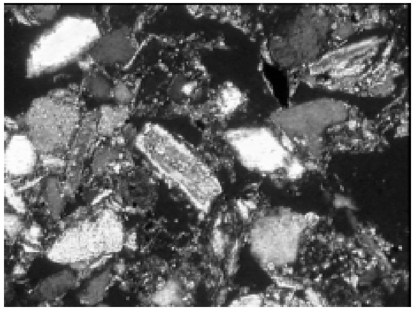
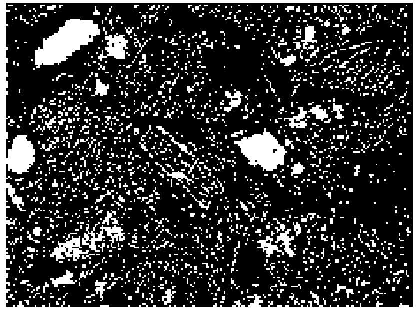
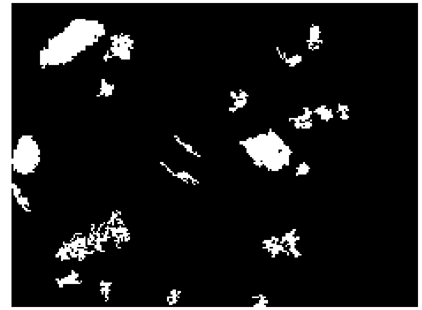
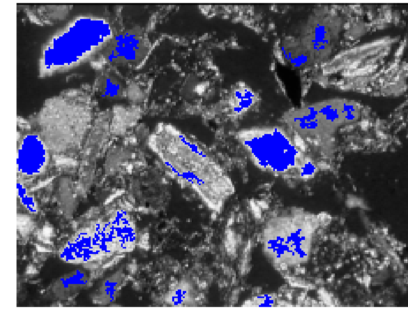
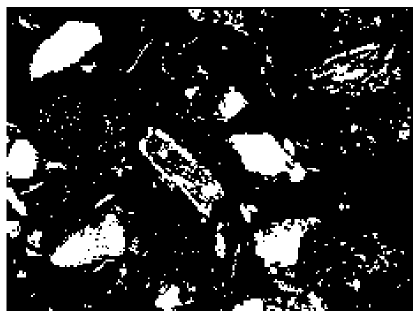
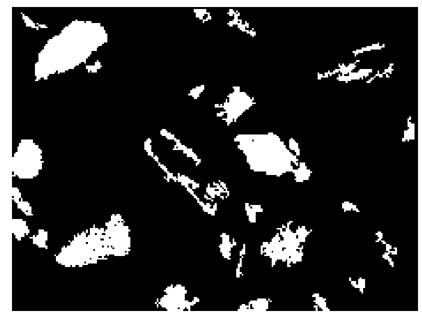
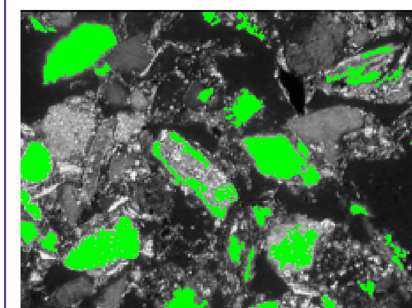
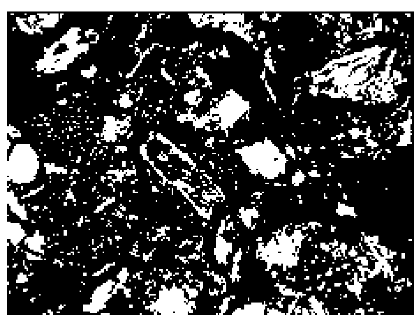
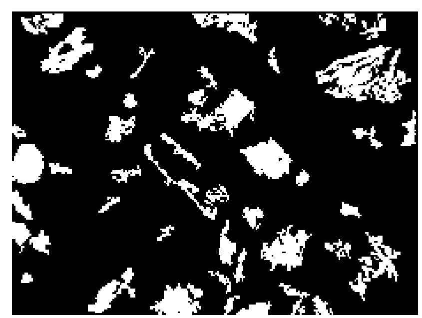
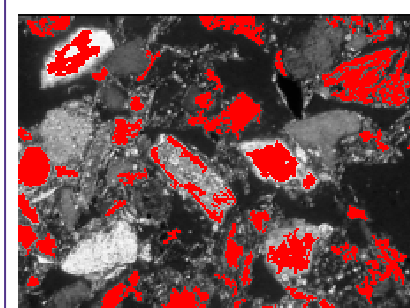
- Generalized Bottom-Hat Transform (valleys)

$$BTH(f) = \varphi(f) - f$$

φ : closing

Depending on the type of opening και closing operators (reconstruction, area, volume) different image areas are extracted with emphasis on different geometrical features.

REGION MARKERS

Reference Image		Marker Set	Refined Marker Set	Marker Set Superimposed on Image
	Intensity Peaks			
	Region Peaks of certain Area			
	Region Peaks of certain Volume			

TEXTURE FEATURES

- Texture Component available via u+v Decomposition
- Modeling of Texture Component as a narrow band 2D AM-FM signal

$$f(x,y)=\sum_{k=1}^{n} a_k(x,y)\cos[\phi_k(x,y)]$$

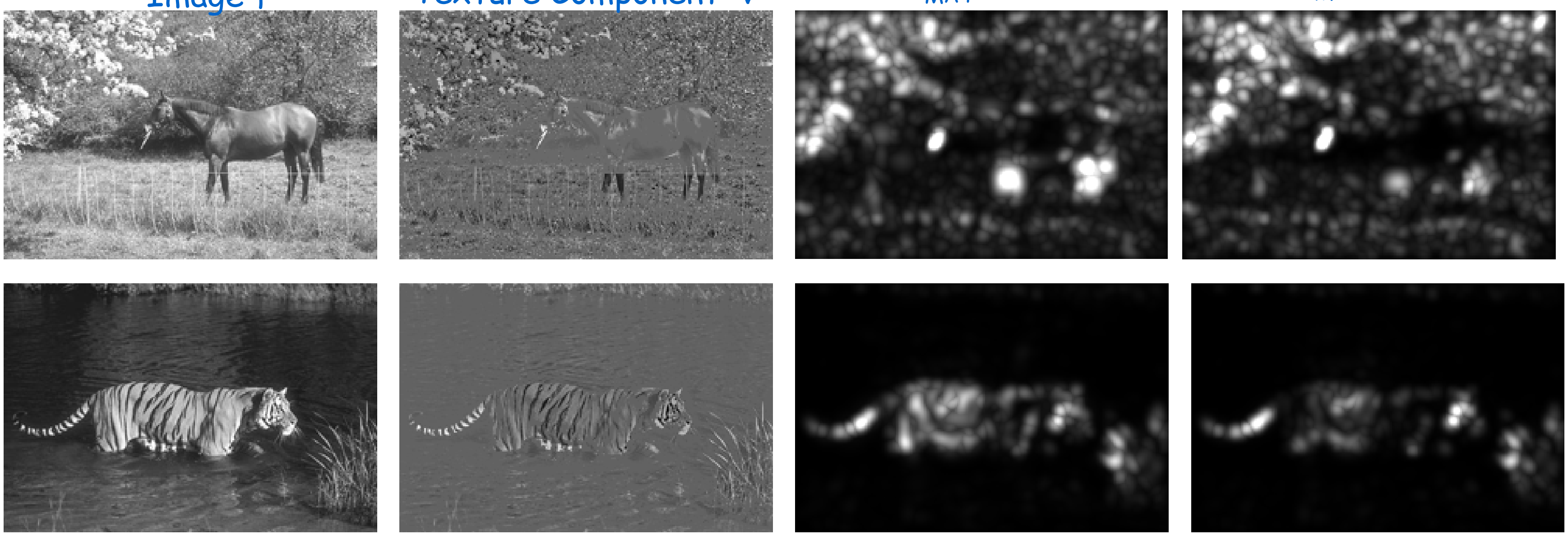
Teager Energy Operator

$$\Psi(f)=\|\nabla f\|^2 - f\nabla^2 f \qquad \Psi[\alpha_k\cos(\phi_k)]\approx \alpha_k^2\|\omega_k\|^2$$

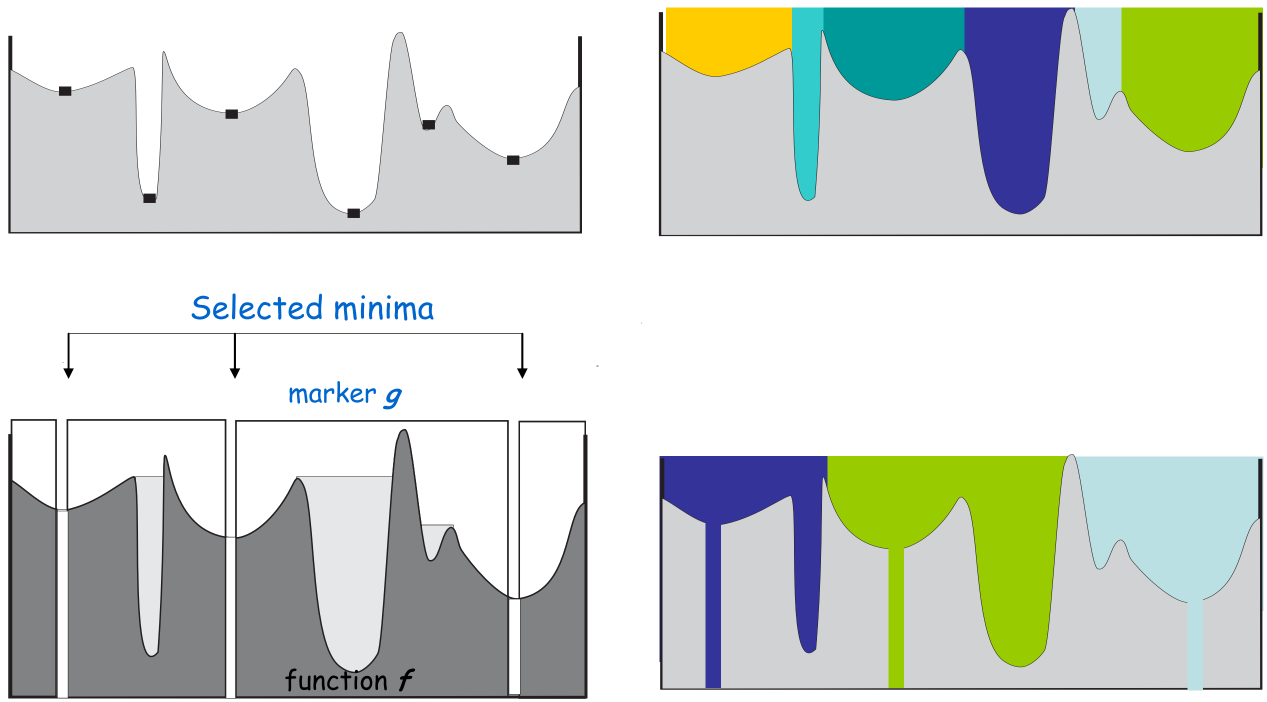
Texture Modulation Energy

$$\Psi_{\mathrm{MAT}}(f(x,y))=\arg\max_k \Psi[(f*h_k)*h_{av}(x,y)]$$

Image f | Texture Component v | $\Psi_{MAT}(f)$ | $\Psi_{MAT}(v)$



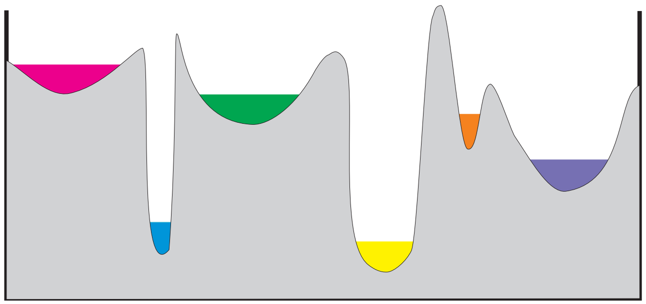
FLOODING PROCESS



- The gradient image is flooded from pre-selected sources (marker set).
- A lake is created from each flooding source.
- The water altitude rises inside each lake.
- The segmentation boundaries are formed at points where the emanating waves meet.

FLOODING CRITERIA AND TYPES OF WATERSHED FLOODING

Flooding Criterion: characteristic that all lakes (associated with the flooding sources) share with respect to water. By varying the flooding criterion different types of segmentation can be obtained.



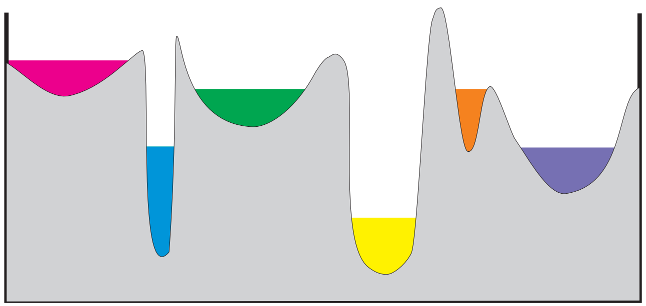
Flooding with constant height criterion

- **Altitude /height** (contrast criteria)

  => Height Watershed Flooding.
- **Area** (size criteria)

  =>Area Watershed Flooding.
- **Volume** (contrast and area criteria)

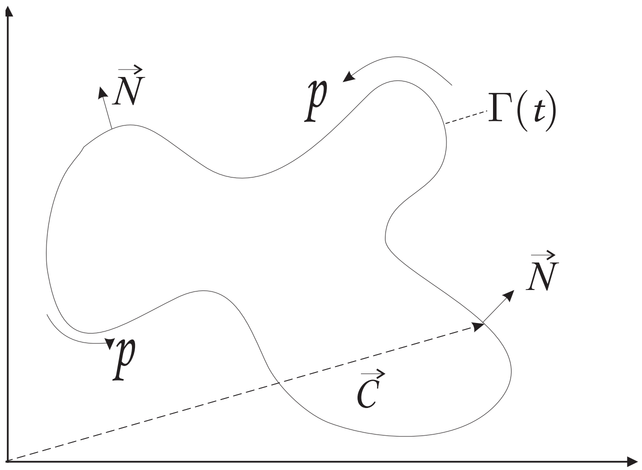
  =>Volume Watershed Flooding.



Flooding with constant volume criterion

ELEMENTS OF FLOODING & CURVE EVOLUTION

- **Marker Points:** source of wave propagation during flooding process.
- **Wave Evolution:** it is determined by flooding criterion
- Modeling of wave propagation is done via Partial Differential Equations (PDEs) and ideas from curve evolution.
- **Flooding Criterion:** it determines the curve's evolution speed



$\Gamma(t)$	Evolving curve $t \geq 0$
$\Gamma(0)$	Simple and smooth closed level curve
$\vec{C}(p,t)$	Position vector
$\vec{N}(p,t)$	Outward Normal Vector
$\upsilon = \vec{C} \cdot \vec{N}$	Evolution speed
$\kappa(p,t)$	Curvature

Curve Evolution PDE

$$\frac{\partial \vec{C}(p,t)}{\partial t} = \upsilon \vec{N}(p,t)$$

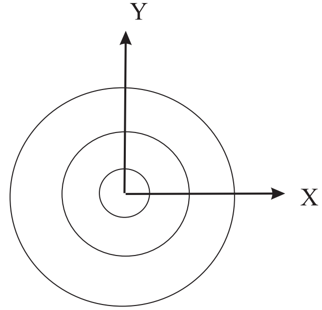
- Constant velocity $\upsilon = 1$ $\Leftrightarrow$ dilation
- Constant velocity $\upsilon = -1$ $\Leftrightarrow$ erosion
- Constant velocity + curvature $\upsilon = 1 - \varepsilon\kappa$

LEVEL SET FORMULATION IN CURVE EVOLUTION
(Osher & Sethian)

Embedding curve $\Gamma(t)$ as the zero level set of function $\Phi(x, y, t)$

$$\boxed{\Gamma(t) = \{(x, y) : \Phi(x, y, t) = 0\}}$$

$\Phi_0(x, y) = \Phi(x, y, 0) = \pm d(x, y)$ from $\Gamma(0)$

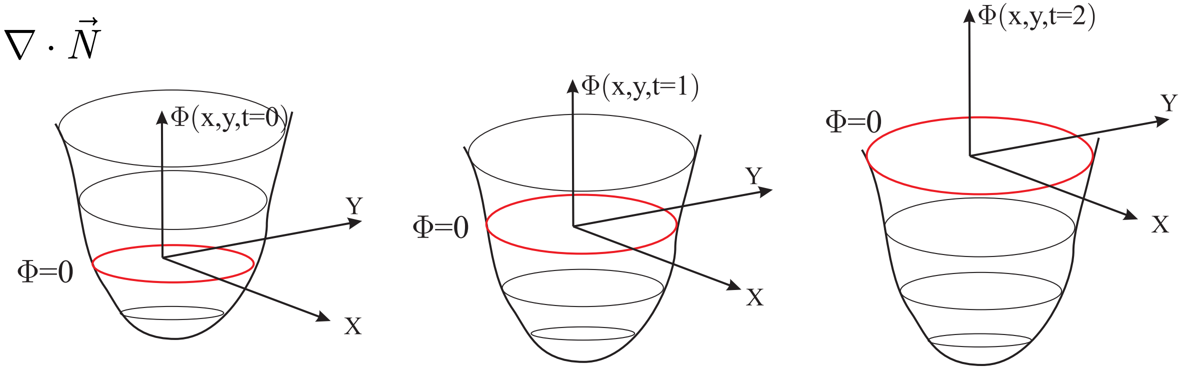


Level Function PDE

$$\boxed{\frac{\partial \Phi}{\partial t} = v \|\nabla \Phi\|}$$

$$K = -\mathrm{div}\,(\nabla\Phi / \|\nabla\Phi\|) = \nabla \cdot \vec{N}$$

$$\vec{N} = -\nabla\Phi / \|\nabla\Phi\|$$

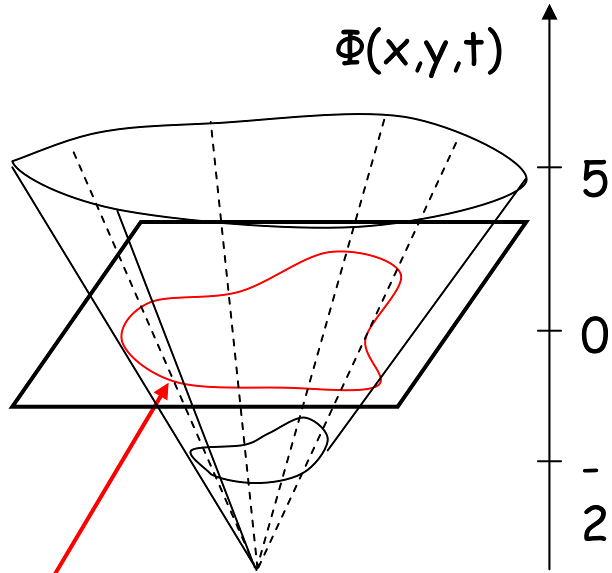


NUMERICAL APPROXIMATION

7	6	5	4	4	4	3	2	1	1	1	2	3	4	5
6	5	4	3	3	3	2	1	0	0	0	1	2	3	4
5	4	3	2	2	2	1	0	-1	-1	-1	0	1	2	3
4	3	2	1	1	1	0	-1	-2	-2	-2	-1	0	1	2
3	2	1	0	0	0	-1	-2	-3	-3	-2	-1	0	1	2
2	1	0	-1	-1	-1	-2	-3	-3	-2	-1	0	1	2	3
2	1	0	-1	-2	-2	-3	-3	-2	-1	0	1	2	3	4
2	1	0	-1	-2	-2	-2	-2	-1	0	1	2	3	4	5
3	2	1	0	-1	-1	-1	-1	-1	0	1	2	3	4	5
4	3	2	1	0	0	0	0	-1	-1	0	1	2	3	4
5	4	3	2	1	1	1	1	0	0	1	2	3	4	5
6	5	4	3	2	2	2	2	1	1	2	3	4	5	6

$\Phi(x,y,t)$

$\Gamma(t)$



$$\Phi(x,y,t+1) = \Phi(x,y,t) + \Delta\Phi(x,y,t)$$

- there is no movement, just change of Φ values
- the position of curve can be in-between samples
- curve topology can change

7	6	5	4	4	4	3	2	1	1	1	2	3	4	5
6	5	4	3	3	3	2	0	-1	0	0	1	2	3	4
5	4	3	2	2	2	1	-1	-2	-1	-1	0	1	2	3
4	3	2	1	1	1	0	-1	-2	-2	-2	-1	0	1	2
3	2	1	0	0	0	-1	-2	-3	-3	-2	-1	0	1	2
2	1	0	-1	-1	-1	-2	-3	-3	-2	-1	0	1	2	3
2	1	0	-1	-2	-2	-3	-3	-2	-1	0	1	2	3	4
2	1	1	0	-2	-2	-2	-2	0	0	1	2	3	4	5
3	2	1	0	-1	-3	-1	0	1	1	1	2	3	4	5
4	3	2	0	-1	-2	0	1	1	0	0	2	2	3	4
5	4	3	2	0	0	1	1	0	-1	0	1	2	4	5
6	5	4	3	2	1	2	2	0	0	1	2	4	5	6

UNIFORM HEIGHT FLOODING - 1D CASE

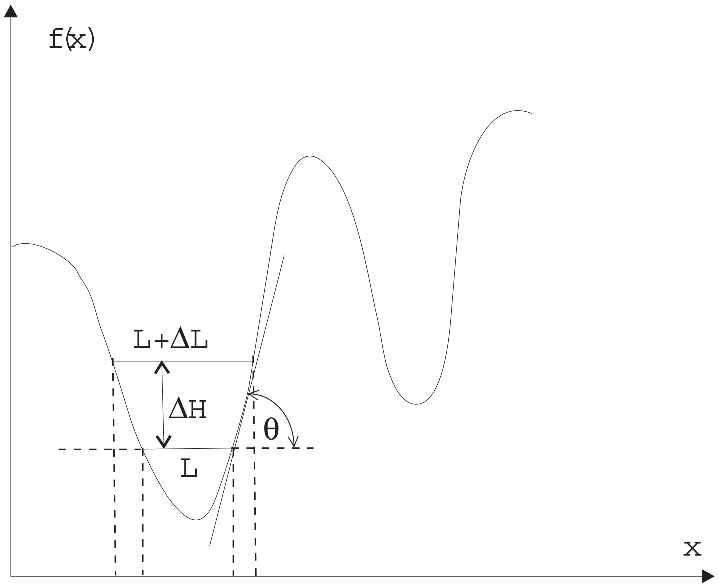
- 1D function f is pierced at one of its regional minima and immersed in water with constant vertical speed
- ΔH : Height difference

Uniform height speed:

$$\frac{\Delta H}{\Delta t} = const = c$$

$$\tan(\theta) = \frac{\Delta H}{\Delta L} = \left|\frac{df}{dx}\right|$$

$$V = \frac{\Delta L}{\Delta t} = \frac{c}{\left|{}^{df}\!/_{dx}\right|}$$

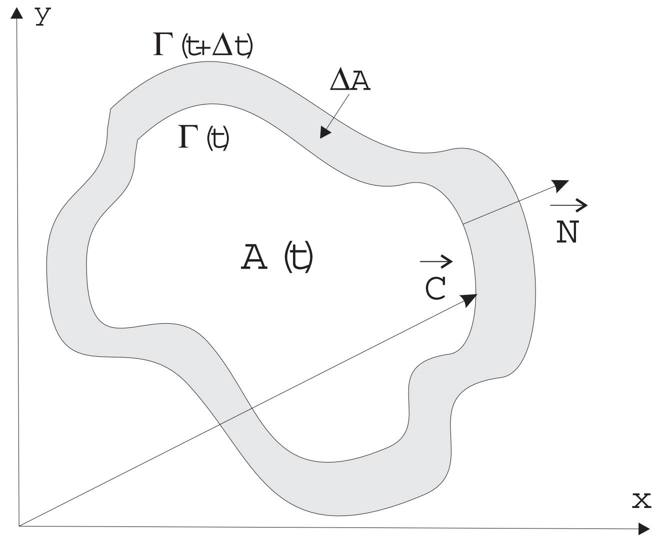


Lakes of 1D function

- V : horizontal velocity by which the level sets of the function f propagate in time
- L(t) : length of level sets

UNIFORM HEIGHT FLOODING - 2D CASE

$\Gamma(t)$: closed planar curve of the lake boundary
$\vec{C}(t)$: position vector of the closed planar curve



Planar projection of a lake of a 2D function

Level Curve Evolution PDE:

$$\boxed{\frac{\partial \vec{C}}{\partial t} = \frac{c}{\left\|\nabla f\right\|} \cdot \vec{N}}$$

Level Set formulation

$$\Gamma(t) = \left\{(x,y) : \phi(x,y,t) = 0\right\}$$

$\phi(x,y,t)$: evolving space function

Level Function Evolution PDE:

$$\boxed{\frac{\partial \phi}{\partial t} = V(x,y)\left\|\nabla \phi\right\|}$$

$V(x,y)$: space-dependent speed function given by

$$V(x,y) = \frac{c}{\left\|\nabla f(x,y)\right\|}$$

Uniform Volume Flooding - 1D Case

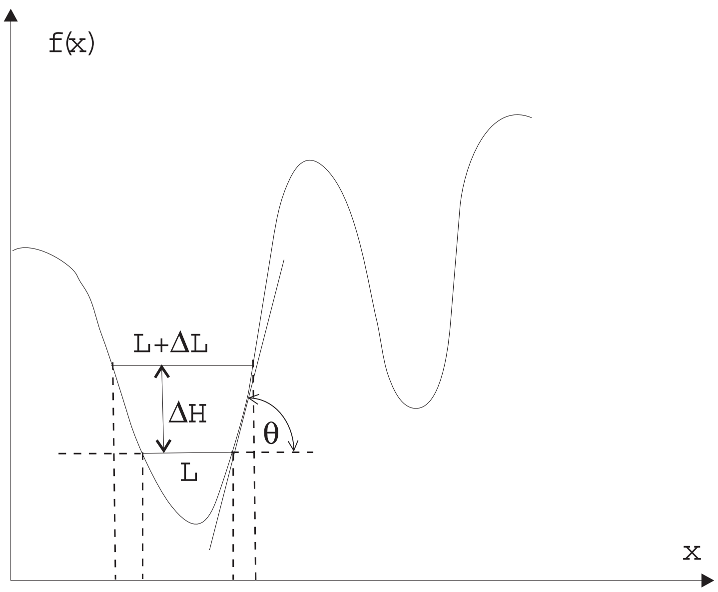
- Flooding is done with uniform volume speed inside all lakes.
- The water height is not at the same level for all lakes.
- The volume change rate of water remains the same
- (variation of water volume is constant).
- Balance between area and contrast.

1D Case:

$$L\frac{\Delta H}{\Delta t} = const = c$$

V: Horizontal velocity

$$\frac{\Delta L}{\Delta t} = \frac{\Delta H}{\Delta t}\frac{1}{\left|{}^{df}\!/_{dx}\right|} \Rightarrow V = \frac{\Delta L}{\Delta t} = \frac{c}{L(t)}\frac{1}{\left|{}^{df}\!/_{dx}\right|}$$



Lakes of 1D function

Uniform Volume Flooding - 2D Case

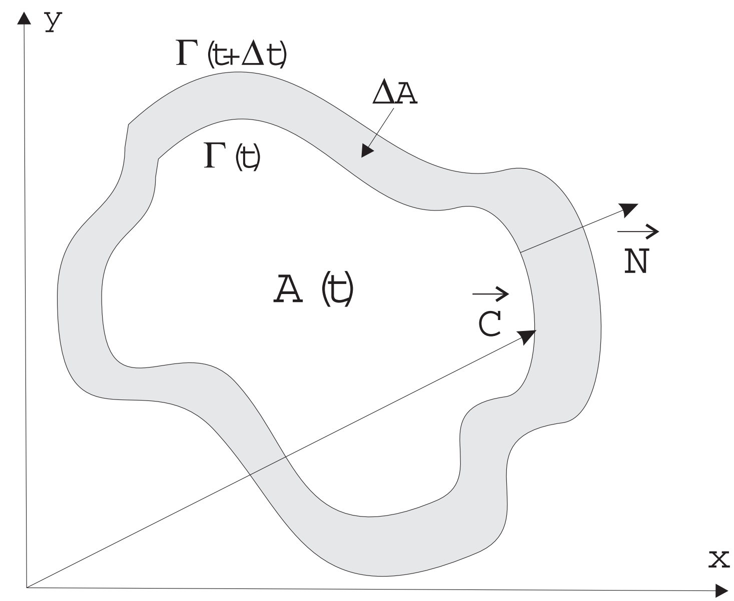
$\vec{C}$: wave emanating from a lake flooded under the constraint of uniform volume speed.
$L(t)$ becomes $A(t)$=> area enclosed by the propagating wave at time *t*

Level Curve Evolution PDE:

$$\boxed{\frac{\partial \vec{C}}{\partial t} = \frac{c}{A(t)\|\nabla f\|} \cdot \vec{N}}$$

Level Function Evolution PDE:

$$\boxed{\frac{\partial \phi}{\partial t} = V(x,y,t)\|\nabla \phi\|}$$



Planar projection of a lake of a 2D function

$$V(x,y,t) = \frac{c}{A(t)\|\nabla f(x,y)\|}$$

time and space dependent speed function

Stationary Eikonal -type PDEs

- Level Curve Evolution PDEs

$$\frac{\partial \vec{C}}{\partial t} = \frac{c}{\|\nabla f\|} \cdot \vec{N}$$

Uniform Height Flooding

$$\frac{\partial \vec{C}}{\partial t} = \frac{c}{A(t)\|\nabla f\|} \cdot \vec{N}$$

Uniform Volume Flooding

- Time dependent PDEs
- One-directional evolving front

$\Rightarrow$ stationary formulation of the embedding level function

$$T(x,y) = \inf\{t : \phi(x,y,t) = 0\}$$

Minimum time of Arrival

$$\|\nabla T(x,y)\| = \frac{1}{V}$$

Eikonal PDE

- Stationary Eikonal-type PDEs for flooding

$$\|\nabla T(x,y)\| = \|\nabla f\| / c$$

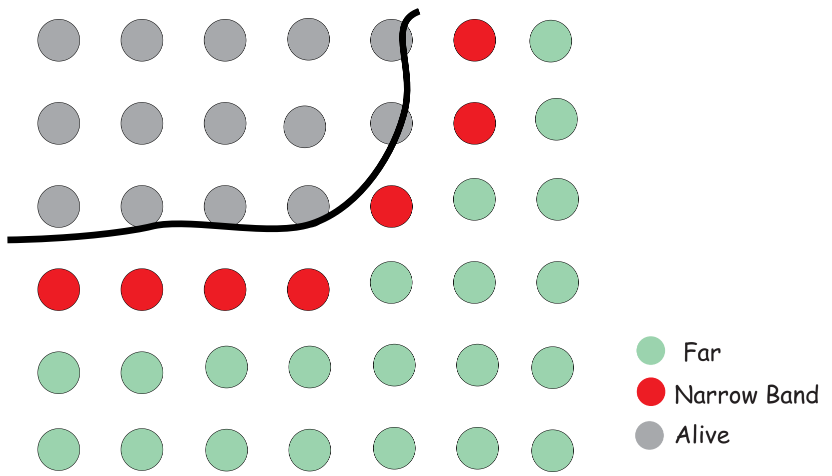
$$\|\nabla T(x,y)\| = A(t)\|\nabla f\| / c$$

FLOODING IMPLEMENTATION USING FAST MARCHING METHOD (FMM)

Fast Marching Method

Narrow Band: pixels 1 grid point away from curve.

- The evolution is towards the pixel with minimum T(x,y).
- The computation of T(x,y) is done by solving a quadratic equation.

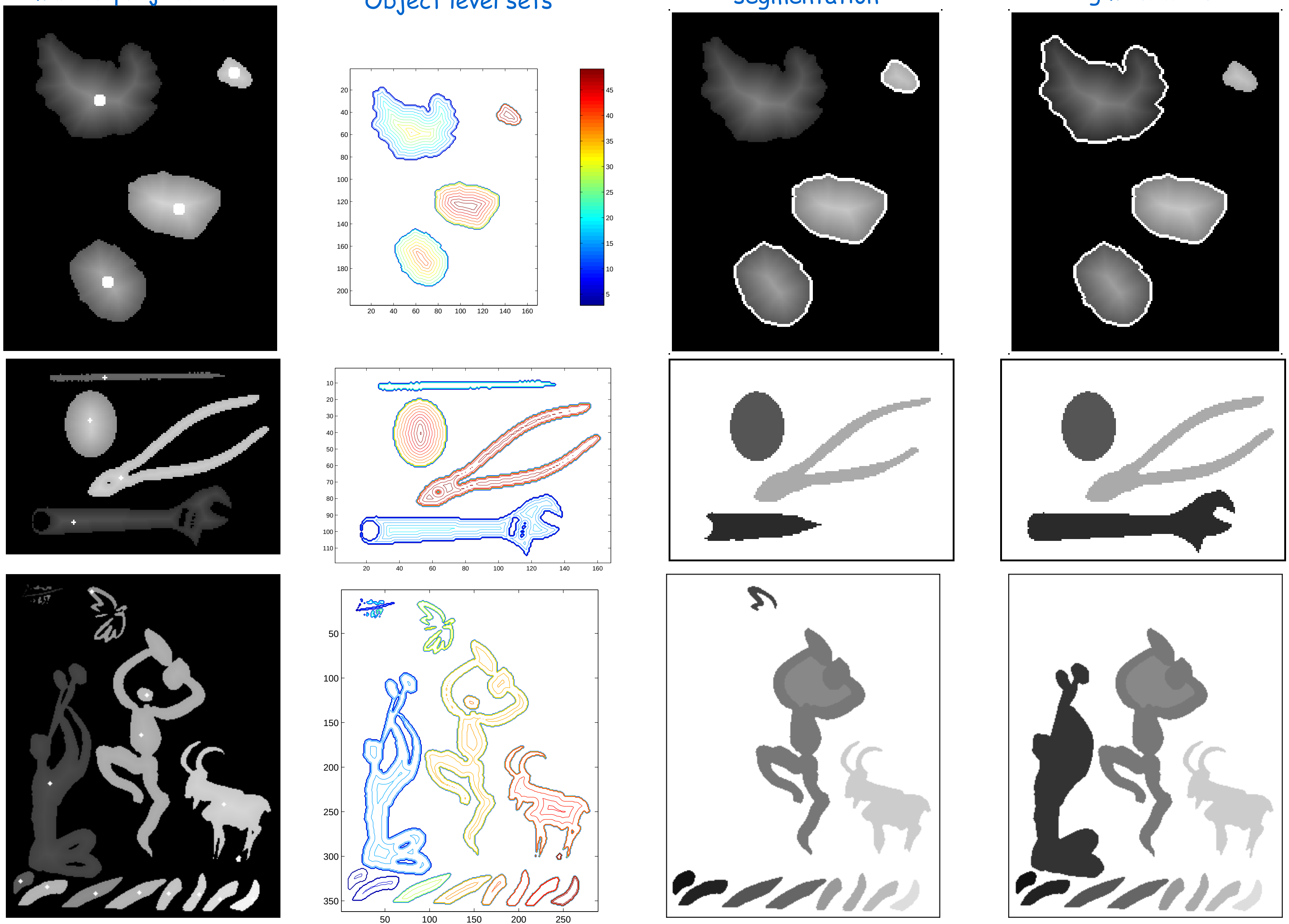


Uniform Volume Flooding

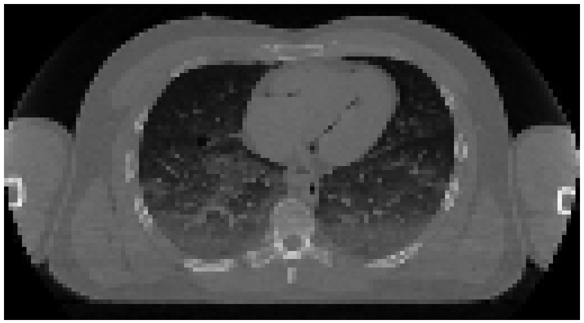
- Simultaneous propagation of different waves
- Update pseudo-time dependent term Area(t) during evolution
- Each grid point can be burnt only once (it cannot be assigned to more than one wave)
- Two or more wave collision $\Rightarrow$ dam erection (segmentation line)

Flooding a Synthetic IMAGE

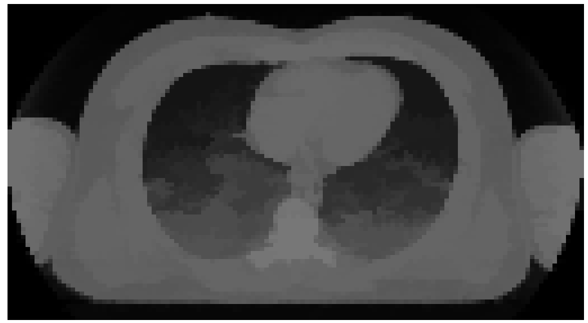
Synthetic image and marker projection | Object level sets | Uniform height flooding segmentation | Uniform volume flooding segmentation



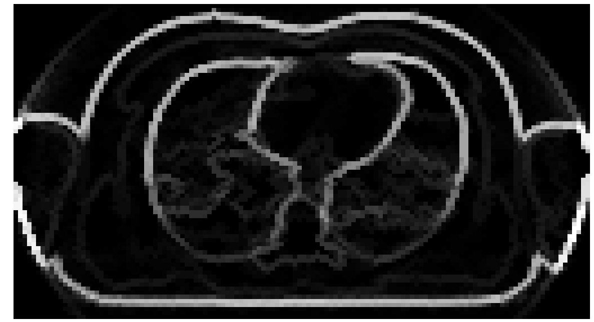
EXPERIMENTAL RESULTS



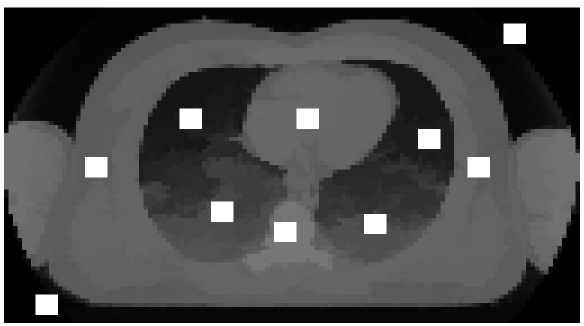
MRI image f



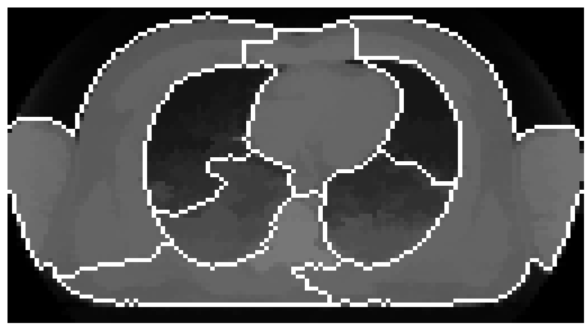
Simplified image g



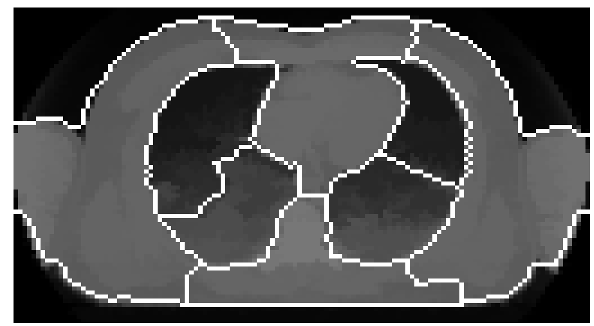
Gradient $\|\nabla g\|$



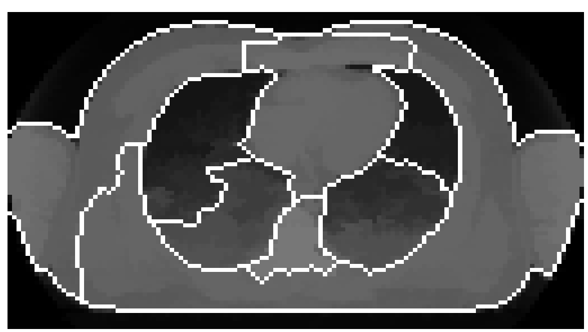
Markers



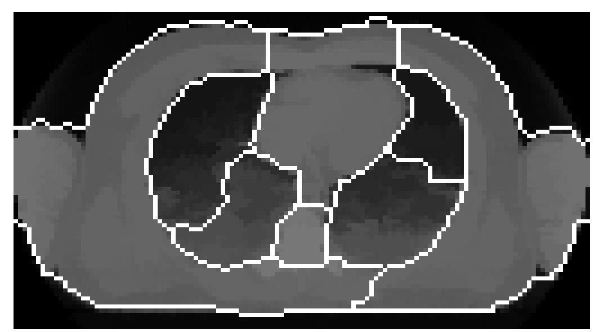
Uniform volume flooding of g



Uniform Volume Flooding of $\|\nabla g\|$

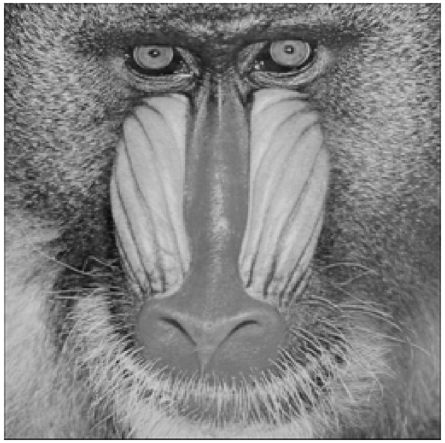


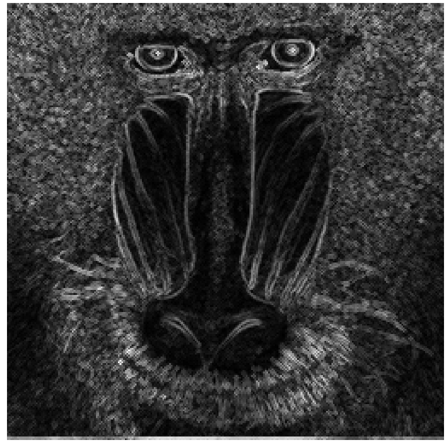
Uniform height flooding of g



Uniform height flooding of $\|\nabla g\|$

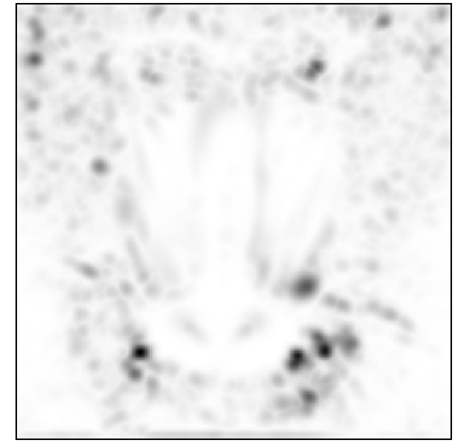
MULTI-CUE SEGMENTATION





Texture Quantification ?

Texture Modulation Energy



$$\frac{\partial \vec{C}}{\partial t} = \frac{c}{A(t)\|\nabla f\|} \cdot \vec{N} + \Psi_{\mathrm{MAT}}(f)$$

texture

size

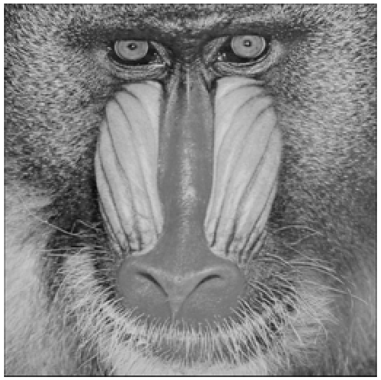
Intensity contrast

- Watershed flooding term (uniform height or volume) stops curve at strong edges
- Texture modulation energy term pushes curve away from areas of high energy without trapping it in-between texture edges

COUPLED MULTI-CUE SEGMENTATION

Component Decomposition $f = u + v$

f

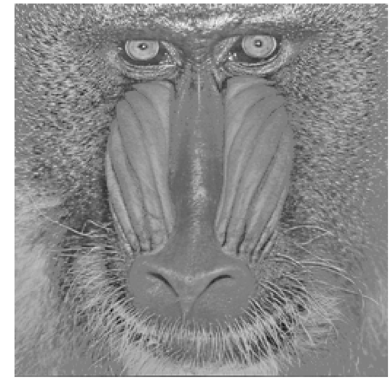


=

u

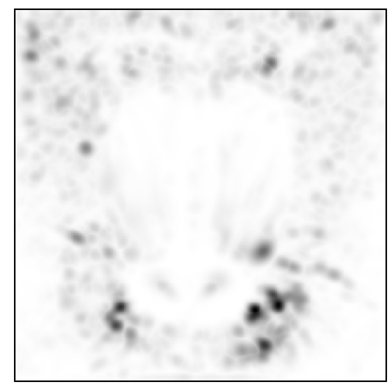
+

v



$\|\nabla(.)\|$

$\Psi_{MAT}(.)$



Coupled Multicue segmentation Scheme

$$\frac{\partial \vec{C}}{\partial t} = \left(\frac{\lambda_1}{\mathrm{Area}(t)\|\nabla u\|} + \lambda_2 \Psi_{MAT}(v) \right) \cdot \vec{N}$$

$$\frac{\partial \Phi}{\partial t} = \left(\frac{\lambda_1}{\mathrm{Area}(t)\|\nabla u\|} + \lambda_2 \Psi_{MAT}(v) \right) \|\nabla \Phi\|$$

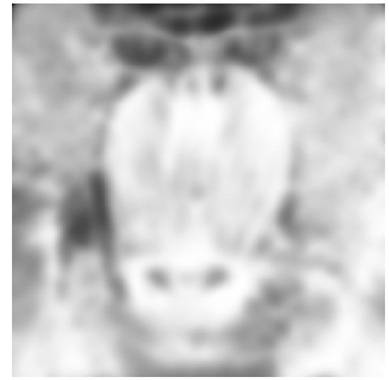
PARAMETER ESTIMATION

$$\frac{\partial \Phi}{\partial t} = \left(\frac{\lambda_1}{\text{Area}(t)\|\nabla u\|} + \lambda_2 \Psi_{\text{MAT}}(v) \right) \|\nabla \Phi\|$$

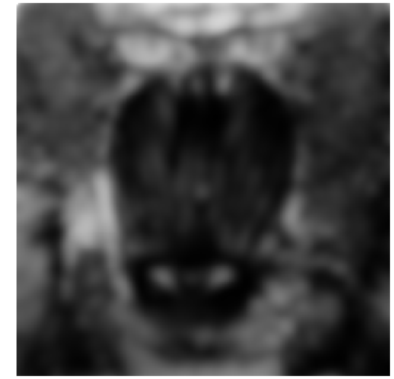
$$\boxed{\begin{aligned} \lambda_1(x,y) &= [G_\sigma * (f-v)^2](x,y) \\ \lambda_2(x,y) &= [G_\sigma * (f-u)^2](x,y) \end{aligned}}$$

$$\boxed{\begin{aligned} \lambda_1(x,y) &= \exp(-[G_\sigma * (f-u)^2](x,y)) \\ \lambda_2(x,y) &= \exp(-[G_\sigma * (f-v)^2](x,y)) \end{aligned}}$$

λ_1



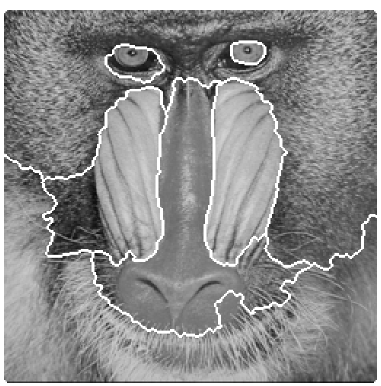
λ_2



Normalization

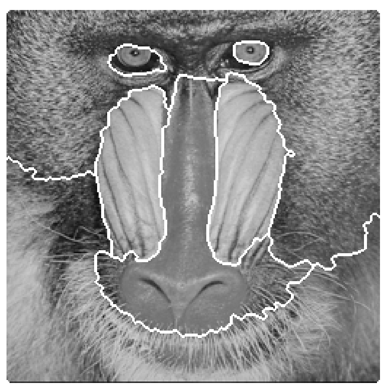
$$\boxed{\lambda_1 + \lambda_2 = 1}$$

$\lambda_1 = 0.3,\ \lambda_2 = 0.7$



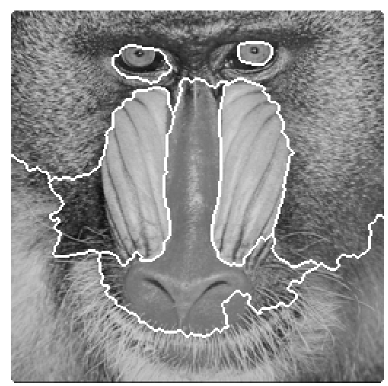
1.258

$\lambda_1(x,y), \lambda_2(x,y)$



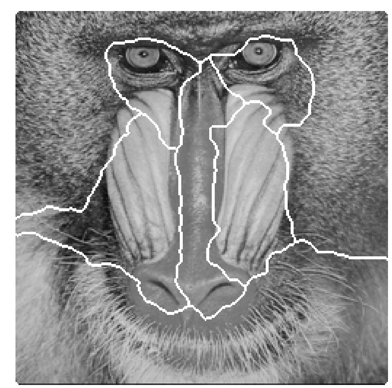
1.200

$\lambda_1 = 1, \lambda_2 = 0$



1.259

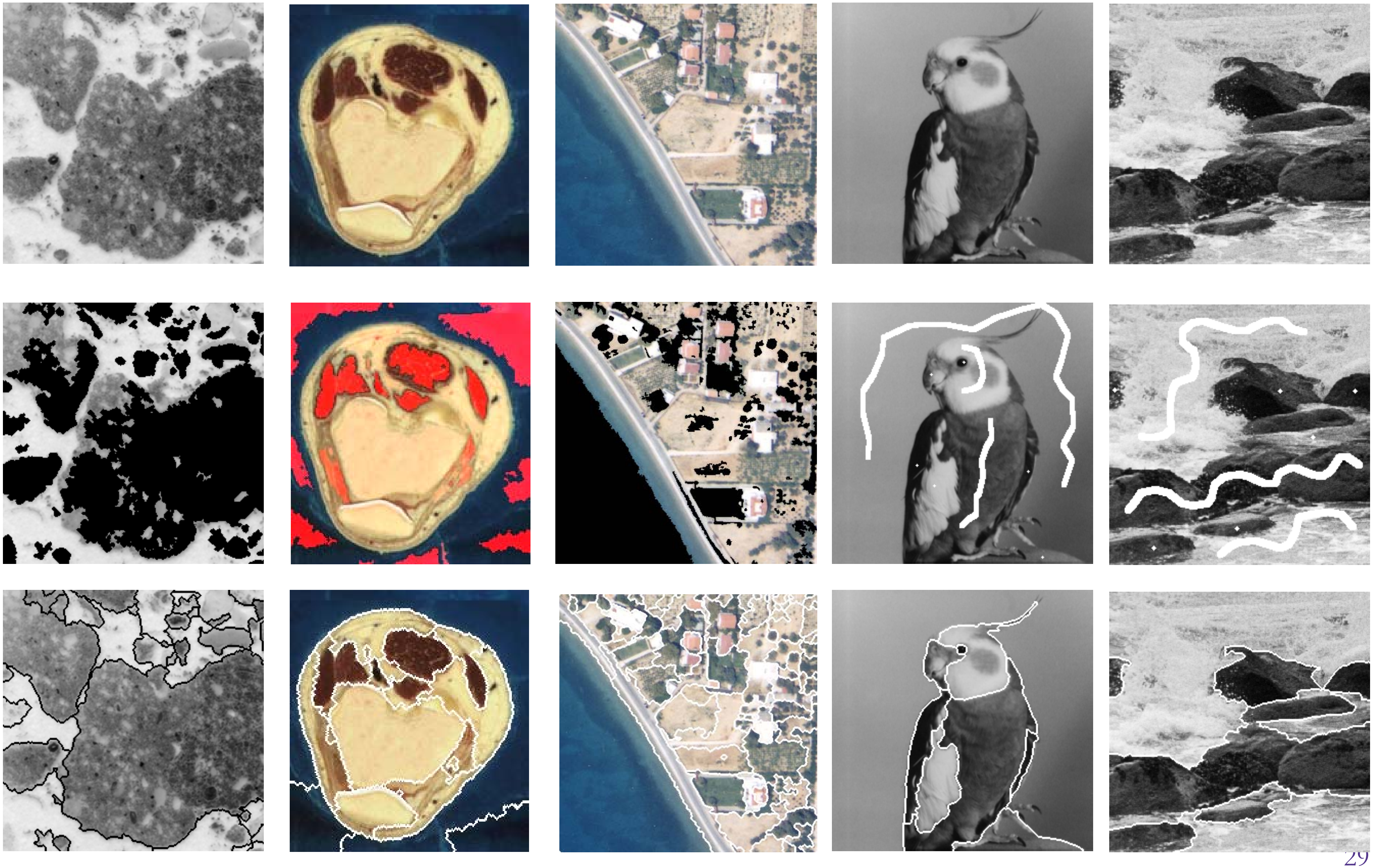
$\lambda_1 = 0, \lambda_2 = 1$



1.315

Mumford- Shah quality criterion

EXPERIMENTAL RESULTS



QUALITY EVALUATION OF SEGMENTATION RESULTS

Liu –Yang Global Cost Function (LY)

$$F = \sqrt{N} \sum_{i=1}^{N} \frac{e_i^2}{\sqrt{\mathrm{Area}_i}}$$

N: number of regions,
$e_i^2 = (f - \mu_i)^2$

- ✓ tradeoff between preservation of level of detail and suppression of non-homogeneity.
- ✓ Punishes small regions, big number of regions and regions with high variance.

Mumford –Shah functional (MS)

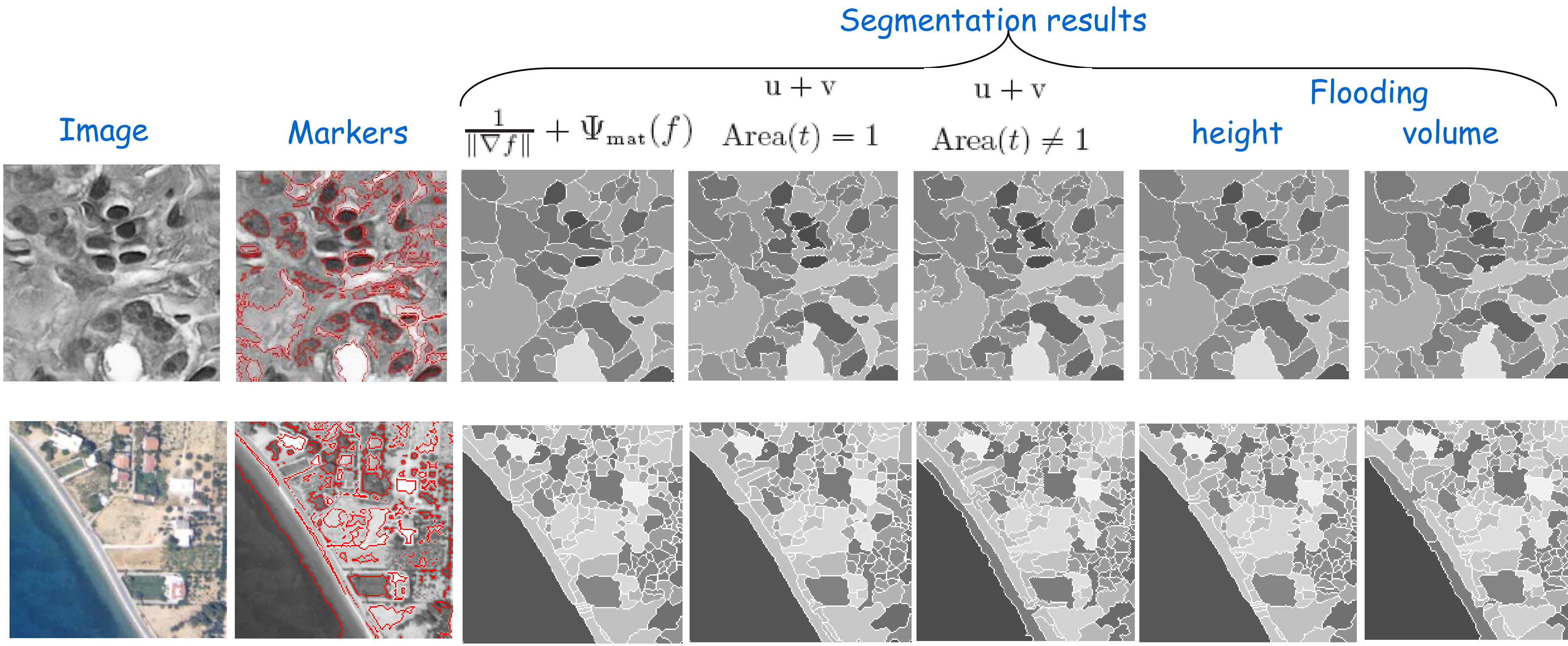
$$E(\Gamma, g) = \mu \iint_R (g - f)^2\, dxdy + \cancel{\iint_{R-\Gamma} \|\nabla g\|^2\, dxdy} + \nu\, |\Gamma|$$

- ✓ region homogeneity
- ✓ smoothness of contours

g: smooth image ⇒ Mosaic Segmentation Image

Γ: region contours

SEGMENTATION RESULTS AND QUALITY MEASURES



		Segmentation Method				
		Multicue Segmentation			Flooding	
	Quality Criterion	$\frac{1}{\|\nabla f\|} + \Psi_{MAT}(f)$	$\frac{\mathrm{u+v}}{\mathrm{Area(t)=1}}$	$\frac{\mathrm{u+v}}{\mathrm{Area(t)}\neq 1}$	**Uniform height**	**Uniform volume**
Tissue image	**LY**	2.44	1.62	1.73	2.41	1.9
	MS	0.156	0.139	0.150	0.151	0.155
Aerial image	**LY**	2.9	2.42	1.11	2.95	1.21
	MS	0.182	0.170	0.182	0.184	0.185

REVISITING QUALITY CRITERIA

Selection of criteria that evaluate geometrical information as well as texture information

Total Variance of Cartoon component

$$\mathrm{var}(u) = \sum_{i=1}^{N} \sigma^2(u(R_i)) = \sum_{i=1}^{N} \frac{(u(x,y) - \overline{u}_i)^2}{|R_i|}$$

$$(x,y) \in R_i$$

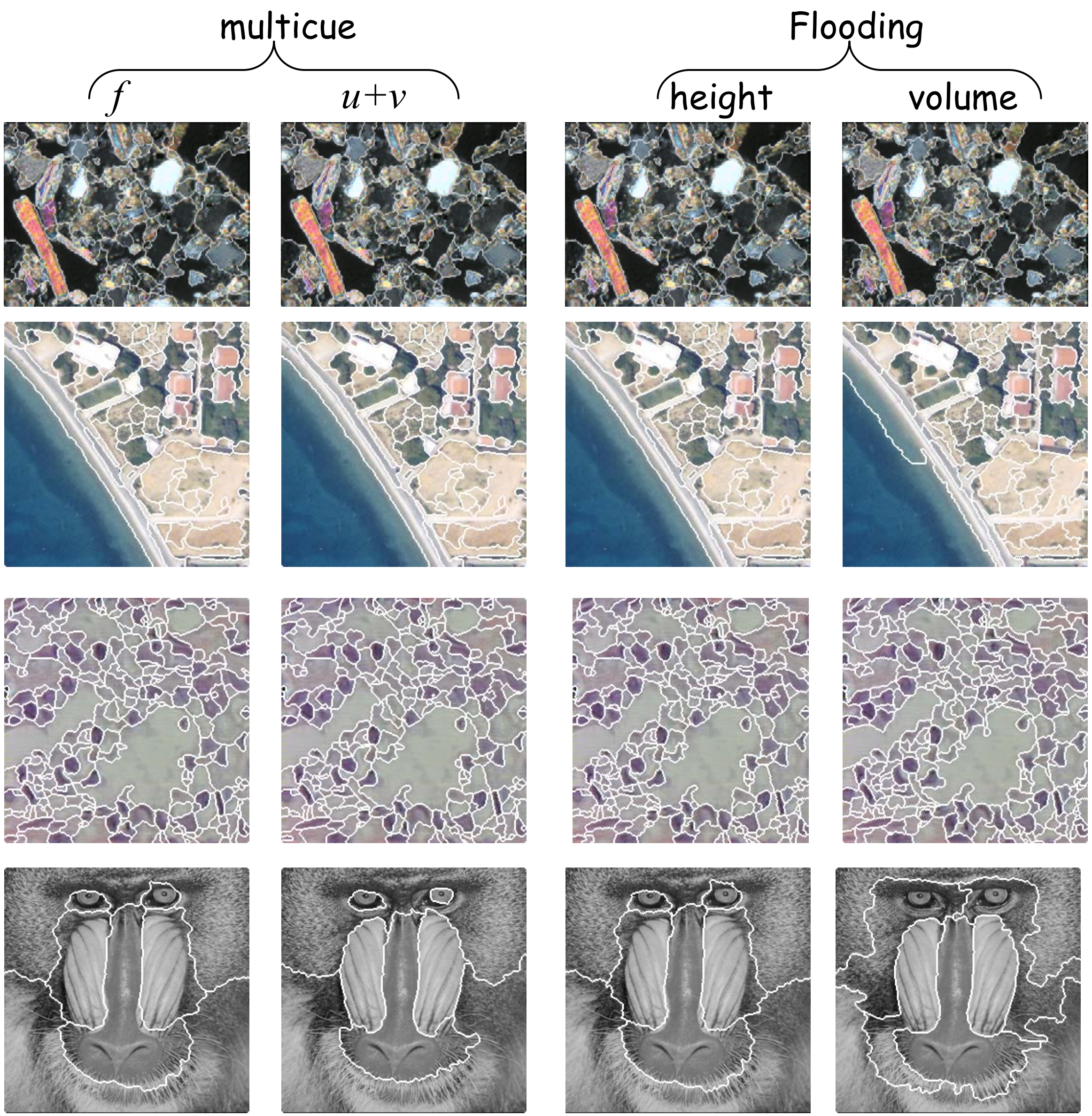
$|R_i|$ Region Cardinality

Total Variance of texture component

$$\mathrm{var}[\Psi_{\mathrm{MAT}}(v)] = \sum_{i=1}^{N} \sigma^2(\Psi_{\mathrm{MAT}}(v)(R_i)) = \sum_{i=1}^{N} \frac{([\Psi_{\mathrm{MAT}}(v)](x,y) - \mu_{\Psi i})^2}{|R_i|}$$

$\mu_{\Psi i}$ Mean texture modulation energy of the i-th region

RESULTS



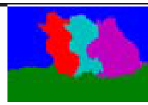
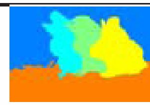
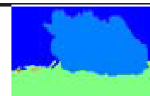
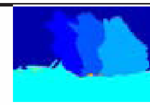
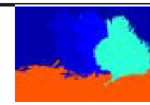
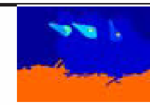
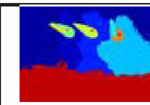
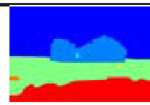
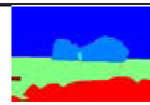
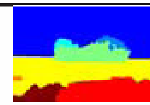
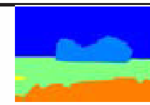
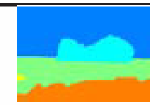
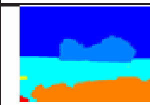
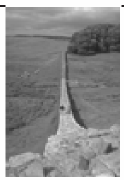
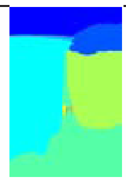
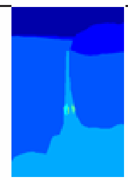
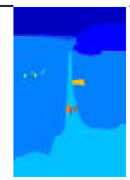
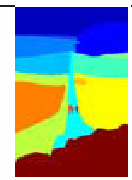
Comparison of region growing watershed-type methods

&

Quality criteria measurements

	Quality Measures	Segmentation Method: Coupled Type I	Coupled Type U+ V	Watershed Flooding Height	Watershed Flooding Volume
soil	var(U)	0.921	0.823	0.893	1.108
	var(Ψ_{mat}(V))	0.280	0.259	0.281	0.254
	length(Γ)	4855	4987	4982	5742
aerial	var(U)	0.335	0.281	0.337	0.383
	var(Ψ_{mat}(V))	0.473	0.468	0.479	0.555
	length(Γ)	3934	4206	4054	4442
biomed	var(U)	0.327	0.294	0.314	0.365
	var(Ψ_{mat}(V))	0.138	0.135	0.140	0.139
	length(Γ)	6529	6630	6728	7593
madrill	var(U)	0.046	0.024	0.046	0.034
	var(Ψ_{mat}(V))	0.272	0.232	0.271	0.285
	length(Γ)	1167	1210	1201	1960

COMPARISONS WITH GROUND TRUTH DATA

image	segmentation	Reference data (ground truth)						BCE
								0.21
								0.061
								0.16

Ground Truth data from Berkeley University Image Database