

# Modelling of Satellite Network Communications using Markov Process

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Satellite systems play a major role in network communications with large geographical coverage, and high-capacity channels for radio, television, mobile telephony and Internet signals transmissions. Markov processes have been widely used to model satellite constellations (Wood (2001), (Usaha and Barria, 2002), Zaim et al. (2003)).

The basic property of a Markov process (Çinlar, 1975) is that its future behavior is conditionally independent of its past provided that their present state is known.

In the present paper, we consider a *continuous time Markov chain* (CTMC) to model a single orbit satellite constellation with four satellites, in which the position of the satellites is fixed in the sky, as in the case of geostationary orbit. In a CTMC, the amount of time the process spends in each state before making a transition to a different state is exponentially distributed. The developed model is based in Zaim et al. (2003), and can easily be extended to more than four satellites in a single orbit.

In this satellite constellation system, we assume that call requests arrive at each satellite according to a Poisson process, and the call holding times are exponentially distributed. Satellites communicate with each other by line of sight using inter-satellite links (ISL). The connection between the earth and the satellite is called UDL, “up-and-down link”. Each satellite’s UDL (ISL) has  $C_{UDL}$  ( $C_{ISL}$ ) bidirectional channels. A call originated at satellite 1 and terminated at satellite 3 is routed through satellite 2, and a call originated at satellite 2 and terminated at satellite 4 is routed through satellite 3. The system is presented in Figure 1.

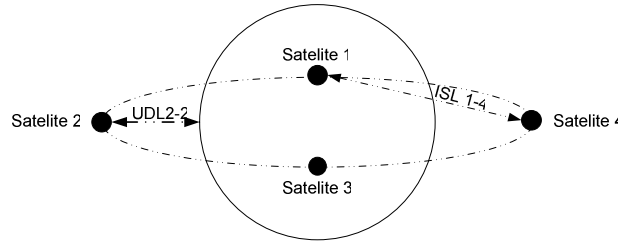


Figure 1: Four satellites in a single orbit.

Let  $n_{ij}$  be a random variable representing the number of active bidirectional calls between satellite  $i$  and satellite  $j$ ,  $1 \leq i \leq j \leq 4$ , regardless whether the calls were originated at satellite  $i$  or  $j$ . Notice that,  $n_{ii}$  represents the number of calls between two customers under satellite  $i$ , and two bidirectional UDL channels are used. The four-satellite system in Fig. 1 can be described by a ten-dimensional CMTC. The set of all possible states of this CMTC is given by:

$$E = \{ (n_{11}, n_{12}, n_{13}, n_{14}, n_{22}, n_{23}, n_{24}, n_{33}, n_{34}, n_{44}) / n_{ij} \in \mathbb{N}; 1 \leq i \leq j \leq 4; i, j \in \mathbb{N}; C_{UDL} \in \mathbb{N}; C_{ISL} \in \mathbb{N}; \quad (2.1)$$

$$2n_{11} + n_{12} + n_{13} + n_{14} \leq C_{UDL}; \quad (2.2)$$

$$n_{12} + 2n_{22} + n_{23} + n_{24} \leq C_{UDL}; \quad (2.3)$$

$$n_{13} + n_{23} + 2n_{33} + n_{34} \leq C_{UDL}; \quad (2.4)$$

$$n_{14} + n_{24} + n_{34} + 2n_{44} \leq C_{UDL}; \quad (2.5)$$

$$n_{12} + n_{13} \leq C_{ISL}; \quad (2.6)$$

$$n_{14} \leq C_{ISL}; \quad (2.7)$$

$$n_{13} + n_{23} + n_{24} \leq C_{ISL}; \quad (2.8)$$

$$n_{24} + n_{34} \leq C_{ISL} \}$$

Constraints (2.1) to (2.8) are due to the fact that some calls share common UDL and ISL. Constraint (2.1) ensures that the number of calls originated (equivalently, terminated) at satellite 1 be, at most, equal to that satellite's UDL capacity. Note that a call that originates and terminates within the footprint of satellite 1, captures two channels, thus the term  $2n_{11}$  in constraint (2.1). Constraints (2.2), (2.3) and (2.4) are similar to (2.1), but correspond to satellites 2, 3 and 4, respectively. Finally, constraints (2.5) to (2.8) ensure that the number of calls using the link ISL between two satellites be, at most, equal to that link capacity.

Let  $\lambda_{ij}$  denotes the arrival rate of calls, and  $1/\mu_{ij}$  the mean holding time of calls between satellites  $i$  and  $j$ . Then, the state transition rates  $r(e, \hat{e})$  from the current state  $e \in E$  to the next state  $\hat{e} \in E$  for this CTMC are given by:

- $r(e, \hat{e}) = \lambda_{ij}, \forall i, j$ , if the transition is due to the arrival of a call between satellites  $i$  and  $j$ . In this case,  $\hat{e}$  is equal to  $e$ , except in the position that corresponds to the element  $n_{ij}$ , that is increased by one;
- $r(e, \hat{e}) = n_{ij}\mu_{ij}, \forall i, j, n_{ij} > 0$ , if the transition is due to the termination of a call between satellites  $i$  and  $j$ . In this case,  $\hat{e}$  is equal to  $e$ , except in the position that corresponds to the element  $n_{ij}$ , that is decreased by one.

In Table 1, we present the size of the CMTC used, considering  $\lambda_{ij} = 1, \mu_{ij} = 2, 1 \leq i \leq j \leq 4$ , and different values of  $C_{UDL}$ . To make it simple, we consider  $C_{ISL} = C_{UDL}$ .

**Table 1 - Number of states and transitions**

| Number of satellites | Number of bidirectional calls for each satellites ( $C_{UDL} = C_{ISL}$ ) | Number of states | Number of transitions |
|----------------------|---|------------------|-----------------------|
| 4                    | 3   | 722              | 4,928                 |
|                      | 5   | 14,138           | 134,970               |
|                      | 10  | 1,960,575        | 25,712,940            |

This table shows that the state space increases exponentially when the number of satellites and/or the capacity of connections in the satellite constellation increase. Therefore the use of this Markov model in constellations with higher capacity of connections, such as, Iridium<sup>1</sup>, which consists of 66 Low Earth Orbit (LEO) satellites, and GLOBALSTAR<sup>2</sup>, which consists of 48 LEO satellites (both aiming at hand held telephony, primarily to remote areas) is not computationally feasible. To deal with these cases the use of decomposition techniques will be studied.

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<sup>1</sup> Iridium Satellite, 2006, <http://www.iridium.com>.

<sup>2</sup> Globalstar, 2006, <http://www.globalstar.com.br>.