Orbit Control for Satellites Using Suboptimal Quadratic Low Thrust

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Abstract: In this paper the problem of spacecraft orbit control with minimum fuel consumption is considered, in terms of simulating low thrust maneuvers for a spacecraft around the Earth. A numerical suboptimal solution is tried, when the direction of the thrust is assumed to be a quadratic form function of a reference angle that specify the position of the satellite in the orbit. The main goal is to develop a software that can calculate the control laws for real missions. The problem is numerically treated by using a direct search approach. The numerical solution of the problem in each iteration is reduced to one of nonlinear programming, which is then solved with the gradient projection method. The spacecraft is supposed to be in Keplerian motion controlled by the thrusts, that are assumed to be of fixed magnitude (either low or high) and operating in an on-off mode. Results of simulations are presented.

Key-Words: astrodynamics, artificial satellites, orbital dynamics, optimal control, transfer orbits.

1 Introduction

This paper is a sequence of previous papers [1], [2], [3], that studied the problem of optimal maneuvers for a spacecraft controlled by the force given by a continuos thrust. In the present paper, the subotimal approach is emphasized, this time using a quadratic form for the control, so we can observe the gain given by this more complex motion of the direction of the thrust. The same mission considered in the references cited above are used for comparison.

R. H. Goddard [4] was one of the first researchers to work on the problem of optimal transfers of a spacecraft between two points. He proposed optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption.

After him comes the important work done by Hohmann [5]. He solved the problem of minimum ΔV transfers between two circular coplanar orbits.

A numerical scheme to solve the transfer between two generic coplanar elliptic orbits is presented by Bender [6]. Another line of research studies the effects of the finite thrust, like the one used in the present paper, in the results obtained from the impulsive model. Zee [7] obtained analytical expressions for the extra fuel consumed to reach the same transfer and for the errors in the orbital elements and energy for a nominal maneuver (a real maneuver that uses the impulses calculated with the impulsive model).

The three-impulse concept was introduced in the literature by Hoelker and Silber [8] and Shternfeld [9]. They showed that a bi-elliptical transfer between two circular orbits has a lower ΔV than the Hohmann transfer, for some combinations of initial and final orbits.

Roth [10] obtained the minimum ΔV solution for a bi-elliptical transfer between two inclined orbits. Following the idea of more than two impulses, we have the work done by Prussing [11], that admits two or three impulses; Prussing [12] that admits four impulses and Eckel [13] that admits N impulses.

Another important assumption is to consider the low thrust. This case is based on the "Primer-Vector" theory developed by Lawden [14], [15]. This situation is considered in several researches, like in the reference [16] [17] and [18].

The third-body perturbation [19] can also be used to help in reducing the costs, like shown in [20], [21] and [22]. In some situations, an onboard orbit determination [23] has to be performed, so you have the actual information about the orbital parameters of the satellite in the moment of starting the maneuver.

2 Definition of the Problem

The basic problem discussed in this paper is the problem of orbit transfer maneuvers. The objective of

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this problem is to modify the orbit of a given spacecraft. In the case considered in this paper, an initial and a final orbit around the Earth is completely specified. The problem is to find how to transfer the spacecraft between those two orbits in a such way that the fuel consumed is minimum. There is no time restriction involved here and the spacecraft can leave and arrive at any point in the given initial and final orbits. The maneuver is performed with the use of an engine that is able to deliver a thrust with constant magnitude and quadratic variable direction. The mechanism, time and fuel consumption to change the direction of the thrust is not considered in this paper.

3 Model Used

The spacecraft is supposed to be in Keplerian motion controlled only by the thrusts, whenever they are active.

This means that there are two types of motion:

i) A Keplerian orbit, that is an orbit obtained by assuming that the Earth's gravity (assumed to be a point of mass) is the only force acting on the spacecraft. This motion occurs when the thrusts are not firing;

ii) The motion governed by two forces: the Earth's gravity field (also assumed to be a point of mass) and the force delivered by the thrusts. This motion occurs during the time that the thrusts are firing.

The thrusts are assumed to have the following characteristics:

i) Fixed magnitude: The force generated by them is always of constant magnitude during the maneuver. The value of this constant is a free parameter (an input for the algorithm developed here) that can be high or low;

ii) Constant Ejection Velocity: Meaning that the velocity of the gases ejected from the thrusts is constant;

iii) Constrained angular motion: This means that the direction of the force given by the thrusts can be modified during the transfer. This direction can be specified by the angles α and β, called pitch (the angle between the direction of the thrust and the perpendicular to the line Earth-spacecraft) and yaw (the angle with the orbital plane). The motion of those angles are constrained to a quadratic form;

iv) Operation in on-off mode: It means that intermediate states are not allowed. The thrusts are either at zero or maximum level all the time.

The solution is given in terms of the constants that specifies the control to be applied and the fuel consumed. Several numbers of "thrusting arcs" (arcs with the thrusts active) can be used for each maneuver. Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the orbital plane) is used as the independent variable.

4 Optimal Control Formulation

The minimum fuel spacecraft maneuver can be treated as a typical optimal control problem, formulated as follows:

Objective Function: Let M_f, the final mass of the vehicle, to be maximized with respect to the control u(.) that is the time to start and to stop the engine and the pitch and yaw angles of the thrust at every instant of time, since the magnitude of the thrust is assumed to be constant. Since we are assuming quadratic variation for the control, all that needs to be specified is the set of six constants: three for the pitch angle and three for the yaw angle.

This system is subject to the following equations of motion:

$$dX_1/ds = f_1 = SiX_1F_1 \quad \textbf{[1]}$$

$$dX_2/ds = f_2 = Si\{[(Ga+1)\cos(s)+X_2]F_1+\nu F_2\sin(s)\} \quad \textbf{[2]}$$

$$dX_3/ds = f_3 = Si\{[(Ga+1)\sin(s)+X_3]F_1 - \nu F_2\cos(s)\} \quad \textbf{[3]}$$

$$dX_4/ds = f_4 = Si\nu F(1-X_4)/(X_1W) \quad \textbf{[4]}$$

$$dX_5/ds = f_5 = Si\nu(1-X_4)m_0/X_1 \quad \textbf{[5]}$$

$$dX_6/ds = f_6 = -SiF_3[X_7\cos(s)+X_8\sin(s)]/2 \quad \textbf{[6]}$$

$$dX_7/ds = f_7 = SiF_3[X_6\cos(s)-X_9\sin(s)]/2 \quad \textbf{[7]}$$

$$dX_8/ds = f_8 = SiF_3[X_9\cos(s)+X_6\sin(s)]/2 \quad \textbf{[8]}$$

$$dX_9/ds = f_9 = SiF_3[X_7\sin(s)-X_8\cos(s)]/2 \quad \textbf{[9]}$$

where:

$$Ga = 1 + X_2\cos(s) + X_3\sin(s) \quad \textbf{[10]}$$

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$Si = (\mu X_1^4)/[Ga^3 m_0(1-X_4)]$ **[11]**

$$F_1 = F\cos(\alpha)\cos(\beta) \quad [12]$$

$$F_2 = F\sin(\alpha)\cos(\beta) \quad [13]$$

$$F_3 = F\sin(\beta) \quad [14]$$

and F is the magnitude of the thrust, W is the velocity of the gases when leaving the engine, ν is the true anomaly of the spacecraft, s is the range angle of the spacecraft, μ is the gravitational parameter of the main body and m_0 is the initial mass of the spacecraft.

In those equations the state was transformed from the Keplerian elements (a = semi-major axis, e = eccentricity, i = inclination, Ω = argument of the ascending node, ω = argument of periapsis, ν = true anomaly of the spacecraft), in the variables X_i, to avoid singularities, by the relations:

$X_1 = [a(1-e^2)/\mu]^{1/2}$ **[15]**

$X_2 = e\cos(\omega-\phi)$ **[16]**

$X_3 = e\sin(\omega-\phi)$ **[17]**

$X_4 = (\text{Fuel consumed})/m_0$ **[18]**

$X_5 = t = \text{time}$ **[19]**

$X_6 = \cos(i/2)\cos((\Omega+\phi)/2)$ **[20]**

$X_7 = \sin(i/2)\cos((\Omega-\phi)/2)$ **[21]**

$X_8 = \sin(i/2)\sin((\Omega-\phi)/2)$ **[22]**

$X_9 = \cos(i/2)\sin((\Omega+\phi)/2)$ **[23]**

$\phi = \nu + \omega - s.$ **[24]**

This system is also subject to the constraints in state, because five of the the Keplerian elements of the initial and the final orbit are fixed: a, e, i, ω, Ω. All the parameters (gravitational force field, initial values of the satellite, etc...) are assumed to be known.

A quadratic parametrization is used as an approximation for the control law (angles of pitch (α) and yaw (β)):

$$\alpha = \alpha_0 + \alpha' * (s - s_0) + \alpha'' * (s - s_0)^2 \quad [25]$$

$$\beta = \beta_0 + \beta' * (s - s_0) + \beta'' * (s - s_0)^2 \quad [26]$$

where α_0, β_0, α', β', α", β" are parameters to be found, s is the instantaneous range angle and s_0 is the range angle when the motor is turned-on.

Considering these assumptions, there is a set of eight variables to be optimized (start and end of thrusting and the parameters α_0, β_0, α', β', α", β") for each "burning arc" in the maneuver. Note that this number of arcs is given "a priori" and it is not an "output" of the algorithm.

By using parametric optimization, this problem is reduced to one of nonlinear programming, which can be solved by several standard methods

5 Numerical Method

To solve the nonlinear programming problem, the gradient projection method was used [24].

It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \nabla\mathbf{f}^{\mathbf{T}}.\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{\mathbf{T}}\right]^{-1}\mathbf{f} \quad [27]$$

where **f** is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \bar{\alpha}\frac{\mathbf{d}}{|\mathbf{d}|} \quad [28]$$

where:

$$\bar{\alpha} = \gamma \frac{J(\boldsymbol{u})}{\nabla J(\boldsymbol{u}).\mathbf{d}} \quad [29]$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla\mathbf{f}^{T}\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{T}\right]^{-1}\mathbf{f}\right)\nabla J(\boldsymbol{u}) \quad [30]$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and γ is a parameter determined by a trial and error technique. The possible singularities in equations **[27]** to **[30]** are avoided by choosing the error margins for tolerance in convergence large enough. This procedure continues until $\left|\boldsymbol{u}_{i+1}-\boldsymbol{u}_i\right|<\varepsilon$ in both equations **[27]** and **[28]**, where ε is a specified tolerance.

6 - Simulations and Numerical Tests

The maneuvers used to validate the method and the software developed are the same ones used in Biggs [16], with the idea of having the possibility of comparing the solutions obtained.

6.1 - MANEUVER 1:
Initial orbit: Semi-major axis: 99000 km, eccentricity: 0.7, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 105 deg.
Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 1.0 N, Initial position: 0, True anomaly: -105 deg, Ejection velocity of the gas: 2.5 km/s.
Condition imposed in the final orbit: Semi-major axis = 104000 km.
Propulsion: 1 arc.
Solution obtained: s_0 = 80.3 deg, s_f = 134.5 deg, α_0 = -3.2, β_0 = 0.0, α' = 0.443, β' = 0.0, α'' = 0.041, β'' = 0.00, Fuel consumed = 2.35 kg, Duration of burn = 6088.4 s.
Final orbit obtained: Semi-major axis: 104000.71 km, eccentricity: 0.712, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 105 deg, True Anomaly = 30.1 deg.

Considering a linear approximation: s_0 = 78.0 deg, s_f = 132.5 deg, α_0 = -8.8, β_0 = 0.0, α' = 0.469, β' = 0.0, Fuel consumed = 2.39 kg, Duration of burn = 6111.6 s.
Final orbit obtained: Semi-major axis: 104000.73 km, eccentricity: 0.714, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 105 deg, True Anomaly = 28.2 deg.

This maneuver changes only the semi-major axis, so the minimum fuel consumption solution is planar, as shown by the results β_0 = 0.0, β' = 0.0.

6.2 - MANEUVER 2:
Initial orbit: Semi-major axis: 99000 km, eccentricity: 0.7, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 105 deg.
Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 1.0 N, Initial position: 0, True anomaly: -105 deg, Ejection velocity of the gas: 2.5 km/s.
Condition imposed in the final orbit: Semi-major axis = 104000 km.
Propulsion: 1 arc, with restriction in applying thrust between the true anomalies of 120.0 deg and 180.0 deg.
Solution obtained: s_0 = 23.1 deg, s_f = 63.1 deg, α_0 = -15.2, β_0 = 0.0, α' = 0.098, β' = 0.0, α'' = 0.034, β'' = 0.00, Fuel consumed = 2.76 kg, Duration of burn = 7001.1 s.
Final orbit obtained: Semi-major axis: 104000.03 km, eccentricity: 0.711, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 100.3 deg, True Anomaly = 318.2 deg.

Considering a linear approximation: s_0 = 25.2 deg, s_f = 65.0 deg, α_0 = -26.3, β_0 = 0.0, α' = 0.179, β' = 0.0, Fuel consumed = 2.80 kg, Duration of burn = 7037.2 s.
Final orbit obtained: Semi-major axis: 104000.06 km, eccentricity: 0.713, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 102.1 deg, True Anomaly = 320.1 deg.

Maneuver 2 considers the same situation simulated before, but there is an extra constraint in the propulsion phase. It is clear that the inclusion of this constraint provided a solution with a larger fuel consumption.

6.3 - MANEUVER 3:
Initial orbit: Semi-major axis: 9900 km, eccentricity: 0.2, inclination: 10 deg, longitude of the ascending node: 0 deg, argument of periapsis: 25 deg.
Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 2.0 N, Initial position: 0, True anomaly: -10 deg, Ejection velocity of the gas: 2.5 km/s.
Condition imposed in the final orbit: Semi-major axis = 10000 km.
Propulsion: 1 arc.
Solution obtained: s_0 = 0.0 deg, s_f = 178.1 deg, α_0 = 0.1, β_0 = 0.0, α' = 0.032, β' = 0.0, α'' = 0.022, β'' = 0.00, Fuel consumed = 3.68 kg, Duration of burn = 4601.1 s.
Final orbit obtained: Semi-major axis: 10000.00 km, eccentricity: 0.2, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 22.9 deg, True Anomaly = 163.2 deg.

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Considering a linear approximation: s_0 = 0.0 deg, s_f = 179.3 deg, α_0 = 2.1, β_0 = 0.0, α' = 0.058, β' = 0.0, Fuel consumed = 3.73 kg, Duration of burn = 4675.3 s.
Final orbit obtained: Semi-major axis: 10000.00 km, eccentricity: 0.2, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 23.2 deg, True Anomaly = 165.9 deg.

Note that the constraint $s_0 \geq 0.0$ is active.

6.4 - MANEUVER 4:
Initial orbit: Semi-major axis: 9900 km, eccentricity: 0.2, inclination: 10 deg, longitude of the ascending node: 0 deg, argument of periapsis: 25 deg.
Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 2.0 N, Initial position: 0, True anomaly: -10 deg, Ejection velocity of the gas: 2.5 km/s.
Condition imposed in the final orbit: Semi-major axis = 10000 km.
Propulsion: 2 arcs.
Solution obtained: First arc: s_0 = 0.0 deg, s_f = 89.7 deg, α_0 = 1.0, β_0 = 0.0, α' = 0.172, β' = 0.0; Second arc: s_0 = 299.1 deg, s_f = 417.6 deg, α_0 = -8.1, β_0 = 0.0, α' = 0.091, β' = 0.0, Fuel consumed = 3.21 kg, Duration of burn = 4009.9 s.
Final orbit obtained: Semi-major axis: 10000.02 km, eccentricity: 0.207, inclination: 10 deg, longitude of the ascending node: 0.0 deg, argument of periapsis: 21.2 deg, True Anomaly = 45.1 deg.

This maneuver shows that the use two arcs for the propulsion reduces the fuel consumption, from 3.73 kg to 3.21 kg in this case.

6.5 - MANEUVER 5:
Initial orbit: Semi-major axis: 4500 km, eccentricity: 0.5, inclination: 8 deg, longitude of the ascending node: -145 deg, argument of periapsis: -20 deg.
Initial data of the spacecraft: Total mass: 11300 kg, Thrust magnitude: 60000 N, Initial position: 0, True anomaly: 170 deg, Ejection velocity of the gas: 4.25 km/s.
Condition imposed in the final orbit: Semi-major axis = 10000 km, eccentricity = 0.122, Inclination = 2.29 deg..
Propulsion: 1 arc and the burn must be completed before the true anomaly of 35.0 deg.
Solution obtained: s_0 = 6.6 deg, s_f = 27.8 deg, α_0 = 0.8, β_0 = 16.5, α' = -0.033, β' = -0.069, Fuel consumed = 5249.9 kg, Duration of burn = 377.4 s.
Final orbit obtained: Semi-major axis: 7435.00 km, eccentricity: 0.122, inclination: 2.290 deg, longitude of the ascending node: 255.2 deg, argument of periapsis: 169.0 deg, True Anomaly = 324.6 deg.

This maneuver considers the case where the thrust is large and that there are three keplerian elements to be changed.

After that, a real mission planned by Brazil is used. For this mission, two kinds of maneuvers will be necessary (in both phases the fuel used is Hydrazine):

i) Initial transfer phase, where the objective is to send the satellite from the parking orbit to the nominal orbit;

ii) Station-keeping, where the objective is to keep the satellite near the nominal orbit.

The transfer phase will occur, in the worst case, with the following data:
i) Initial orbit: Semi-major axis of 6768.14, eccentricity of 0.00591, inclination of 97.44 degrees, ascending node of 67.27 degrees, argument of perigee of 97.66 degrees, mean anomaly of 270 degrees;
ii) Final orbit: Semi-major axis of 7017.89, eccentricity of 0.000, inclination of 97.94 degrees, free ascending node, free argument of perigee, free mean anomaly;
iii) Initial mass of 170 kg;
iv) Thrust level of 4.0 N.

The station-keeping phase will correct the semi-major axis only, and this will occur when its value gets 1.26 km below the nominal value. Using these values, a typical maneuver will increase the semi-major axis from 7016.63 km to 7017.89 km and it will keep the eccentricity in zero and the inclination in 97.94 degrees. The initial mass is 150 kg and the thrust level is 4.0 N.

Considering these values, the solutions obtained are compared with Hohmann Transfer. Initially, the suboptimal method was applied in the transfer phase, with 2, 4 and 8 "thrusting arcs" and no constraints in the control. The results are shown in Table 1

Table 1
Suboptimal initial transfer phase with 2, 4 and 8 "thrusting arcs"

Arc	s_0(deg)	s_f(deg)	α_0(deg	β_0(deg)	α'	β'	Fuel-kg
1	459.9	721.9	11.4	-60.2	0.029	0.501	------
2	963.2	1184.5	17.2	49.5	-0.111	-0.049	14.24
1	498.0	603.3	0.2	-25.5	0.018	-0.054	------
2	1025.5	1125.7	10.2	41.1	-0.160	-0.190	------
3	1590.1	1697.9	3.0	-51.2	-0.010	0.498	------
4	2105.9	2206.7	10.0	40.5	-0.151	-0.187	12.14

1	527.3	576.8	1.3	-16.6	-0.099	-0.055	------
2	1055.4	1105.5	6.9	36.3	-0.153	-0.111	------
3	1622.2	1672.9	2.6	-39.9	-0.005	0.561	------
4	2135.4	2187.5	6.0	35.6	-0.140	-.089	------
5	2327.4	2377.6	1.8	-16.9	0.099	-0.107	------
6	2855.5	2905.8	6.0	35.3	-0.150	-0.111	------
7	3422.1	3473.1	2.5	-39.9	-0.006	0.564	------
8	3935.5	3987.8	6.9	35.8	-0.191	-0.099	11.92

In a second set of simulations the same maneuvers were performed with the additional constraints that the control angles must be fixed ($\alpha' = \beta' = 0$); and, in a third set, the constraint $\alpha_0 = 0$ was added (only β_0 is a free parameter for the control law). The objective is to know how much more fuel is required to compensate a more simple implementation of the control device and to satisfy the constraints of keeping some equipment (antennas, for example) pointed toward Earth. The same maneuvers were simulated with the optimal control approach in Prado and Rios-Neto [3]. Table 2 shows the comparison in fuel expenditure for all cases studied. The value obtained by considering a Hohmann Transfer is about 12.00 kg of fuel.

Table 2
Fuel expenditure (kg) for all maneuvers simulated

Method	2 arcs	4 arcs	8 arcs
Suboptimal	14.24	12.14	11.92
Suboptimal ($\alpha'=\beta'=0$)	21.39	17.06	12.88
Suboptimal($\alpha'=\beta'=\alpha_0=0$)	-------	17.98	13.42
Optimal	13.04	12.09	11.87

For the station-keeping phase, the suboptimal and optimal methods were applied with no constraints in control, and with 1, 2, 3 and 4 "thrusting arcs" applied in different positions of the orbit. The results showed that, due to the small magnitudes involved, there is no difference in all methods tested. As an example, the results for the suboptimal and optimal methods with 1 "thrusting arc" are shown in Table 3.

Table 3
Station-keeping with sub-optimal (top) and optimal (bottom) methods

Arc	s_0(deg)	s_f(deg)	A_0(deg)	B_0(deg)	A'	B'	Fuel(kg)
1	0.0	1.56	0.0	0.0	0.0	0.0	47.0
Arc	x_s(deg)	x_e(deg)	A(const.)	B(const.)			Fuel(kg)
1	0.0	1.56	0.0	0.0			47.0

7 - CONCLUSIONS

Suboptimal control was explored to generate algorithms to obtain solutions for the minimum fuel maneuvers for a spacecraft.

By comparing the results obtained with the algorithms developed and those found in the literature [16] it seems that the suboptimal solutions is very adequate, specially when a large number of "thrusting arcs" is used.

The results obtained here are very close to the ones available in the literature, including an old implementation of this method made by the in reference [1].

The method have a good numerical behavior, but it can not be used in real time. Process time (CPU) is short (less than a minute, in a PC computer) for simple maneuvers, but when several constraints and/or "thrusting arcs" are present the process time can be large (more than one hour, in some cases).

Optimization techniques are not required when station-keeping maneuvers are considered..

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