Design Optimization of Lithium Niobate Modulators using a Genetic Algorithm

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Abstract

This work presents the optimization of some design parameters of a lithium niobate modulator using a genetic algorithm to iteratively refine candidate set of parameters. Each of these candidate solutions is evaluated by applying the Finite Element Method. Numerical results show that the genetic algorithm is able to optimize design parameters of the lithium niobate modulator. Test cases include optimization of single and multiple parameters of the modulator. The set of characteristics include the half wave voltage, the electrode characteristic impedance, and the resulting bandwidth of the device.

Keywords *- Electrooptical modulators, optimization, genetic algorithms, finite element methods.*

1. Introduction

The demand for telecommunications services and bandwidth has increased since the past decade. Integrated optic devices are currently employed to increase transmission bandwidth and electrooptical modulators are among the most important devices, since they can provide external modulation, one of the main requests in fiber optics communication systems. Particularly, the lithium niobate ($LiNbO_3$) modulators are compatible with the bandwidth requirements of these systems and minimize fiber dispersion. Additionally lithium niobate presents advantages in comparison with other electrooptical materials. The investigation of materials, components and physical configuration of integrated optic devices presents aspects that not always allow an analytical evaluation of the device performance, requiring the manufacturing of several prototypes for experimental evaluation. This lengthens the design, evaluation and optimization cycle. The use of a computational model for simulate the electrooptic behavior of these devices significantly reduces their development cost and time. For a given choice of materials and configuration, the software would allow to analyze the performance of the device.

This is a typical optimization problem and stochastic methods are often employed in this kind of problems due to the easy of implementation and good trade-off between the quality of the solution and processing time. In the case of the more complex multiobjective problems, specific optimization techniques were developed.

In this work, the computational analysis is performed by using a numerical optimization technique, in order to explore the effect of some geometric parameters of the coplanar waveguide on the performance of the modulator. A genetic algorithm (GA) is used for the design optimization of the electrooptical modulator and the finite-element method (FEM) is applied on each candidate solution generated by GA to study both the optical properties of the diffused channel ($Ti:LiNbO_3$) waveguide and the quasi-static propagation transverse electromagnetic (TEM) modes of the coplanar waveguides (CPW).

To minimize the computational execution time, the optimization software was implemented in order to be executed in a distributed memory parallel environment. This environment is composed by a cluster of PC's microcomputers interconnected by a switch Fast-Ethernet network. The parallelization of the software employed the the MPICH implementation [1] of the Message Passing Interface communication library (MPI).

It will be presented in the section 2, the parameters often used as figure of merit of an electrooptical modulator. The FEM simulation technique used to solve the optical and electric fields is shown in section 3. Section 4 describes, briefly, the genetic algorithms and presents the optimization approach used in this work. The results of the optimization are discussed in section 5, and section 6 presents the main conclusions and the future work associated to the analysis and optimization of this sort of devices.

2. Mach-Zehnder modulators on lithium-niobate

The cross section of a Mach-Zehnder modulator with CPW is presented in Figure 1. In this figure, Tb is the thickness of the buffer layer, Te is the electrode thickness, G is the gap between the hot electrode and the ground electrode and W_H is the width of the hot electrode.

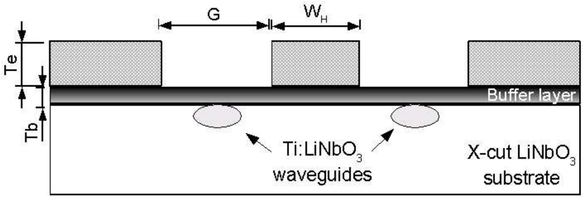

Figure 1. Cross section of the Mach-Zehnder modulator.

An electrooptical modulator can be characterized by analysis of the some electrical parameters such as: the characteristic impedance (Z_c), the half-wave voltage (V_π), the effective index of the transverse electromagnetic wave (N_{eff}) and the bandwidth (Δf). The definitions of these parameters are:

$$Z_c=\frac{1}{c}\frac{1}{\sqrt{C C_1}} \quad (1)$$

$$V_\pi L=\frac{\lambda_0 G}{n_{be}^3 r_{33}\left(|\Gamma_1|+|\Gamma_2|\right)} \quad (2)$$

$$N_{eff}=\sqrt{\epsilon_{eff}}=\sqrt{\frac{C}{C_1}} \quad (3)$$

$$\Delta f\ L=\frac{1{,}4c}{\pi\left|\sqrt{\epsilon_{eff}}-n_{eff}\right|} \quad (4)$$

In the equations (1)-(4), C and C_1 are the capacitance per unit length of the CPW calculated with the dielectric materials and with the materials replaced by vacuum, respectively, c is the free-space light velocity, n_{eff} is the effective index of the optical wave, V is the applied voltage, L is the length of CPW electrodes, λ_0 is the free-space optical wavelength, n_{be} is the extraordinary refractive index of the substrate, r_{33} is the pertinent electrooptic coefficient and Γ is the overlap factor between the electric field of the (optical wave) lightwave and the electric field induced by the electrodes. For x-cut Mach-Zehnder modulators, the optical waveguides are in the middle of the gap G and the overlap factor is equal in both waveguides $|\Gamma|=|\Gamma_1|=|\Gamma_2|$. The overlap factor for each waveguide is defined as:

$$\Gamma=\frac{G}{V}\frac{\iint E_{op}^2(x,y)E_{el}(x,y)\,dx\,dy}{\iint E_{op}^2(x,y)\,dx\,dy} \quad (5)$$

where E_{op} is the optical modal electric field and E_{el} is the electric field of the electrode structure (TEM wave and E_x component for x-cut configuration).

The optical waveguides used in this sort of devices are built by diffusing ions (usually Ti, or protons) in a dielectric substrate. One of the most used substrate in integrated optical circuits is the $LiNbO_3$, because it presents appropriate optical and mechanical properties. In the fabrication process, a diffused channel, in which the refractive index varies continuously, is produced.

3. Simulation of microwave properties using finite element method

The finite element method (FEM) is based on the minimization of a functional whose solution is equivalent to the original differential equation. An approximated solution for a set of differential equations can be obtained by applying the Weighted Residual Method (WRM) or by applying variational principles. The finite element formulations adopted in the computations performed in this work were used in previous works [2] and are briefly presented to follow.

The microwave properties of the CPW have been simulated using a two-dimensional finite-element method. The E_{el} is computed by the FEM in the quasi-static approximation (TEM modes). The TEM modes are related to the solutions of the Laplace equation for the electric potential Φ:

$$\nabla(\epsilon_r\nabla\Phi)=0 \quad (6)$$

where the diagonal relative permittivity tensor is given by:

$$\epsilon_r=\begin{bmatrix}\epsilon_{xx} & 0\\ 0 & \epsilon_{yy}\end{bmatrix} \quad (7)$$

The FEM applied to (6) yields the matrix equation:

$$[S]\{\Phi\}^T=\{b\}^T \quad (8)$$

where :

$$[S]=\int_\Omega\left(\epsilon_{xx}\frac{\partial\{N\}^T}{\partial x}\frac{\partial\{N\}}{\partial x}+\epsilon_{yy}\frac{\partial\{N\}^T}{\partial y}\frac{\partial\{N\}}{\partial y}\right)dx\,dy \quad (9)$$

$$\Phi=\{N\}\{\Phi\}^T \text{ and } E=-\nabla\Phi$$

{ } represents a row matrix, $\{\ \}^T$ stands for a transposed matrix and $\{N\}$ represents a complete set of base functions for the used finite elements.

4. Design optimization approach

The design optimization process is composed, basically, by two mechanisms: the analysis engine and the optimization/search algorithm. The optimization design is an iterative process and candidate solutions are successively generated by the optimization algorithm and evaluated by the analysis engine.

The genetic algorithms [3], which mimics survival of the fittest among competing organisms, have seen wide application in recent years. The algorithm used in this study is standard in most respects. An initial population of N_p chromosomes (individuals) is constructed randomly, with each group of bits in a chromosome corresponding to a geometric parameter. The geometric model of a modulador is then configured according to each chromosome and the fitness of each configuration is evaluated by computing the characteristic electric parameter. A selection process follows, during which a new population of size N_p is created. During selection, individuals are chosen from the old population with a probability equal to the ratio of the individual fitness to the total fitness of the old population. Regardless of the result, the individuals with the highest fitness values are selected to guarantee their genetic information will be passed along to the next generation. This scheme is known as elitism. After selection, individuals are paired off as parents and crossed over, each with a crossover probability. Each time crossover occurs, the chromosomes are split at a random bit location and their partial bit strings (all bits before the random bit) are swapped between the parents. Following crossover, a mutation is performed by negating each bit on each chromosome with a mutation probability. The fitness of each resulting individuals is evaluated, and the process is repeated for a specific number of generations or until that a convergence criterion is reached.

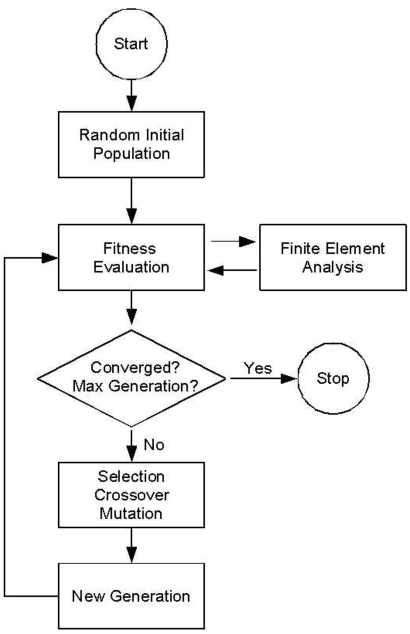

Figure 2. Flowchart of the design optimization process.

In this work, the GA is responsable for the search operation and the FEM carries out the analysis. Figure 2 presents a diagram of the optimization process used in this work.

The size of the GA initial population is 22 individuals. The individuals of the initial population are generated randomly. Two design parameters (gap and thickness of the electrodes) are coded into a 8 bit string segments using binary coding. These segments are appended one after the other to form a chromosome. The roulette wheel technique is used for the reproduction operator, with one-point crossover. The occurrence probability for the crossover is set to 0.8 and for mutation is set for 0.05. Elitism of 4 individuals is applied.

The GA works with the values of electrode cross-sectional parameters (gap G, buffer thickness Tb, electrode thickness Te and width W). The boundaries parameters used in the optimization process are in the range that can be manufactured by the current processing technology (thickness up to 30 μm and 1 μm to electrode and buffer, respectively) [4].

These parameters are summarized as:

$$6\ \mu m \le W_H \le 36\ \mu m$$

$$5\ \mu m \le G \le 35\ \mu m$$

$$15\ \mu m \le Te \le 30\ \mu m$$

$$Tb = 0.2\ \mu m$$

In all optimization cases, it was assumed a value of $n_{eff} = 2.1423$ for the effective index of the optical wave.

5. Optimization results

In order to test the proposed design optimization method, only two parameters of the modulator were optimized, to obtain the better characteristic impedance (Z_c). The primary objective of the optimization is to find a modulator configuration that offers a Z_c as close as the impedance of commercial microwave sources, namely 50 Ω .The result is shown in Figure 3.

The result shows that was a good choice in the set of parameters for the several configurations of the geometric model that had been tested by the optimization process, almost reaching the desired value (0.55%). And the convergence of the process was fast (15^{th} generation).

This set of values allowed to reach better values for the objective function of that a tested set previously [5].

Another test of convergence was realized involving the effective index of the transverse electromagnetic wave (N_{eff}) and the result is shown in Figure 4. The convergence of the process was fast, too (15^{th} generation). The reached result was 0.26% of the intended value n_{eff} .

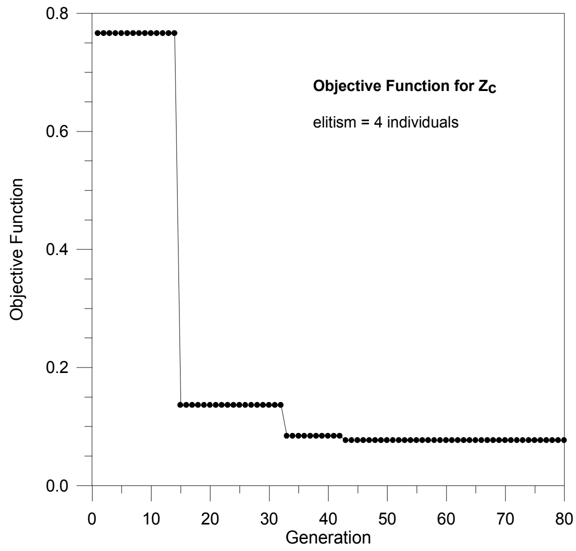

Figure 3. Change of fitness over generations for characteristic impedance (Z_c) of the modulator.

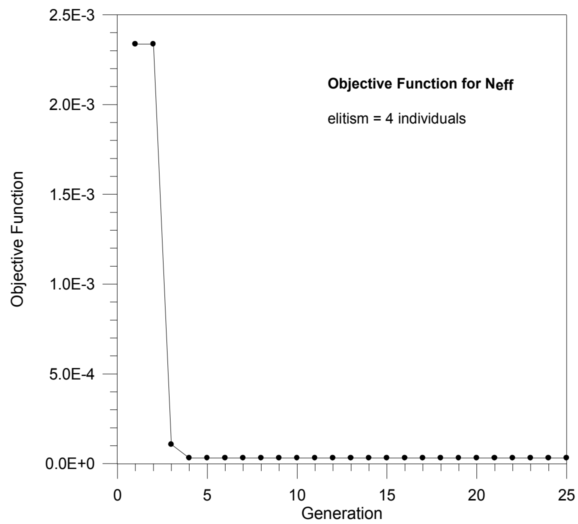

Figure 4. Change of fitness over generations for effective index of the electromagnetic wave (N_{eff}).

This optimization process was used to explore other electrical parameters, such as the half-wave voltage (V_π), and the effective index of the transverse electromagnetic wave (N_{eff}). The best results that were obtained by the optimization process are present in Tables 1, 2 and 3. Also, the numbers presented of the right side of the tables are of the previous work [5], for comparison with this.

Table 1. Optimization result for Z_c.

	This work	Previous work
	Z_c = 49.7240 Ω	Z_c = 32.1091 Ω
Electrode gap (*G*)	35 μm	25 μm
Electrode thickness (*Te*)	26 μm	20 μm
Electrode width (W_H)	6 μm	10 μm
Buffer layer thickness (*Tb*)	0.2 μm	0.5 μm
V_π	19.3898 V.cm	12.4182 V.cm
N_{eff}	2.4096	2.5202

Table 2. Optimization result for V_π.

	This work	Previous work
	V_π = 3.2636 V.cm	V_π = 3.9946 V.cm
Electrode gap (*G*)	5 μm	5 μm
Electrode thickness (*Te*)	30 μm	30 μm
Electrode width (W_H)	36 μm	30 μm
Buffer layer thickness (*Tb*)	0.2 μm	0.5 μm
Z_c	12.1288 Ω	12.0506 Ω
N_{eff}	1.8720	1.8236

Table 3. Optimization result for N_{eff}.

	This work	Previous work
	N_{eff} = 2.1367	N_{eff} = 2.1462
Electrode gap (*G*)	7 μm	11 μm
Electrode thickness (*Te*)	23 μm	29 μm
Electrode width (W_H)	18 μm	24 μm
Buffer layer thickness (*Tb*)	0.2 μm	0.5 μm
Z_c	16.4559 Ω	18.7686 Ω
V_π	3.9043 V.cm	6.1782 V.cm

From these results, one can see that for this type of device, the optimum configuration obtained for one parameter is not the best for the others.

In reason of these conflicting parameters, an multiobjective approach for the optimization process must be considered for these kinds of problems.

In order to analyze the behavior of the modulator, subject to multiobjective optimization, had been implemented two functions, that used the electrical parameters - the characteristic impedance and the half-wave voltage - in the optimization process.

The first function is a composite objective function formed by the weighted sum of the objectives, where a weight for an objective is proportional to the preference assigned to that particular objective. This method of scalarizing the several objectives into a single composite objective function converts the multiobjective optimization problem into a singleobjective optimization problem. When this function is optimized, in most cases it is possible to obtain one particular trade-off solution. This procedure is called *preference-based multiobjective optimization* [6]. The best trade-off found for the function for this two parameters, that is the convergence, is shown in Figure 5.

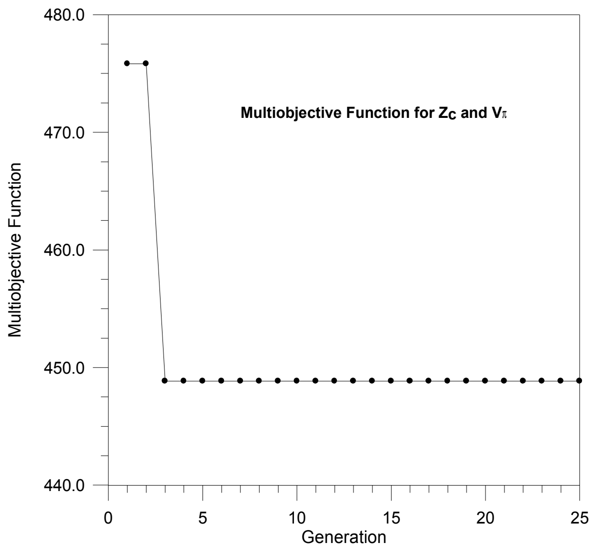

Figure 5. Change of fitness over generations for multiobjective function for Z_c and V_π .

The multiobjective function that has been used into optimization is presented below, where the two parameters of interest are calculated by means of the quadratic difference, being Z^* and V^*_Π the desirables values. Both functions have the same weight into the multiobjective function.

$$\mathbf{F}=[(Z^*-Z)^2]+[k*(V^*_\pi-V_\pi)^2] \qquad (10)$$

where k is a number that normalizes a function, with respect to another one (k = 1.0e5).

The best value that the multiobjective function could reach for the electrical parameters is shown in the Table 4.

The second function (equation 11) is formed by the driving power of operation of the microwaves source (equation 12), that express the relationship between the characteristic impedance and the half-wave voltage, and the characteristic impedance. Again, the quadratic difference is applied and both functions have the same weight.

Table 4. Optimization result for multiobjective approach

Z_c	**32.5254 Ω**
V_π	**8.788 V.cm**
Electrode gap (*G*)	15 µm
Electrode thickness (*Te*)	19 µm
Electrode width (W_H)	26 µm
Buffer layer thickness (*Tb*)	0.2 µm

$$\mathbf{F}=[(Z^*-Z)^2]+[k*(P^*_{in}-P_{in})^2] \qquad (11)$$

$$P_{in}=\frac{V_\pi^2}{8Z_s\{1-[(Z_s-Z_c)/(Z_s+Z_c)]^2\}} \qquad (12)$$

where Z^* and P^*_{in} are the desirables values and k is a number that normalizes a function, with respect to another one (k = 5.0e7).

The best trade-off found for the function, using the equation (11) as fitness function, is shown in Figure 6.

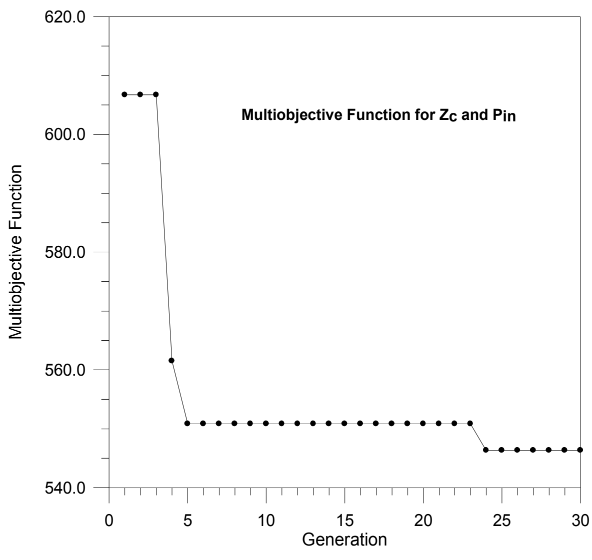

Figure 6. Change of fitness over generations for multiobjective function for Z_c and V_π , using equation (12) as function.

With this function, the best value that the multiobjective function could reach for the electrical parameters is presented in the Table 5.

Both the functions had obtained its better value, for the set of parameters in study, but for engineering problems the use of Pareto-optimal solutions is more convenient, where the optimization process supplies to a set of acceptable solutions trading-off differently between the two objectives. Then, the decision marker can choice the feasibles configurations for manufacturing the prototypes.

Table 5. Optimization result for multiobjective approach

Z_c	**23.95654 Ω**
V_π	**5.3889 V.cm**
Electrode gap (*G*)	13 µm
Electrode thickness (*Te*)	15 µm
Electrode width (W_H)	28 µm
Buffer layer thickness (*Tb*)	0.2 µm

6. Conclusion

In this work, a design optimization method was proposed for Mach-Zehnder modulators. The Finite Element Method was employed to model the device and to analyze the performance of the electrooptical modulator for each candidate solution. A Genetic Algorithm performed the optimization. In a first approach, the optimization process was applied by changing two parameters of the modulator since it is difficult to model all the parameters of a real device. Results show the feasibility of the proposed approach. The use of parallel computing was required in order to reduce the computational time.

The next step on this work includes the implementation the Pareto-optimal concept in the optimization process in order to give a more versatility in to obtain a set of solutions for the problem.

7. References

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