A meshfree PIC model for non-collisional plasma simulation with the element-free Galerkin method

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The Particle-in-Cell (PIC) methodology has been widely used for the simulation of several plasma phenomena (Birdsall and Langdon, 1985), (Kruer, 1998), which are of high interest in industry for the development of plasma-based devices (Cheng *et al.*, 2004), (Hermannsfeldt, 1987) and materials (Lieberman and Lichtemberg, 1994) in many different manufacturing sectors (Lean, 1998), (Matyash *et al.*, 2005), and in research (Scales *et al.*, 1997). The applications of plasma-based technologies cover a wide range of scientific research areas such as medicine, material processing, aeronautics and aerospace research (Shang, 2001), energy sources research (Rantamäki *et al.*, 1999) , and others.

Some common difficulties arise when studying plasma phenomena. In general, the presence of regions with strong-gradient fields is a common issue in the majority of the problems encountered in all of the above mentioned research areas. Particularly, the simulation of plasma-based devices frequently involves complex geometries, as well as regions with strong-gradient electric and magnetic fields.

The PIC methodology is very versatile and allows treating all of these difficult tasks, considered the limits of validity of the approximations. PIC models have been studied since the sixtieth; however its development was delayed for many years because of the limited capabilities of memory and processor. In other words, PIC models require a certain amount of memory and processor speed that, most of time, was only practical in parallel machines for the past decades, but nowadays most of these models may run in a single processor machine, depending the size of the problem. The huge range of strategic technologies of high interest has stimulated the advances in this research area (Hewett, 2003), (Shon *et al., 2001*), which has been supported by the increasing availability of high performance computing architectures at low cost, such as PC clusters (Passaro *et al.,* 2004) that enables both the inclusion of more detailed physical description and the choice of more robust numerical methods.

One can say that the capability of a PIC model with a specific setting of numerical methods is subjected to the capabilities of these methods. For instance, when using a finite difference method (FDM) for computing the electric field, it is hard to deal with any complex geometry. Similarly, when using the finite element method (FEM) (Paes *et al.,* 2003) for this task, it is well known that complex three-dimensional geometries are difficult to deal because of mesh requirements. Additionally, the presence of strong space-dependent variations of the field in the plasma requires a well refined mesh in these locations in order to fairly reproduce the field gradients, and to correctly calculate the particle acceleration. When these strong-gradient field regions change their location during the simulation, the mesh should adapt accordingly to the field changes. In few words, there exist many situations where it would be convenient the method used to solve the field problem allows the easily inclusion of *h*-adaptive algorithms (*mesh-adaptive*). Although adaptive mesh refinement procedures in two-dimensional domains are a well studied issue in many research areas, it is of common sense that it becomes a difficult and time consuming task as the complexity of the geometry increases. Indeed, this task becomes much more complex, and sometimes prohibitive, for three-dimensional complex geometries. Thus, a proper choice of these numerical methods may not only improve numerical accuracy and computational efficiency but it may also dictate the practical applicability of the code for simulating a given problem.

Meshfree methods (Belytschko *et al.,* 1994), (Cingoski *et al.,* 1998), (Ho *et al.,* 2001) are a recent class of numerical methods alternative to mesh based methods, such as the FEM. The main interest in this class of methods is that the construction of the approximation do not depend on mesh elements, but only on discretization nodes and weight functions.

In this work we introduce a meshfree formulation based on the interpolating element-free Galerkin method (EFGM) (Marques *et al.,* 2005), (Verardi *et al.,* 2003) for simulating plasmas. Differently from PIC-

FEM and PIC-FDM, the present formulation modifies the geometrical concept of the cells in the standard PIC methodology, leading to a numerical model closer to the physical modeling of plasmas.

The implementation of the proposed approach requires different computational procedures, since the cell topology is generalized, for instance in the management of particle locations with respect to the cell structure. Another interesting feature is the high degree of continuity of the EFGM approximations and their derivatives, which enables the field variations inside the cells to be fairly represented. Moreover, another motivation of this approach relies in the high efficiency of the EFGM with respect to its *h*-adaptive property (Häussler-Combe and Korn, 1998), which is performed by adding nodes rather than remeshing a region of the domain. This is a very desirable property, and it is required for accurately simulating a huge variety of problems commonly encountered in frontiers of scientific and technological researches.

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