# Lagrangean Bounds for the Daily Photograph Scheduling of an Earth Observation Satellite

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The daily photograph scheduling problem (DPSP) is an important task for several earth observation satellites. This problem consists in defining a subset of photographs from a set of candidate photographs which will be effectively taken by satellite. However, this subset must respect a large number of hard constraints but maximizing a profit function.

The profit function reflects several criteria such as client importance, demand urgency, meterological forecasts and so on (Vasquez & Hao, 2001). The constraints include both physical constraints such as the recording capacity on board of the satellite and logical constraints such as non overlapping trials and meeting the minimal transition time between two successive trials on the same camera.

In the literature, Bensana *et al* (1999) proposed benchmark problems obtained from *SPOT* satellites which constitute a family of earth optical observation satellites that are developed by the *CNES* (French *Centre National d'Etudes Spatiales* www.cnes.fr) and exploited by the *SPOT Image* company (www.spotimage.fr).

Vasquez & Hao (2001), based on the benchmark problems, proposed a mathematical formulation that is a generalized version of the well-known knapsack model, which includes large number of binary and ternary logical constraints. This formulation is presented bellow:

$$Max\ v(DPSP) = \sum_{i=1}^{m} g_i x_i \qquad (1)$$

$$\text{Subject to} \quad \sum_{i=1}^{m} c_i x_i \le Max\_Capacity \qquad (2)$$

$$x_i + x_j \le 1 \quad \forall (x_i, x_j) \in C_2 \qquad (3)$$

$$x_i + x_j + x_k \le 2 \quad \forall (x_i, x_j, x_k) \in C_3 \qquad (4)$$

$$x_i + x_j + x_k \le 2 \quad \forall (x_i, x_j, x_k) \in C_4 \qquad (5)$$

$$x_i \in \{0,1\} \quad \forall i = 1...m$$

Where:

- $m$ is the total number of candidate photographs considering camera and type of photograph (mono or stereo);
- $g_i$ is the profit assigned to candidate photograph $x_i$;
- $c_i$ is the size assigned to candidate photograph $x_i$;
- *Max_Capacity* is the maximal recording capacity. Constraints (2) state that the sum of the sizes of photographs cannot exceed *Max_Capacity*;
- $C_2$ is the set of all pairs ($x_i$, $x_j$) whose photographs cannot be obtained simultaneous. Constraints (3) involve the non overlapping of two trials, the minimal transition time between two successive trials of a camera, and also limitations on instantaneous data flow;
- $C_3$ is a set of three candidate photographs ($x_i$, $x_j$, $x_k$) that cannot appear simultaneous. Constraints (4) are known as ternary constraints, involve limitations on instantaneous data flow;
- $C_4$ is a set of three mono candidate photographs ($x_i$, $x_j$, $x_k$) that must be obtained by only one camera on the satellite. Constraints (5) ensure that each mono photograph must be taken a once.

Considering the formulation above, some relaxations were tested to verify the quality of the bounds generated. We performed experiments with linear and lagrangean relaxations and compared our results with the literature. For lagrangean relaxations, the best results were found with a lagrangean relaxation with clusters (LagClus) (Ribeiro & Lorena, 2005; 2006) that is obtained after a graph representation of the DPSP that is partitioned into clusters. The main results are shown bellow.

**Table 1. Main results**

| Instance | Optimal/Best Solution ($v^*$) | Linear relaxation | | Column Generation (Gabrel, 1990) | | Partitioning (Vasquez & Hao, 2003) | | LagClus | |
|---|---|---|---|---|---|---|---|---|---|
| | | Bound ($v$) | GAP | Bound ($v$) | GAP | Bound ($v$) | GAP | Bound ($v$) | GAP |
| 54 | 70 | 83.00 | 15.66 | 71.44 | 2.02 | 70 | 0.00 | 70 | 0.00 |
| 29 | 12032 | 13057.00 | 7.85 | - | - | 12032 | 0.00 | 12032 | 0.00 |
| 42 | 108067 | 190567.50 | 43.29 | 108067.00 | 0.00 | 108067 | 0.00 | 108067 | 0.00 |
| 28 | 56053 | 221090.50 | 74.65 | 70053.00 | 19.98 | 58053 | 3.45 | 62053 | 9.67 |
| 5 | 115 | 315.00 | 63.49 | 119.00 | 3.36 | 116 | 0.86 | 115 | 0.00 |
| 404 | 49 | 96.00 | 48.96 | 53.66 | 8.68 | 49 | 0.00 | 49.99 | 0.00 |
| 408 | 3082 | 5188.00 | 40.59 | 3091.75 | 0.32 | 3083 | 0.03 | 3082.89 | 0.00 |
| 412 | 16102 | 31323.50 | 48.59 | 16433.97 | 2.02 | 16102 | 0.00 | 16103 | 0.01 |
| 414 | 22120 | 40416.00 | 45.27 | - | - | 22120 | 0.00 | 22122.27 | 0.01 |
| 503 | 9096 | 12637.50 | 28.02 | 9595.00 | 5.20 | 9096 | 0.00 | 9096 | 0.00 |
| 505 | 13100 | 22236.00 | 41.09 | 14107.75 | 7.14 | 13103 | 0.02 | 13100 | 0.00 |
| 507 | 15137 | 27361.50 | 44.68 | 17639.22 | 14.19 | 15137 | 0.00 | 15138 | 0.01 |
| 509 | 19125 | 36394.00 | 47.45 | - | - | 19125 | 0.00 | 19125 | 0.00 |
| 1401 | 176056 | 300000.00 | 41.31 | - | - | 180062 | 2.22 | 192530.9 | 8.56 |
| 1403 | 176140 | 300149.00 | 41.32 | - | - | 180160 | 2.23 | 195654.9 | 9.97 |
| 1405 | 176179 | 300207.00 | 41.31 | - | - | 179226 | 1.70 | 198952.1 | 11.45 |
| 1407 | 176245 | 300385.00 | 41.33 | - | - | 177304 | 0.60 | 189080.2 | 6.79 |
| 1502 | 61158 | 64160.50 | 4.68 | - | - | 61158 | 0.00 | 61158 | 0.00 |
| 1504 | 124243 | 191279.00 | 35.05 | - | - | 124258 | 0.01 | 124243.6 | 0.00 |
| 1506 | 168247 | 276863.00 | 39.23 | - | - | 168294 | 0.03 | 179699.5 | 6.37 |

GAP = $(v-v^*)/v$ (%) and "-" means no result is available.

Table 1 shows that LagClus provided better bounds than linear relaxation and Gabrel´s column generation. Comparing our results with the partitioning of Vasquez & Hao (2003), only in some cases, LagClus provided better bounds, however LagClus in the worst case, elapsed 53,955 seconds to conclude whereas the partitioning elapsed days to weeks to finish. It shows that LagClus can be used in a branch-and-price algorithm for example, to prove the optimality of some instances.

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