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CSI DUE TO SLOSHING MOTION ON LEO LSS

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This paper discusses the mathematical modeling approaches to better represent the sloshing dynamics, the effect of sloshing motion on spacecraft system stabilization, the problem of internal energy dissipation associated with rotating spacecrafts, and existing software tools for dynamics analysis of the sloshing problem. The sloshing phenomenon is presented and discussed in the scope of microgravity environment that characterizes the on orbit space vehicles. In this work the LSS physical model is approached by a large tubular platform containing two large and flexible solar arrays and a large water tank. The actuators for the sake of the attitude control are composed by a set of reaction wheels and a set of thrusters. The gravity-gradient torque is considered as the most important disturbing torque acting on the low Earth orbit (LEO) LSS. The mathematical model includes the solar arrays first mode of vibration and the sloshing effect of the large water tank. The sloshing is mathematically represented by a set of equivalent spring-mass systems. The assumed mode method is used for the mathematical modeling of the elastic displacement of a C-F-F-F plate like solar panel. The proportional integral derivative (PID) control approach is used to derive the control law. The whole linear system of equations is simulated via computer by using the MATLAB software package. The sloshing motion is excited to analyze the impact of the sloshing on the flexibility of the solar arrays. The thrusters are fired to control the LSS attitude causing simultaneously structural vibration and fluid excitation. The impact of the attitude control actuation on the structural and fluid excitation is analyzed. On the other hand the simultaneous structural and liquid excitation impact in the control performance is also studied. The results show that the sloshing excites the solar attitude and attitude rates.

INTRODUCTION

The Control Structure Interaction (CSI) has been subject of studies for many years aiming a better understanding of the interaction between the attitude and orbit control actions and the structural response in addition to that stabilization which should be provided by the attitude and orbit control subsystem. That interaction may occur for passive and/or active control subsystems. The effect goes from internal energy dissipation to unbalance and destabilization effects due, for instance, to sloshing motion. The matching of control frequencies and frequencies of flexible components of spacecraft as well as other moving parts can destabilize and risk space missions. CSI technology involves the use of active and/or passive systems to assure the stability and pointing

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accuracy of disturbed spacecraft. Such subsystems involve the usage of hardware (HW) as accelerometers, star sensors, horizon sensors, sun sensors and actuators as thrusters, reaction wheels, and torque coils. The structural components that are more susceptible to CSI includes flexible solar arrays, antennas, heat pipes, nutation dampers, partially filled tanks with fuel, water or any other liquid. It is well known the spin stabilization accomplishment in presence of perturbations by using partially filled ring nutation dampers or other types of internal energy dissipation devices. Also, it is well know the case of the Explorer I satellite whose mission failed due to interaction of the flexibility of antennas and the passive spin stabilization around the axis of minimum moment of inertia. The ATS satellite mission has been lost due to interaction of the heat pipes thermal control subsystem and the satellite active control subsystem to maintain the satellite spinning around the axis of minimum moment of inertia during orbit transfer. If we go though the literature we will find more CSI problems and, sometimes solutions. If the CSI is an effect well known, then, for a particular mission, it can be used for the sake of attitude control, as is the case of passive spin stabilized satellite technique combined with the usage of nutation dampers.

A typical case of CSI study and analysis was conducted at NASA Langley Research Center aimed at improving a better understanding of the flexible behavior of the Shuttle Flexible Remote Manipulator System (RMS)[1]. The RMS dynamics is characterized by low frequency and lightly damping mainly due to the flexibility of the RMS two long links. The easy structure dynamics excitation causes vibratory oscillations following attitude maneuvers and payload handling operations. The control technology applied to the Shuttle RMS aiming stability of the spacecraft used additional end-points accelerometers to the RMS in order to obtain the low frequency flexible modes of interest about three axes. The signal is then fedback by using a closed-loop control law which commands the RMS joint servos to actively damp the RMS flexibility motion exited by normal payload maneuvers and Shuttle thruster firing.

Sloshing is another source of CSI and has been the subject of research and tests since the early days of the space era. The sloshing impacts not only the attitude dynamics and control but also the navigation and control of launcher vehicles. It was in the area of rockets that the research, studies and tests came first. Then studies and analysis have been conducted for satellites containing liquid fuel such as hydrazine. With the advent of Large Space Structures (LSS) as the Space Stations the sloshing come to light again because of the large tanks of water, fuel and operations such as space docking that requires high accuracy control to avoid collision and successful docking. The space rendezvous and docking are not new at all, for the Moon Mission required those orbital operations and they were accomplished successfully at that time. However, nowadays those operations are becoming more and more automated and the idea is the use of spacecraft-robot based on sensors, actuators, and software (SW). In this scenario the Automated Transfer Vehicle (ATV) comes to light as a good example of automated rendezvous & docking operations whose analysis took into account the sloshing phenomenon in addition to the flexibility of solar arrays. The ATV is basically a cylindrical shaped spacecraft containing a docking port and a propulsion module. Its structure is composed by a rigid body, four flexible solar panels. For the purpose of guidance navigation and control the dynamics analysis took into account the sloshing perturbation induced by the propellants contained in several tanks of the vehicle. The sloshing modes (from 0.08 Hz to 0.2 Hz) had been taken into account in the controller design as they may have an important impact on the docking accuracy.[2]

Figure 1. ATV Approaching the ISS

This ATV dynamical analysis is a recent study and analysis considering sloshing. However the base of knowledge of the sloshing impact on attitude dynamics, control and navigation in space had been launched in the literature since the 60's. Two considerations are very important when analyzing the sloshing dynamics for space applications: one refers to the non-inertial base that characterizes space vehicles dynamics. The other refers to fluid dynamics behavior in zero-g environment. The subject of low g and microgravity has been presented in one of the earliest studies of the liquid behavior with focus in modeling approaches and space applications, the NASA SP 106, entitled "The dynamic behavior of liquids in moving containers with applications to space vehicle technology"[3]; This publication became so referenced and important for those who have to go through sloshing in space that it has been reviewed and re-published taking into account the computer program SLOSH. This program predicts the equivalent mechanical model and natural frequencies of linear sloshing for any axisymmetric tank. It is written in the Visual Basic language and contains on-screen instructions. The new version is entitled "The New Dynamic Behavior of Liquids In Moving Containers"[4].

PROBLEM FORMULATION

The problem consists of the LSS mathematical modeling and a proper control law formulation, followed by computer simulations and the resulting analysis with focus on the effect of the sloshing of a partially filled liquid tank and its interaction with the attitude control subsystem and consequent impact on attitude angles and rate control. The sloshing motion is approached by equivalent spring-mass systems without any damping device to suppress the sloshing. The physical model is similar to that presented in reference[7]. The main differences are that the mathematical model for the present study includes the sloshing motion and does not include both the solar panel rotation and the stepper motor. The actuators chosen for the attitude control subsystem are thrusters and reaction wheels. The main idea of the work is let the liquid free for oscillation once it is excited to check the effect in the attitude angles and rates. The PID control technique is used for the sake of the attitude control. Figure 2 shows the physical model adopted for this study.

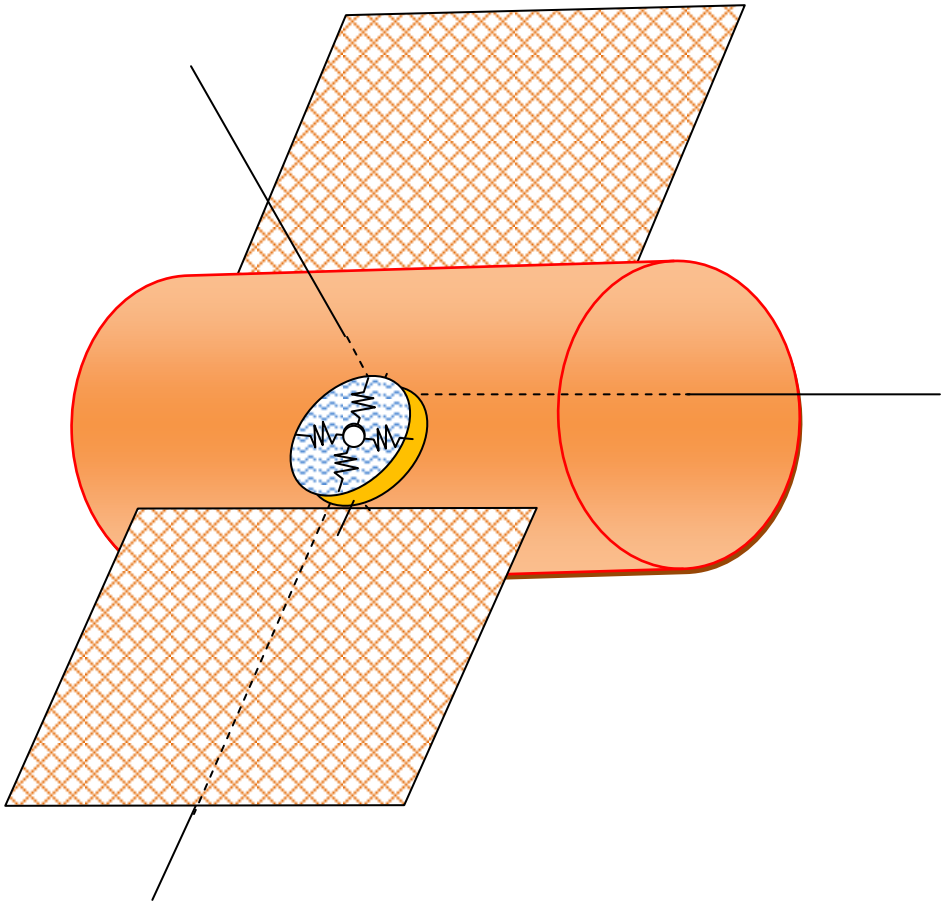

Figure 2. Physical Model for the LSS

The central tubular structure is assumed as rigid body while the solar arrays are considered as flexible components of the large orbital structure. The sloshing is represented by two equivalent models of spring-mass systems. The objective is to simulate via computer the attitude dynamics and control of the rotational/vibrational motion of the LSS taking into account the sloshing oscillations without any damping device to suppress the liquid motion. The space structure is mathematically modeled by using the Lagrangian formulation for generalized coordinates combined with the Lagrangian formulation for quasi coordinates[8] so as to obtain the modified Euler Equations of motion accounting for in a unique process the rotational/vibrational motion of the structure in orbit[8]. The assumed modes method has been used to represent the elastic displacement in terms of shape functions multiplied by time-dependent generalized coordinates[9-11].

The main steps of the mathematical modeling by using the Lagrangian formulation and the assumed modes method as outlined here are:

- Define the physical model and system of axes in which the equations are to be written. Define the vector position of an elemental mass for each part of the spacecraft, including the solar panel and the spring-mass systems.
- Write the vibrational displacements in terms of a set of admissible functions for plates, which can be somewhat arbitrary as long as they satisfy at least the geometric boundary conditions.
- Derive the kinetic energy for each part of the LSS, including the spring-mass model to represent the sloshing. Then write the system total kinetic energy. Usage of symbolic manipulators is strongly recommended for the operations of integrations and matrix manipulations.
- Derive the elastic potential energy for the flexible solar panels and springs Usage of symbolic manipulators is strongly recommended to perform the mathematical differentiations and integrations involved in the modeling process.
- Write the Lagrangian function in terms of the components of the angular velocity vector and the generalized coordinates associated with the elastic displacements.
- Use the Lagrange's formula for quasi-coordinates to obtain the modified Euler equations.
- Use the Lagrange's formula for generalized coordinates to derive the equations of motion in the librational degrees of freedom (solar arrays and springs).
- Derive the expression for the gravity-gradient torque[12].
- Linearize the equations about the gravity-gradient stabilized configuration and the non-deformed structure (elastic displacement).
- Write the system state equations.
- Design an appropriate controller
- Implement the attitude dynamics and control by using appropriate software/computer.
- Analyze the results.

In the present work the MatLab® software package symbolic toolbox has been used for algebraic manipulation, differentiation and integration operations. Then the software control toolbox combined with our own computer programs was used to execute the attitude dynamics and control simulations.

The details of the mathematical modeling[13] for the LSS without spring-mass systems are shown in reference 13. Some considerations and features about the spring-mass model are presented here. Consider figure 3 that illustrates spring-mass systems:

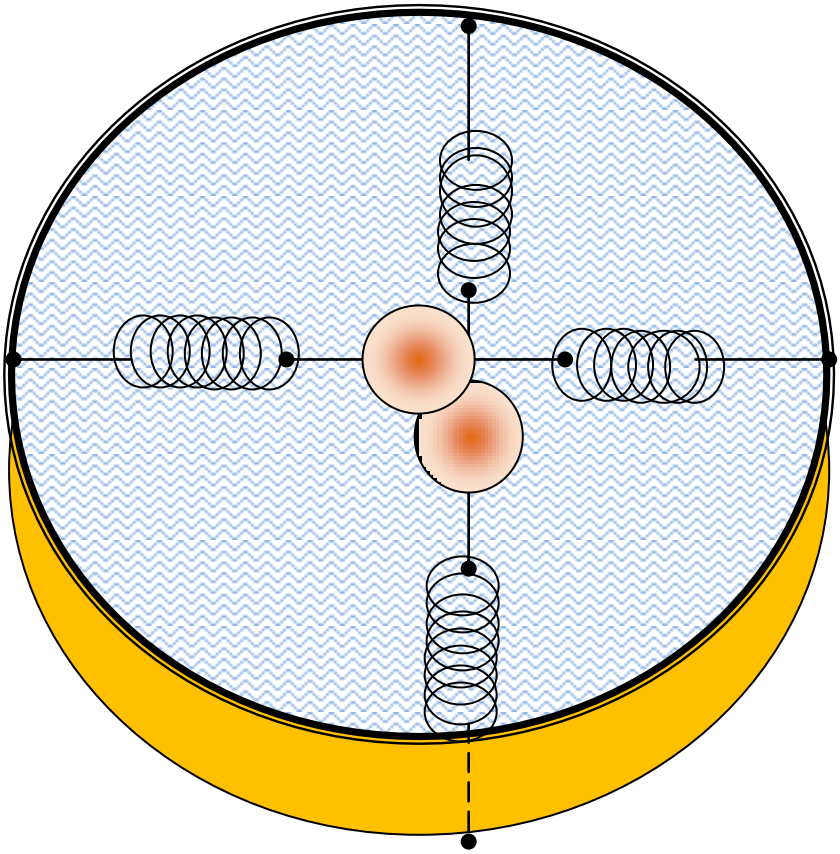

Figure 3. Two sets of independent spring-masses for the equivalent models of the sloshing

The tank shown in figure 3 was assumed to have 3 meter diameter with a height of'1.5 meters. The frequency for the system has been chosen to be 0.5 Hz. This frequency is typical of sloshing and is associated with low Bond numbers. Low Bond numbers are appropriate for system containing liquid under low level of acceleration as it is in the microgravity environment.

Kinetic Energy

The kinetic energy for the whole system can be written as

$$T = \frac{1}{2}\{\omega\}^T[J]\{\omega\} + \frac{1}{2}\{\Omega\}^T[J_{RW}]\{\Omega\} + \{\omega\}^T[J_{RW}]\{\Omega\} + \frac{1}{2}\{\dot{q}\}^T[M_p]\{\dot{q}\} + \frac{1}{2}\{\dot{r}_{1s}\}^T[M_s]\{\dot{r}_{1s}\} + \frac{1}{2}\{\dot{r}_{2s}\}^T[M_s]\{\dot{r}_{2s}\} + 2\{\omega\}^T[P]\{\dot{q}\} + \frac{1}{2}\{\omega\}^T[J_s + J_p + J_{RW}] \quad (1)$$

Where $[J]$, $[J_{RW}]$, $\left[M_p\right]$, $[M_s]$ are the main bus inertia matrix, the reaction wheel inertia matrix, the solar panels mass matrix, and the spring-mass matrix. $\left[M_p\right]$ and $[M_s]$ are time dependent on the solar panels elastic coordinates $q_i(t)$ and the spring positions $x_s(t)$ and $y_s(t)$. The technique to obtain the expressions for the total kinetic energy involves defining the vector positions for elemental masses of the main bus, reaction wheels, and the solar panels, and the vector position for each of the spring masses (assumed to be the same, that is $m_{1s}=m_{2s}$). All the vectors shall be defined with respect the center of mass of the whole system. Once the vectors are properly defined, the velocities of each part of the system are derived. The crossing terms between velocities appears in this phase. Then the kinetic energy can be written following the kinetic energy definition. One spring-mass is defined with the spring line towards the *x* axis (yaw) and the other along the *y* axis (Roll). The idea is to represent the fluid motion in two directions through the spring-mass systems. The derivation of matrix $\left[M_p\right]$ is not trivial since it involves integrating the products of the shape functions over the solar panel domain[10-11]

Potential Energy

The potential energy associated with the solar panel and the springs are:

$$V = \frac{1}{2}\{q\}^T\left[K_p\right]\{q\} + \{r_{1s} + r_{2s}\}^T\left[K_p\right]\{r_{1s} + r_{2s}\} \tag{2}$$

The matrix $\left[K_p\right]$ involves the integration over the solar panel domains of the product of second derivatives of shape functions[10-11]. It is hard to work with this formulation without a symbolic manipulator.

Once the *T* and *V* are obtained the Lagrangian function can be written by following the definition *L = T- V*. In this paper the Gravity-Gradient torque is considered. There are two options to take it into account. The gravitational potential can be derived and included in V or the torque can be derived and then use the Lagrangian formulation for external torques and includes the gravity-gradient torque as an external torque in the right side of the Lagrangian formula for quasi-coordinates[8]. In this paper this second option has been adopted and the gravity-gradient torque for first order approximation has been derived by following the Kaplan derivation[12].

Equations of Motion

The equations of motion can be derived by using the Lagrange's formulas for quasi-coordinates and for generalized coordinates[9].

$$\frac{d}{dt}\left\{\frac{\partial L}{\partial \omega}\right\} + [\tilde{\omega}]\left\{\frac{\partial L}{\partial \omega}\right\} = \{\tau_g + \tau_C\} \tag{3}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}_i}\right) - \left(\frac{\partial L}{\partial \xi_i}\right) = Q_i \tag{4}$$

The ξ_i are the q_i (solar panel degrees of freedom), *x* and *y* (associated with the spring-mass system degrees of freedom).

Eq.(3) yields the Euler modified equations of motion as

$$[J_T]\{\dot{\omega}\} + \left([\dot{J}_p + \dot{J}_s]\right)\{\omega\} + [\tilde{\omega}]([J_T]\{\omega\} + [J_{RW}]\{\Omega\} + 2[P]\{\dot{q}\}) + [P]\{\ddot{q}\} = \{\tau\} \quad (5)$$

By using Eq.(4) the equations for the solar panels can be derived as:

$$[M_p]\{\ddot{q}\} + [K_p]\{q\} + [c]\{\dot{q}\} + 2[P]^T\{\dot{\omega}\} = 0 \quad (6)$$

Again, using Eq.(4) the equivalent equations by using the spring-mass systems can be written as

$$[M_s]\{\ddot{r}_s\} + [K_s]\{r_s\} = Q_s \quad (7)$$

In Eq.(7) the $[K_s]$ is obtained through the Bond number[6] for low frequencies for the sloshing so that the equation for the springs use a frequency typical of the sloshing for low Bond number. This is reasonable for sloshing under low acceleration as is the case of sloshing in microgravity environment. In this work the frequency of 0.2 Hz has been used for the $[M_s]^{-1}[K_s]$ that defines the frequency in Eq.(8). The sloshing frequency in microgravity environment can vary from 0.08 to 0.2 Hz without causing significant perturbation[2]. The significance, of course, depends on the mission requirements. For Hubble Telescope, for instance, pointing error must be very close to zero.

Control Design

The control law is derived by using the standard PID (Proportional, Integral and Derivative actions). The proportional action provides a contribution which depends on the instantaneous value of the control error. The integral action, on the other hand, gives a controller output that is proportional to the accumulated error, which implies that it is a slow reaction control mode. And finally, the derivative action acts on the rate of change of the control error. The mathematical equation of a PID controller can be written as[13]

$$N_c = K_{pr}\theta + K_{der}\dot{\theta} + K_{int}\int \theta dt \quad (8)$$

Where θ is the pointing error and the *pr=proportional, der=derivative, and int=integral. K is the gain.*

A simple gain selection strategy based on the Ziegler-Nichols rule has been implemented. The proportional and derivative gains are obtained by taking into account the inertia of the system, the pulsation and the desired control damping. The gains are obtained by

$$K_{pr} = diag([J])\varpi^2 \quad (9)$$

$$K_{der} = -2diag([J])\varpi\zeta \quad (10)$$

$$K_{int} = \frac{K_{pr}^2}{2K_d} \tag{11}$$

where $[J]$, ϖ, and ζ is the inertia matrix, pulsation and damping, respectively.

Thruster Control

The thruster control law is based on bang-bang control where the thruster characteristics are taken into account [14]. The thruster control law is written as

$$\tau_{th} = Fd\frac{tOn}{dt}sign(\tau_d) \tag{12}$$

Where τ_{th} is the thruster torque, $tOn = \left|\frac{\tau_d dt}{Fd}\right|$, τ_d is desired torque, *F* is the thruster force, *d* is the distance from the center of mass, *dt* is the sampling time.

The Linearized Equations of motion have been written in state form by defining the state vectors as

$$\begin{aligned} y = \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} &= \{\theta_1 \quad \theta_2 \quad \theta_3 \quad q_1 \quad q_2 \quad q_3 \; x_s \; y_s \; \dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3 \; \dot{q}_1 \quad \dot{q}_2 \quad \dots \; \dot{y}_s \quad \}^T \\ &= \{x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \; x_7 \; x_8 \; x_9 \quad x_{10} \quad x_{11} \; x_{12} \quad x_{13} \quad \dots \; x_{16} \quad \}^T \end{aligned} \tag{13}$$

RESULTS

The system equations have been simulated via computer aiming the check of the sloshing effect on the attitude angles and rates. Figure 4 shows the effect of the sloshing on the attitude angles. Note that the pitch angle, after the transient phase, exhibits a permanent oscillation which the control system cannot damp. It is necessary to design some devive such as baffles to supress or damp the sloshing so as the attitude control can perform as shown in Figures 8. Figures 5 to 6 show a zoom of figure 4 to illustrate the small effect of the sloshing of order of 10^{-3} degrees. Figure 7 shows residual angular rates due to sloshing. Figure 8 shows the case where no sloshing perturbance impacts the attitude. Figure 9 shows details of figure 8 considered until 120 seconds. Figure 10 shows the results for the attitude rates where no sloshing is considered. Comparing figure 10 with figure 7 its is possible to evaluate the effect of the sloshing in the control performance. The steady state close to zero exhibs a residual angle about 10^{-5} degrees associaated with the solar panel vibration effect on the attitude. The control excitation of the sloshing depends on the usage of thrusters. If reaction wheels are used to control the attitude, the sloshing oscillation is not significant and does not affect the permanent response of the system. The sloshing appears to not be disturbed by the solar panels vibration for the linear model subject of this study. Even when large amplitudes of the solar panels (see figure 11) the liquid appears to be not disturbed.

Perhaps the simulation of the non linear system of equations would show the coupling between the elastic and sloshing modes. A frequency of 0.5 Hz was set for the sloshing. The

sloshing frequency can be very low as to 0.08 Hertz. In this case the equivalent system of spring-mass that frequency implies much sensitivity of the system to any disturbance and it becomes out of control about pitch axis. More analysis and computer simulations is necessary to clarify the behavior of the fluid represented by the spring-mass systems as modeled here.

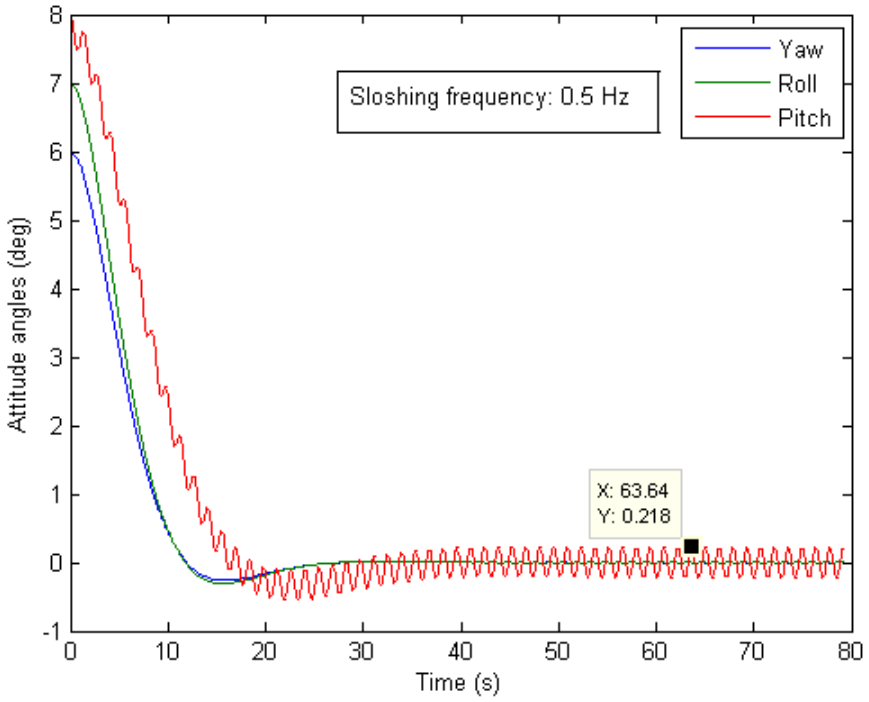

Figure 4. Residual oscillation affecting the pitch angle.

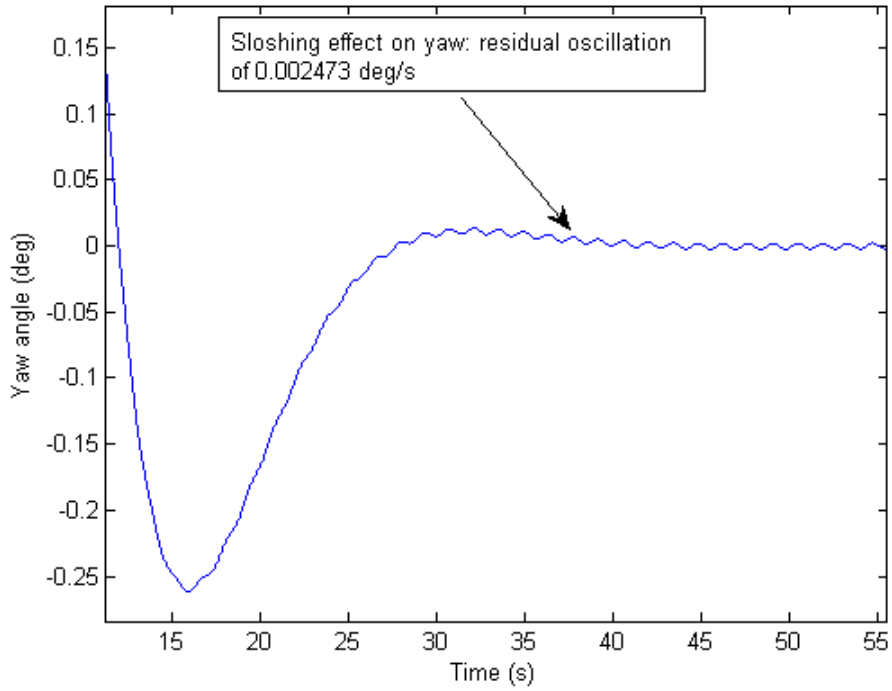

Figure 5. Detail of yaw angle from figure 4, showing the sloshing disturbance on the yaw angle

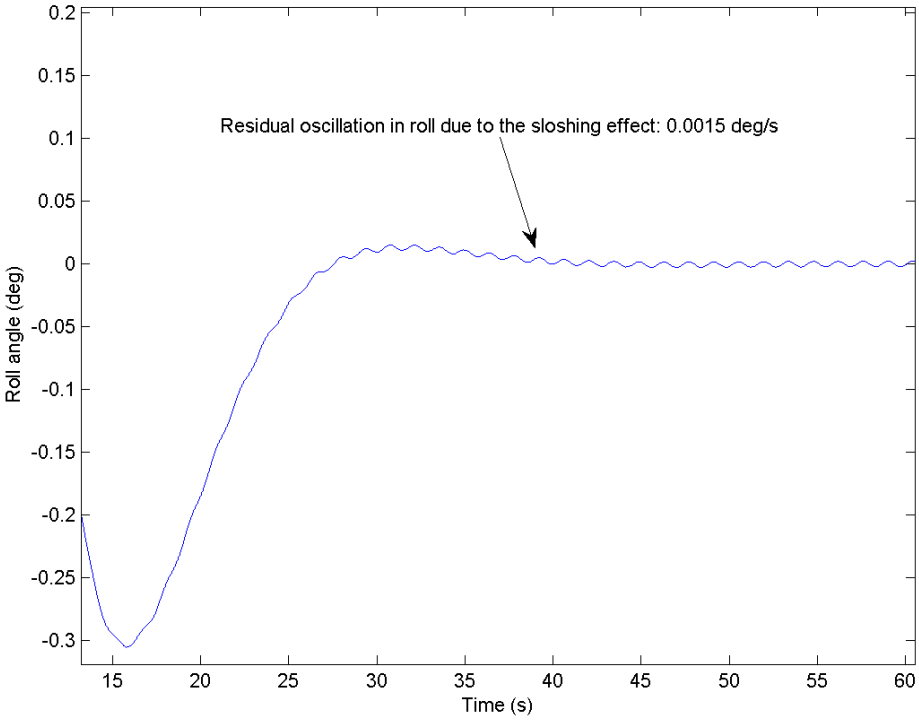

Figure 6. Detail of roll angle from figure 4, showing the sloshing disturbance on the roll angle

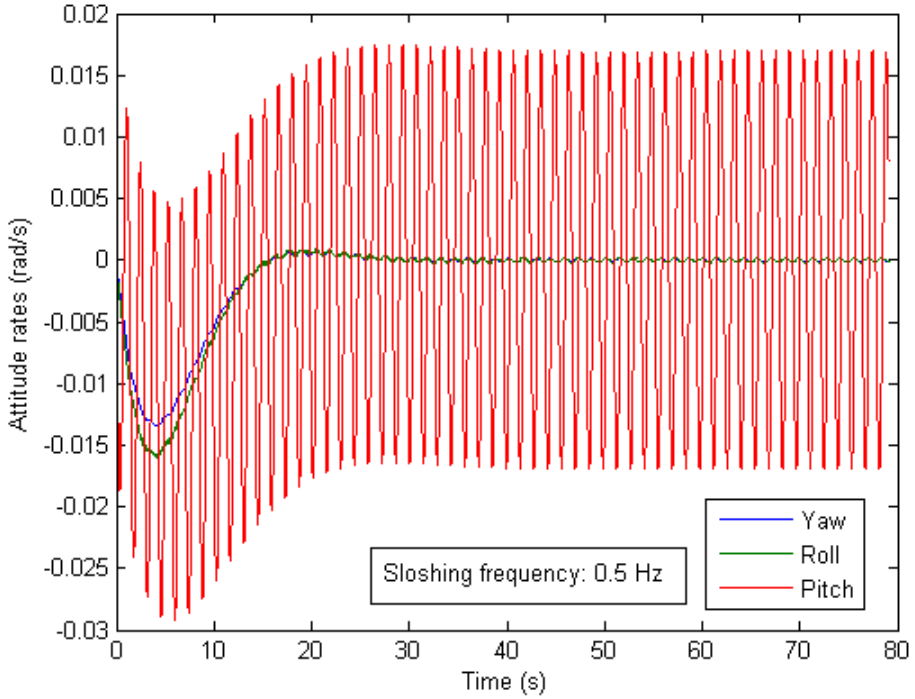

Figure 7. Attitude rates enhancing the sloshing impact on the pitch rate

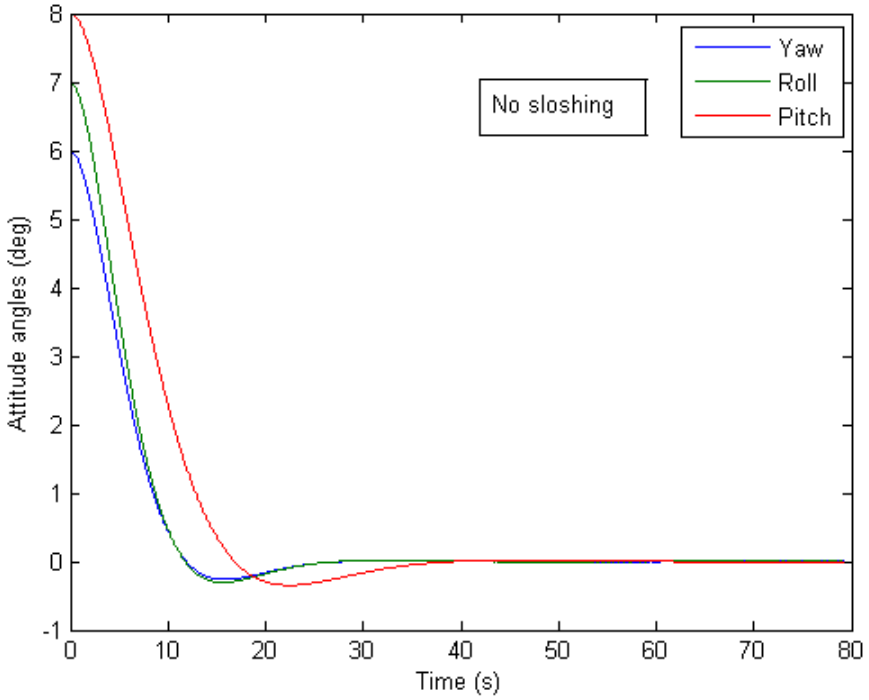

Figure 8. No attitude angles – no sloshing effect on the attitude

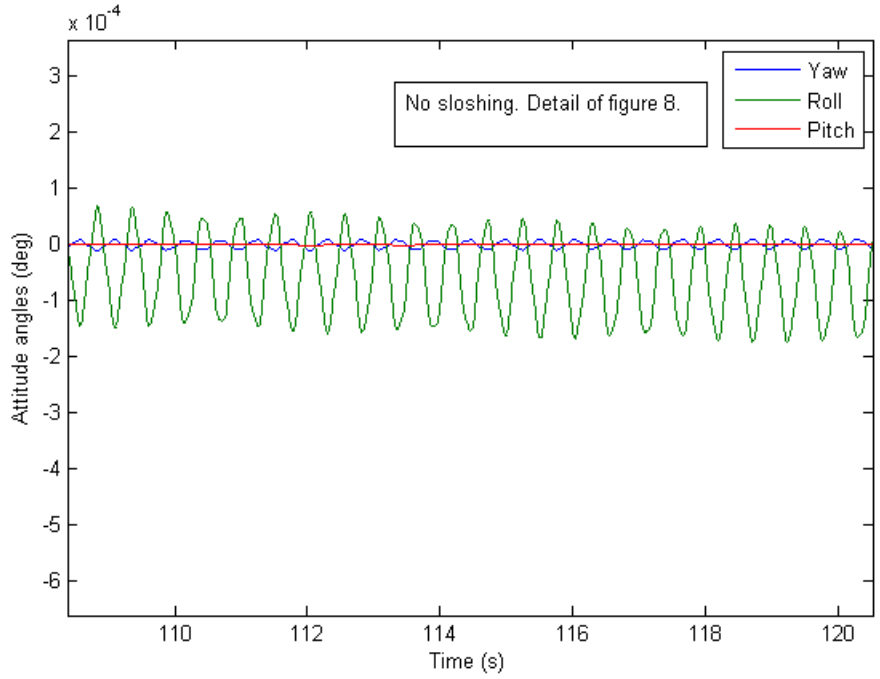

Figure 9. Detail for the case where no sloshing is considered

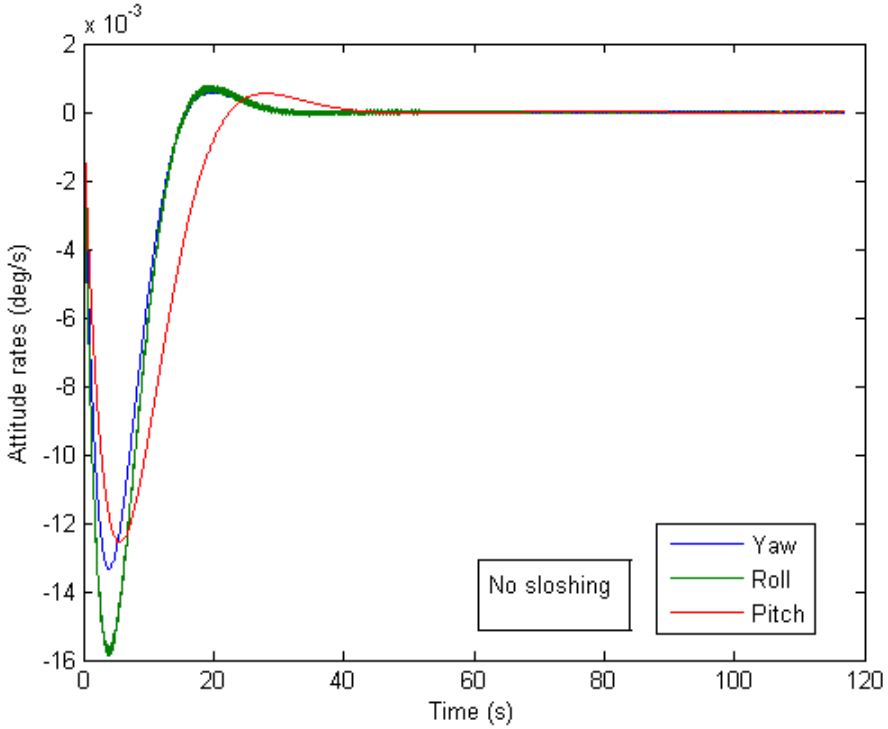

Figure 10. Attitude rates without the sloshing disturbance

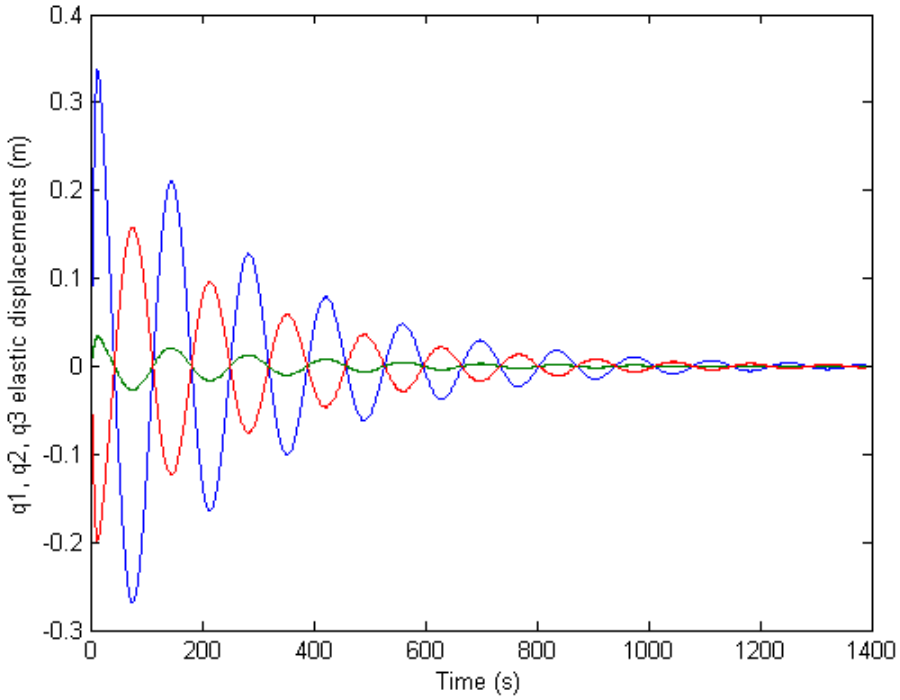

Figure 11. Solar panels elastic displacements

CONCLUSION

Computer simulations of a mathematical model of a large space structure containing a large tank of liquids are presented. A typical sloshing frequency of 0.5 Hertz has been chosen for the spring-mass equivalent systems. Based on that frequency and the fluid mass the equivalent spring constant has been calculated and the result is consistent with low Bond numbers. The low Bond numbers are appropriate for the study of sloshing in microgravity environment when the level of acceleration is very low. It was verified that the excitation of the fluid in the circular tank caused the permanent response in pitch to exhibit a residual oscillation that could not be acceptable for pointing systems. The linear equations may have hidden the effect of coupling between the elastic vibration of the solar panels. It is worse to note that the non linear system is not invariant with respect to time for the coefficients of the derivatives in the elastic coordinates as well as in the spring-mass coordinates are time dependent. However the terms were canceled according to the linearization criteria. More simulations and analysis are necessary to accomplish the goal of studying the effect of the sloshing on the attitude control and vice versa.

ACKNOWLEDGEMENTS

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