

# Satellite Simulator Control System Design Using SDRE Method

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Received: March 29, 2011 / Accepted: April 28, 2011 / Published: June 25, 2011.

**Abstract:** This paper presents the application of the State-Dependent Riccati Equation (SDRE) method in conjunction with Kalman filter technique to design a satellite simulator control system. The performance and robustness of the SDRE controller is compared with the Linear Quadratic Regulator (LQR) controller. The Kalman filter technique is incorporated to the SDRE method to address the presence of noise in the process, measurements and incomplete state estimation. The effects of the plant non-linearities and noises (uncertainties) are considered to investigate the controller performance and robustness designed by the SDRE plus Kalman filter. A general 3-D simulator Simulink model is developed to design the SDRE controller using the states estimated by the Kalman filter. Simulations have demonstrated the validity of the proposed approach to deal with nonlinear system. The SDRE controller has presented good stability, great performance and robustness at the same time that it keeps the simplicity of having constant gain which is very important as for satellite onboard computer implementation.

**Key words:** Satellite simulator, state-dependent Riccati equation (SDRE) method, nonlinear dynamics.

## 1. Introduction

There are several methodologies to investigate the satellite Attitude Control System (ACS) performance [1], depending on the investigation objectives; computer simulation cannot be the appropriate one. The use of experimental platforms has the important advantage of allowing the satellite dynamics representation in laboratory, from which is possible to accomplish experiments and simulations to evaluate satellite ACS. Experimental test has the possibility of introducing more realism than the simulation; however, it has the difficulty of reproducing zero gravity and torque free space condition.

Examples of experimental platforms for investigating different aspects of the satellite dynamic

and control system can be found in [2-3]. A classic case not investigated experimentally before launch was the dissipation energy effect that has altered the satellite Explorer I rotation [4]. An important aspect that must be first investigated through experimental procedure is the platform inertia parameters identification [5]. When inertia parameters are not well known the system can present some source of uncertainty [6]. An algorithm based on the least square method to identify mass parameters of a space vehicle in rotation during attitude maneuvers has been developed by [7]. The H-infinity control technique [8] was used to design robust control laws for a satellite composed of rigid and flexible panels.

The SDRE method is a new approach that can deal with non-linear plant, and is considered as the non-linear counterpart of LQR control. It linearizes the plant around the instantaneous point of operation and produces a constant state-space model of the system where the LQR control technique can be applied and a

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specific controller calculated. The process is repeated in the next sampling periods therefore producing and controlling several state dependent linear models out of a non-linear one. The SDRE method has been applied for controlling a non-linear system similar to the three degree of freedom satellite model considered in this paper [9]. Kalman filter technique, when applied as state observer in conjunction with the SDRE method, allows the incorporation of non-linearities in the filter process. The uncertainties of the system can be represented by process and measurements noise.

In this paper the standard LQR linear controller [10] and the SDRE controller associated with Kalman filter are applied to the same non-linear plant, in the presence of noise and uncertainties, to investigate the performance and robustness of both methods. Results have proven the reliability of SDRE as a control method to be applied in the satellite attitude simulator. Comparing with traditional non-linear control methods [11], the SDRE method has the advantage of avoiding intensive calculation, resulting in simpler control algorithms more appropriate to be implemented in satellite on-board computer.

The paper is organized as follows: Section 2 presents the general mathematical simulator model. Section 3 introduces the SDRE plus Kalman filter control system design methods. Simulations results and discursion are presented in Section 4. Section 5 ends the paper with the conclusions.

## 2. Simulator Mathematical Model

The general 3-D simulator model is similar to the INPE's simulator shows in Figure 1. It has a disk-shaped platform, supported on a plane with a spherical air bearing. The platform can accommodate various satellites components; like sensors, actuators, computers and its respective interface and electronic. Apart from the difficulty of reproducing zero gravity and torque free condition, modeling a 3-D simulator,



Fig. 1 INPE 3-D satellite simulator.

basically, follows the same step of modeling a rigid satellite with rotation in three axes free in space. The inertial reference system  $F_i (I_1, I_2, I_3)$  is located in the centre of the spherical bearing. The orientation of the platform is given by the body reference system  $F_b$  with respect to inertial system considering the principal axes of inertia. To describe the orientation of  $F_b$ , one uses Euler angles  $(\theta_1, \theta_2, \theta_3)$  in the sequence 3-2-1, to guarantee that there is no singularity in the simulator attitude rotation.

The equations of motion are obtained using the Euler's angular momentum theorem given by

$$\dot{\vec{h}} = \vec{g} \quad (1)$$

where  $\vec{g}$  and  $\vec{h}$  are the total torque and angular momentum of the system, respectively.

The total angular momentum of the system is given by

$$\vec{h} = I\vec{\omega} + I_w(\vec{\Omega} + \vec{\omega}) \quad (2)$$

where  $I = \text{diag}(I_{11}, I_{22}, I_{33})$  is the system matrix inertia moment,  $\vec{\omega}$  is the angular velocity of  $F_b$ ,  $\vec{I}_w = \text{diag}(I_{w1}, I_{w2}, I_{w3})$  is the reaction wheel matrix inertia moment and  $\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$  are the reaction wheel angular velocity. Since  $\vec{I}$  and  $\vec{I}_w$  are constants in  $F_b$ , it is convenient to express (1) in this system.

Differentiating (1) and considering the external torque equal to zero, one has

$$\dot{\vec{h}} + \vec{\omega}^x \vec{h} = 0 \quad (3)$$

Substituting (2) into (3) the system angular velocity equation is given by

$$\dot{\vec{\omega}} = (I + I_w)^{-1} \left[ -\vec{\omega} (I + I_w) \vec{\omega} - \vec{\omega} I_w \vec{\Omega} - I_w \dot{\vec{\Omega}} \right] \quad (4)$$

The simulator attitude as function of the angular velocity is

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} 0 & \sin \theta_3 / \cos \theta_2 & \cos \theta_3 / \cos \theta_2 \\ 0 & \cos \theta_3 & -\sin \theta_3 \\ 1 & \sin \theta_3 \sin \theta_2 / \cos \theta_2 & \cos \theta_3 \sin \theta_2 / \cos \theta_2 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \quad (5)$$

### 3. SDRE and Kalman Filter Methods

The SDRE linearizes the plant around the current operating point and creates constant state space matrices so that the LQR method can be used. This process is repeated in all steps, resulting in a point wise linear model from a non-linear model, so that the Riccati equation is solved and a control law is calculated also in each step.

Here the SDRE method is applied to design a satellite ACS simulator, where  $A(x)$  is the only non-linear state dependent matrix. Therefore, the non-linear state model given by (4) and (5) can be written in the linear-like state dependent coefficient (SDC) form given by

$$\begin{cases} \dot{x} = A(x)x + Bu \\ y = Cx \end{cases} \quad (6)$$

where  $A(x)$  represents the part of the system with non-linear dynamics,  $B$  and  $C$  are the input and output matrices, which define the position and the type of actuator and sensor. After some manipulations, matrices  $A(x)$  and  $B$  are given by

$$A(x) = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\sin \theta_3}{\cos \theta_2} & \frac{\cos \theta_3}{\cos \theta_2} \\ 0 & 0 & 0 & 0 & \cos \theta_3 & -\sin \theta_3 \\ 0 & 0 & 0 & 1 & \frac{\sin \theta_3 \sin \theta_2}{\cos \theta_2} & \frac{\cos \theta_3 \sin \theta_2}{\cos \theta_2} \\ 0 & 0 & 0 & 0 & \frac{I_{22}\omega_3}{I_{11} + I_w} & \frac{-I_{33}\omega_2}{I_{11} + I_w} \\ 0 & 0 & 0 & \frac{-I_{11}\omega_3}{I_{22} + I_w} & 0 & \frac{I_{33}\omega_1}{I_{22} + I_w} \\ 0 & 0 & 0 & \frac{I_{11}\omega_2}{I_{33} + I_w} & \frac{-I_{22}\omega_1}{I_{33} + I_w} & 0 \end{pmatrix} \quad (7)$$

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{-I_w}{I_{11} + I_w} & 0 & 0 \\ 0 & \frac{-I_w}{I_{22} + I_w} & 0 \\ 0 & 0 & \frac{-I_w}{I_{33} + I_w} \end{pmatrix} \quad (8)$$

The SDRE method has also a performance criterion  $J$  to be minimized given by

$$J = \frac{1}{2} \int_0^\infty [y^T Q(x) y + u^T R(x) u] dt \quad (9)$$

The stability condition requires that the weighting matrix  $Q$  be positive semi-definite and  $R$  be positive definite. The state variable feedback control law is given by

$$u = -K(x)x \quad (10)$$

with the state-dependent gain given by

$$K(x) = -R^{-1} B^T P(x) \quad (11)$$

and  $P(x)$  is solution of the Riccati equation given by

$$A^T(x)P(x) + P(x)A(x) - P(x)BR^{-1}B^T P(x) + Q = 0 \quad (12)$$

The simulator model state vector  $x$  consists of the angle and angular velocity given by

$$x = (\theta_1 \ \theta_2 \ \theta_3 \ \omega_1 \ \omega_2 \ \omega_3)^T \quad (13)$$

The SDRE theory presented up to here allows designing the control law without estimating the states; but for the case when some states are not available, one uses the Kalman filter, which also is adequate for on-board implementations because the estimation of the next state depends only on the previous and actual state, without keeping in the memory the history of all the states calculated. Besides, the Kalman filter takes into account the process noise  $w$  and the measures noise  $z$ . As a result, the system becomes

$$\begin{cases} \dot{\hat{x}} = A(x)\hat{x} + Bu + w \\ y = Cx + z \end{cases} \quad (14)$$

The Kalman filter algorithm consists basically of two steps; the time and the measurement update, which is given by the following two sets of equations

$$\begin{cases} \hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1} \\ P_k^- = AP_{k-1}A^T + S_w \end{cases} \quad (15)$$

$$\left. \begin{aligned} K_k &= P_k^- C^T (C P_k^- C^T + S_z)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k (y_k - C \hat{x}_k^-) \\ P_k &= (I - K_k C) P_k^- \end{aligned} \right\} \quad (16)$$

The matrix  $P_k$  represents the covariance of the estimated state  $\hat{x}$ , and  $K_k$  is the Kalman gain. One observes now that the matrix  $A(x)$  is calculated in every step of estimating the states  $\hat{x}$ ,  $P_k$  and  $K_k$ .

#### 4. Simulations Results and Discursion

Initially, one performs simulations to evaluate the performance and robustness of the SDRE technique to control the non-linear plant in comparison with the LQR technique. In the sequel, the Kalman filter efficiency to estimate the states in the presence of noise is also shown. In order to stress the non-linear terms of the plant, one considers very large attitude maneuvers. The controller performance requirements are small overshoot and short time of response.

The 3-D simulator Simulink model develop allows to study diferents configuration even for the number and positions of the reaction wheel. Here one uses typical values for the simulator and reaction wheel inertia moments, as shown in Table 1. The values for sensor noises are also typical, they are  $\theta = 0.2$  deg and rate  $\dot{\theta} = 0.1$  deg/s.

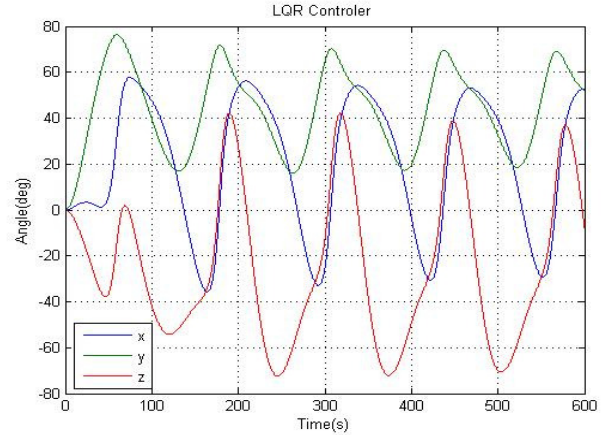
Figures 2 and 3 show the LQR and SDRE controllers without noise for the simulator attitude maneuver from zero to (50, 20, -30) deg emphasizing the plant non-linear terms.

It is possible to observe that the SDRE controller performance is better than LQR. However, it is important to mention that although both performances are function of the weighting matrices  $R$  and  $Q$  of the LQR and SDRE controllers. During the design it was not possible to find the  $R$  and  $Q$  matrices for the LQR controller that could be able to control the nonlinear plant properly. Besides, in this first design one has considered that all states are available for feedback, which is not always true.

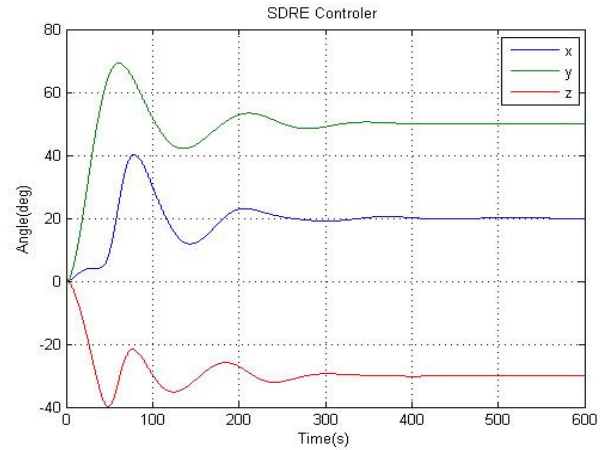
Figure 4 shows SDRE controllers without noise for

**Table 1** Typical values used in the simulations.

Simulator	Kg.m <sup>2</sup>	Wheel	Kg.m <sup>2</sup>
I <sub>11</sub>	1.17	I <sub>x</sub>	0.0018
I <sub>22</sub>	1.17	I <sub>y</sub>	0.0018
I <sub>33</sub>	1.13	I <sub>z</sub>	0.0018



**Fig. 2** LQR controller for large maneuvers.

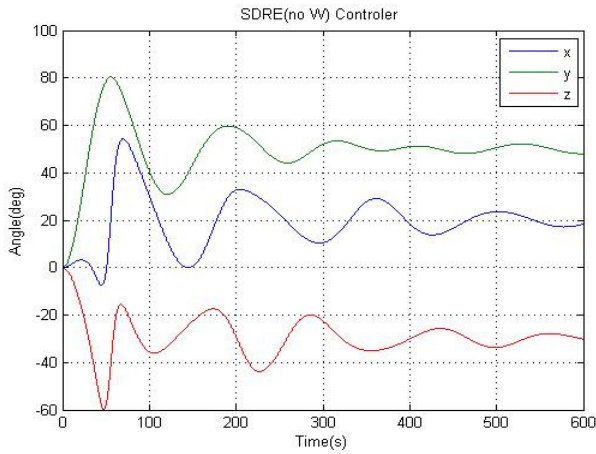


**Fig. 3** SDRE controller for large maneuvers.

the previous maneuver; having only the angles to fed back. Without angular velocity sensor, the SDRE controller performance is degraded showing the need for Kalman filter to estimates the states that are not available.

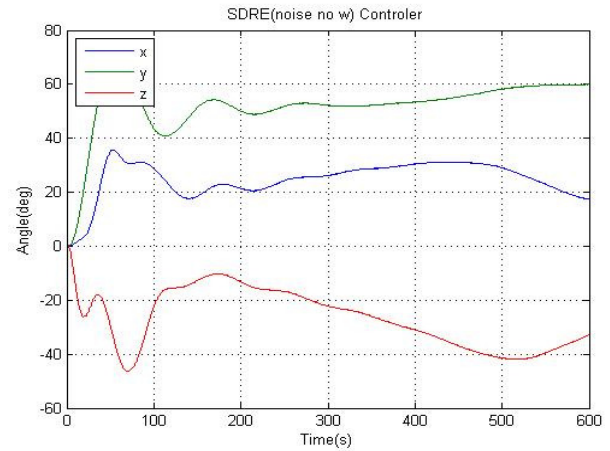
Figure 5 shows the SDRE controllers in the presence of noise for the same large attitude maneuvers as before. It is observed that the SDRE controller performance is quite similar when there is no noise. This results show that the SDRE controller has the same robust properties of the LQR controller when all

the states are available for feedback. One should keep in mind that noise effect

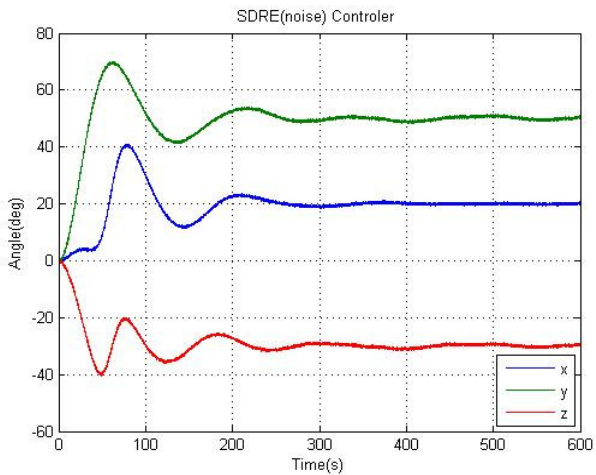


**Fig. 4** SDRE controller only angle to feedback.

Finally, Figure 8 shows the torques of the SDRE controllers in the presence of noise for the same large



**Fig. 6** SDRE controller with only angle to feedback.

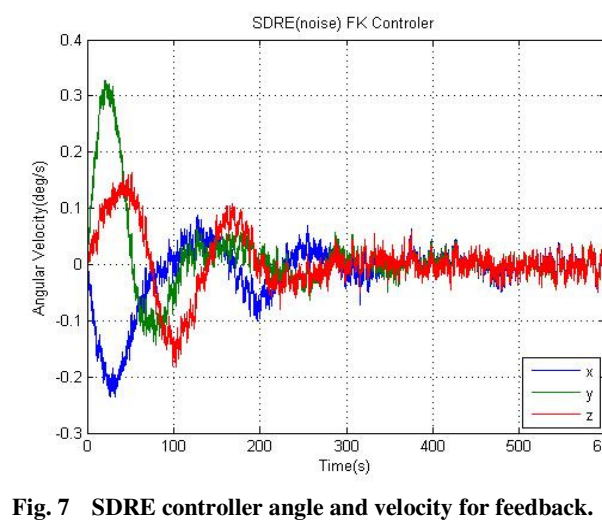
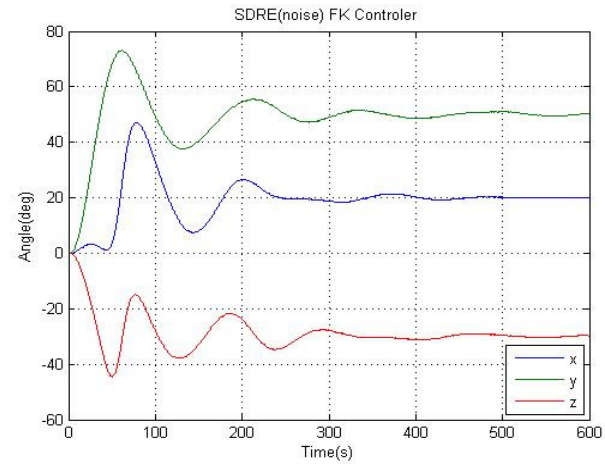


**Fig. 5** SDRE controller with system noises.

is very important when the controllers pointing precision is very demanding.

Figure 6 shows the SDRE controllers in the presence of noise for the same large attitude manoeuvres, having only the angular measurements for feedback. One observes that the SDRE controller performance has been degraded due to the noise presence and the unavailability of all the states. In this case the introduction of the Kalman filter in the control system to estimates all the states is fundamental.

Figure 7 shows the angles and the angular velocities behaviors for the SDRE controllers in the presence of noise for the same large atHowever, having angle and velocities for feedback.

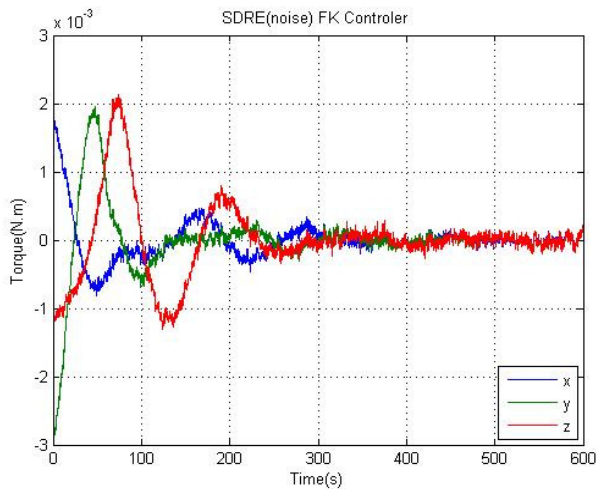


**Fig. 7** SDRE controller angle and velocity for feedback.

attitude maneuvers as before. For the three previously figures, one observes that using the Kalman filter to



estimate angle and angular velocities is fundamental to improve the SDRE controller performance and



**Fig. 8 SDRE Torque to control large maneuver.**

robustness. This is a key point to have better measurements for feedback mainly when the satellite mission involves very large maneuvers ending with a stringent pointing accuracy.

## 5. Conclusions

This paper presents the design of the Attitude Control System for a 3-D satellite simulator based on the SDRE method in conjunction with Kalman filter technique. The satellite simulator model is generic since its equation of motion depends only on its inertia parameters. Simulation has shown that the SDRE controller has superior performance than the LQR controller for large angle maneuvers when the plant non-linear terms are relevant. The LQR controller performance reaches instability where the final attitude angle is far from the origin. On the other hand, the SDRE controller is able to maintain the same level of performance in any region, demonstrating its ability to control non-linear plants. Regarding the presence of noise and Kalman filter implementation together with SDRE technique, one observes that the noise levels affect the SDRE controller performance when the pointing accuracy is stringent, being less relevant when the simulator performs large angle maneuvers. In

general, one observes that the SDRE method keeps the same robust properties of the LQR; however, losing performance when there is no availability of all states for feedback. Comparing with traditional non-linear control methods [12], the SDRE method has the advantage of avoiding intensive calculation, resulting in simpler control algorithms more appropriate to be implemented in satellite on-board computer.

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