A Four-Antenna Transmit Diversity Scheme with Quantized Feedback

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***Abstract*—We propose a full-diversity scheme for four transmit antennas and quantized feedback channel. We present a pre-processor design that enables this scheme to achieve full diversity order as well as a coding/array gain. A quantized feedback analysis of the proposed scheme for quasi-static flat Rayleigh fading channels is also performed. Through simulation results, we show that the proposed scheme has a performance gain of about 1.6dB and 2dB over ACBS scheme for 3 and 4 feedback bits, respectively.**

***Index Terms*—Array/coding gain; MIMO systems; quantized feedback; transmit diversity.**

I. Introduction

It is well known that multiple-input multiple-output (MIMO) wireless communication systems can exploit the spatial dimension to improve capacity and reduce sensitivity to fading. Looking to the receiver, several techniques, such as selection diversity combining (SDC), equal gain combining (EGC), and maximum ratio combining (MRC) have been used to obtain receive diversity gain.

Space-time codes are an effective way to exploit spatial diversity in MIMO wireless communication systems and since the work of Alamouti [1], the spatial diversity could be obtained in multiple-input single-output (MISO) wireless communication systems.

The orthogonal space-time block codes [2] require no knowledge of the channel state information (CSI) at the transmitter to provide full diversity gain. While the orthogonality constraint is required in order to achieve maximum diversity advantage, it has been shown [3], [4] that by relaxing the orthogonality requirement, higher transmission rates can be obtained at the cost of loosing some degree of diversity order.

MIMO communication systems can obtain significant performance improvements if a feedback channel exists so that the CSI is known at the transmitter [5], [6]. If the CSI is perfectly known at the transmitter, then maximum ratio transmission (MRT) [7] is the optimum beamforming. If the antenna amplifiers are not required to modify the transmit power of the transmitted signals, then equal gain transmission (EGT) [8] with any combining scheme (SDC, EGC or MRC) achieves full diversity order over MIMO flat fading channels.

However, a quantized feedback channel is a more realistic assumption, where only few bits can be sent to the emitter. Considering this scenario, Machado and Uchôa-Filho [9] have proposed a hybrid transmit antenna/code selection scheme that chooses from a list of space-time block codes the best code to be used with a subset of transmit antennas. The code selection is based on the instantaneous error probability minimization criterion. This idea has been refined later in [10], [11].

The study and analysis of quantization methods for MIMO channels using a low-rate feedback channel is a well-studied research problem and several papers on this area can be found [12]–[19].

In [19], Choi *et al.* have proposed an interesting phase-feedback-assisted scheme with four transmit antennas which uses a pre-processor for combining two Alamouti codes in terms of Frobenius norm maximization. They have shown that full diversity is achieved by the combining effect at the pre-processor. Also the phase feedback is utilized to increase a coding/array gain. Herein we refer to this scheme as Alamouti-code based scheme (ACBS).

Following these ideas, in this paper, we propose a four-antenna transmit diversity scheme with quantized feedback. We design a pre-processor which combines four “trivial” codes. The pre-processing of the proposed scheme is based on the designs presented in [11], [16], [19]. As a result, we observe an increase of the coding/array gain. A compromise of this proposal is that the receiver needs to feed, at least, 3 bits back to the emitter to ensure a full spatial diversity order.

In this paper, we also present a procedure for determining the feedback information which maximizes the signal-to-noise ratio (SNR) at the receiver. Another important feature of the proposed scheme is that the maximum likelihood detector is based on linear processing, which leads to a very simple receiver. Simulation results reveal that the proposed scheme yields the maximum diversity advantage and also outperforms the ACBS scheme.

The remainder of this paper is organized as follows. Section II presents the channel model considered in this work. Section III addresses the proposed transmit diversity scheme with quantized feedback. In Section IV, simulation results are presented. Finally, Section V presents some concluding and final remarks.

II. System model

Consider a MIMO system with M_T transmit and M_R receive antennas, and that the channels have a flat Rayleigh fading and remains constant over τ symbol intervals. The transmission model consists of *linear processing*, as described in [19]. With some slight modifications, we arrive at

$$\mathbf{Y} = \gamma_0 \mathbf{X}\mathbf{H} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ is the $\tau \times M_R$ matrix of the received signals and $\mathbf{X}$ is the $\tau \times M_T$ matrix of transmitted signals with unit average energy. Let $\mathcal{CN}(0, \mathbf{R})$ represent the joint p.d.f. (probability density function) of a zero-mean circularly symmetric complex normal random vector with covariance matrix $\mathbf{R}$. Then, $\mathbf{N}$ is the $\tau \times M_R$ matrix $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{\tau M_R})$ representing the joint p.d.f. of the i.i.d. (independent and identically distributed) additive Gaussian noise samples with unit variance, $\mathbf{H}$ is the $M_T \times M_R$ MIMO channel characterized by the p.d.f. $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{M_T M_R})$, and $\gamma_0 = \sqrt{\rho}$, with ρ the average SNR at each receive antenna.

We assume that there are Q QAM data symbols $\{s_q\}$, $q = 1, \ldots, Q$, with unit average energy to be transmitted over τ symbol intervals, and a reliable feedback channel through which b bits can be sent to the emitter.

Throughout this paper, we consider that the receiver has only one receive antenna. Normal letters represent scalar quantities, boldface lowercase letters indicate vectors, and boldface uppercase letters indicate matrices. The superscripts $(\cdot)^T$ and $(\cdot)^*$ represent the transpose and the complex conjugate operation, respectively.

III. A Four-Antenna Transmit Diversity Scheme with Quantized Feedback

In this section, we present how to perform the proposed scheme. The transmitter has four transmit antennas and the data symbol s_1 is pre-processed by $\mathbf{p}$, resulting in the complex transmitted signal vector $\mathbf{x}$:

$$\begin{aligned} \mathbf{x} &= \mathbf{p} s_1 \qquad (2)\\ &= \left[\mathbf{v}_\varphi \left(\begin{bmatrix} \cos(\theta) & 0 \\ 0 & \sin(\theta) \end{bmatrix} \otimes \mathbf{I}_2 \right)\right] s_1, \\ &= \begin{bmatrix} \cos(\theta)\exp\{j0\} \\ \cos(\theta)\exp\{j\varphi_1\} \\ \sin(\theta)\exp\{j0\} \\ \sin(\theta)\exp\{j\varphi_2\} \end{bmatrix}^T s_1, \\ &= \begin{bmatrix} s_1\cos(\theta) \\ s_1\cos(\theta)\exp\{j\varphi_1\} \\ s_1\sin(\theta) \\ s_1\sin(\theta)\exp\{j\varphi_2\} \end{bmatrix}^T, \end{aligned}$$

where $\otimes$ is the kronecker product, $\mathbf{I}_n$ is the n-by-n identity matrix, s_1 is the information symbol, and

$$\mathbf{v}_\varphi = \left[\exp\{j0\} \quad \exp\{j\varphi_1\} \quad \exp\{j0\} \quad \exp\{j\varphi_2\}\right].$$

From the Equations (1) and (2), the received signal y can be written as

$$y = \gamma_0 h_e s_1 + \eta, \tag{3}$$

where

$$\begin{aligned} h_e &= \mathbf{p}\mathbf{h}, \\ &= h_{e1}\cos(\theta) + h_{e2}\sin(\theta), \end{aligned}$$

$\mathbf{h} = [h_1 \ \ldots \ h_4]^T$, η is the additive white Gaussian noise, h_i denotes the path gain from the j-th transmit antenna to the receive antenna, and h_{e1} and h_{e2} are given by

$$\begin{aligned} h_{e1} &= h_1 + h_2 \exp\{j\varphi_1\}, \\ h_{e2} &= h_3 + h_4 \exp\{j\varphi_2\}. \end{aligned}$$

In order to guarantee full diversity order, the receiver needs to inform the appropriate phases, φ_1, φ_2, and θ, to the transmitter. Next, we present the expression for determining the phases that provide the maximum instantaneous SNR at the receiver.

A. Instantaneous SNR Analysis

Consider the received signal in Equation (3). The following linear processing produces the desired inputs to the maximum-likelihood detection:

$$\begin{aligned} \tilde{s}_1 &= y h_e^* \qquad (4) \\ &= \left(|h_{e1}\cos(\theta)|^2 + |h_{e2}\sin(\theta)|^2 + h_{Re12}\right) s_1 + h_e^* \eta, \end{aligned}$$

with

$$\begin{aligned} |h_{e1}\cos(\theta)|^2 &= \left(|h_1|^2 + |h_2|^2 + h_{Re1}\right)\cos^2(\theta) \\ &= |h_{e1}|^2 \cos^2(\theta), \end{aligned}$$

$$\begin{aligned} |h_{e2}\sin(\theta)|^2 &= \left(|h_3|^2 + |h_4|^2 + h_{Re2}\right)\sin^2(\theta) \\ &= |h_{e2}|^2 \sin^2(\theta), \end{aligned}$$

and

$$h_{Re12} = 2\Re\left\{h_{e1}h_{e2}^*\right\}\cos(\theta)\sin(\theta)$$

where $|\cdot|^2$ denotes the modulus squared of a complex number and $\Re\{\cdot\}$ its real part.

$$h_{Re1} = 2\Re\left\{h_1 h_2^* \exp\{-j\varphi_1\}\right\}$$

and

$$h_{Re2} = 2\Re\left\{h_3 h_4^* \exp\{-j\varphi_2\}\right\}.$$

As we can observe, the detection can be performed with low complexity.

It can be shown that the SNR at the receiver is given by

$$\mathrm{SNR} = \left(|h_{e1}|^2\cos^2(\theta) + |h_{e2}|^2\sin^2(\theta) + h_{Re12}\right)\frac{\sigma_x^2}{\sigma_n^2} \tag{5}$$

where σ_x^2 is the total power of the estimated signal, and σ_n^2 is the noise power at the receiver.

Now, we can determine the optimal phases by differentiation of the Equation (5), yielding a maximum signal-to-noise ratio and full diversity order. Nevertheless, full diversity can also be achieved with quantized feedback. In this regard, the feedback information is used to guarantee that each one of the terms $|h_{e1}|^2$, $|h_{e2}|^2$, and h_{Re12}, is a positive number in every transmission frame. Moreover, with this feedback, the instantaneous SNR is maximized.

The phases φ_1, φ_2, and θ, are the variables taking into account in this optimization process. Since there is dependence among these variables, we need to solve this problem in steps. In this paper, we consider a two-step solution.

TABLE I
QUANTIZED FEEDBACK: CRITERIA FOR THE PHASE SELECTION.

Number of feedback bits	$2\Re\{h_1 h_2^*\}$	$2\Re\{h_3 h_4^*\}$	φ_{1q} / φ_{2q}
$b = 2$ bits	> 0	> 0	0 / 0
	> 0	< 0	0 / π
	< 0	> 0	π / 0
	< 0	< 0	π / π
+	ν	κ	θ_q
1 bit	> 0	$\sharp$	$\frac{\pi}{4}$
	< 0	$\sharp$	$\frac{-\pi}{4}$
or +	ν	κ	θ_q
2 bits	> 0	> 0	$\frac{2\pi}{6}$
	> 0	< 0	$\frac{\pi}{6}$
	< 0	> 0	$\frac{-2\pi}{6}$
	< 0	< 0	$\frac{-\pi}{6}$

First, taking the inner cross terms in $|h_{e1}|^2$ and $|h_{e2}|^2$, i.e., h_{Re1} and h_{Re2}, respectively, we easily verify that these terms are maximized when

$$\varphi_1 = \xi_1 - \xi_2 \quad \text{and} \quad \varphi_2 = \xi_3 - \xi_4$$

where, $h_i = \alpha_i \exp\{j\xi_i\}$.

Second, we differentiate the SNR expression in terms of θ, following a similar procedure adopted in [19]. The first and second differentiations are given by

$$SNR' = 2\kappa\left(\cos(\theta)\sin(\theta)\right) + 2\nu\left(\cos^2(\theta) - \sin^2(\theta)\right) \quad (6)$$

and

$$SNR'' = 2\kappa\left(\cos^2(\theta) - \sin^2(\theta)\right) - 8\nu\left(\cos(\theta)\sin(\theta)\right), \quad (7)$$

respectively.

Solving (6) and (7) under the conditions $SNR' = 0$ and $SNR'' < 0$, we obtain the following optimal theta phase[1] [19]

$$\theta_{opt} = \arctan\left(\frac{\kappa + \sqrt{\kappa^2 + 4\nu^2}}{2\nu}\right), \quad (8)$$

with

$$\kappa = \left(|h_3|^2 + |h_4|^2 + h_{Re2}\right) - \left(|h_1|^2 + |h_2|^2 + h_{Re1}\right)$$

and $\nu = \Re\left(h_{e1}h_{e2}^*\right)$.
We decide to omit the term $\frac{\sigma_x^2}{\sigma_n^2}$ in (6) and (7), since it is irrelevant to the phase optimization.

B. Quantized Feedback

In this section, we present how to use the proposed scheme with quantized feedback. We assume two uniform phase quantization: φ_{1q} and $\varphi_{2q} \in [0 \;\; \pi]$, as adopted in [17], [18], and $\theta_q \in [-\pi/2 \;\; \pi/2]$. The receiver needs to send, at least, three feedback bits (three-phase information) to the transmitter (see Table I).

Table I defines the criteria used to choose the quantized phases. Two bits are used to guarantee that the terms h_{Re1} and h_{Re2} are positive. The third bit ensures full diversity order, and the other "extra" ones are used to improve the SNR gain. In Table I, the symbol $\sharp$ specifies the "do not care state". In other words, when $b = 3$ bits, the signal of κ is not taken into consideration for feeding the phase information back to the transmitter.

[1] Optimal in the sense of maximizing the instantaneous SNR.

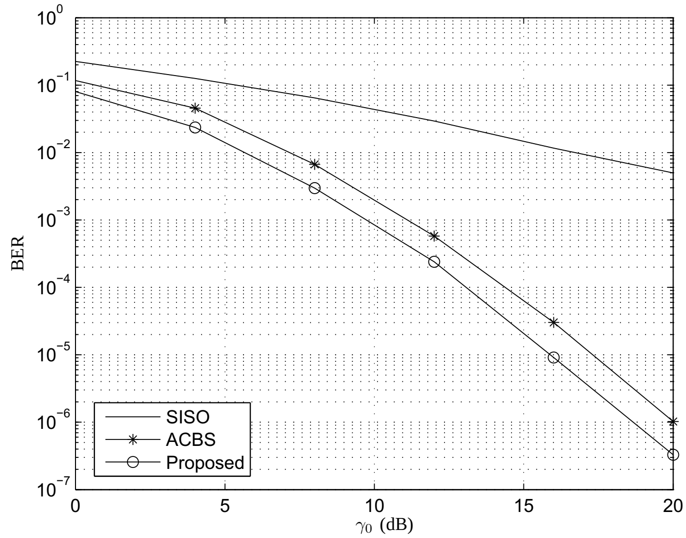

Fig. 1. BER performance of the proposed scheme. M_T=4 antennas, M_R=1 antenna, and b=3 bits.

IV. SIMULATION RESULTS

In this section, we present some simulation results to illustrate the performance of the proposed scheme. In order to assess the coding/array gain of the proposed scheme, we compare the BER of the proposed scheme to the Alamouti-code based scheme [19]. The performances are compared in terms of bit error rate (BER) versus SNR (γ_0) over quasi-static flat Rayleigh fading channels.

In Figures 1 – 3, the results are given for $M_T = 4$ transmit antennas, $M_R = 1$ receive antenna, 4-QAM, 130 symbols per frame, and considered as stopping criterion the occurrence of 300 symbol errors per each SNR. The proposed and ABCS schemes are full-rate, i.e., we compare two unitary spatial transmit rate schemes ($R = Q/\tau = 1$). The BER for the no-diversity scenario (SISO) is also plotted, used as a reference.

The BER performance is presented in Figure 1 for $b = 3$ feedback bits, φ_{1q} and $\varphi_{2q} \in \{0, \; \pi\}$, and $\theta_q \in \{-\pi/4, \; \pi/4\}$, and in Figure 2, for $b = 4$ feedback bits, φ_{1q} and $\varphi_{2q} \in \{0, \; \pi\}$, and $\theta_q \in \{-2\pi/6, \; -\pi/6, \; \pi/6, \; 2\pi/6\}$. The quantization adopted in this paper is different from the one presented in [19]. The quantization used here (for the case $b = 4 = 2 + 2$ bits) seems to be slightly better than the one presented in [19]. However, it is just an inference and we did not prove it. Figure 3 presents the BER performance of the proposed scheme for different levels of feedback quantization.

In Figure 1, we observe that the proposed scheme has a performance gain of about 1.6dB over ACBS scheme for $b = 3$ feedback bits. Figure 2 illustrates that the proposed scheme has a performance improvement of about 2dB over ACBS scheme for $b = 4$ feedback bits. Figure 3 shows a performance loss due to the quantization on the phase feedback.

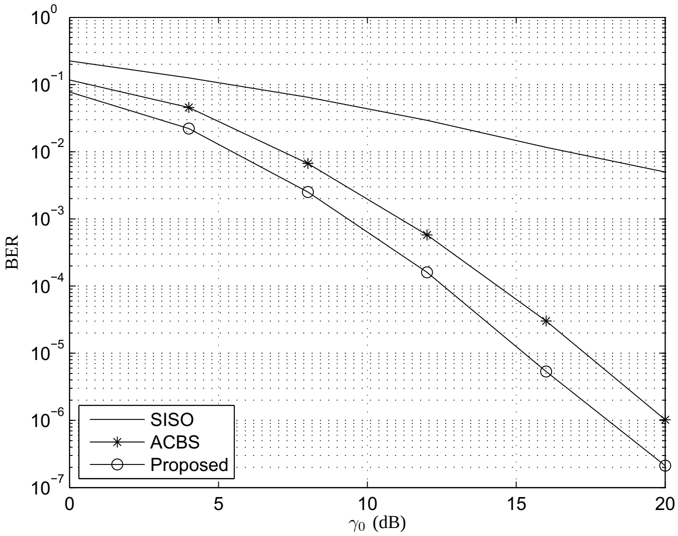

Fig. 2. BER performance of the proposed scheme. M_T=4 antennas, M_R=1 antenna, and b=4 bits.

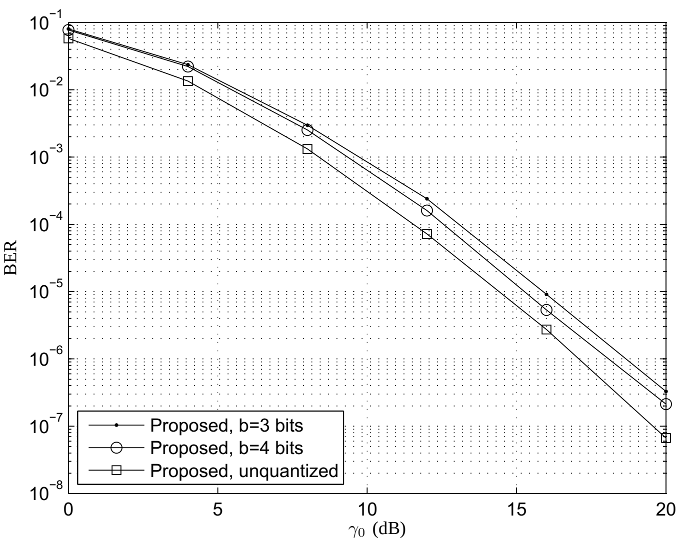

Fig. 3. BER performance of the proposed scheme for different levels of feedback quantization.

V. Conclusions and Final Remarks

In this paper, a four-antenna transmit diversity scheme with quantized feedback was proposed. The main idea backing the proposed scheme was to explore the benefits of the equal gain transmission scheme [8], and Alamouti-code based scheme [19] in a new transmit diversity strategy. A quantized feedback analysis for the quasi-static flat Rayleigh fading channels was performed, and their error performance was evaluated through computer simulations. It was observed that the proposed scheme outperforms the ACBS scheme in terms of coding/array gain.

It is worthy mentioning that the proposed scheme adopts a very simple linear decoding method with a small decoding delay ($\tau = 1$), while the ABCS has a decoding delay equal to two ($\tau = 2$). A compromise of the proposed scheme is that the receiver needs to feed, at least, 3 bits back to the transmitter to ensure full transmit diversity.

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References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun*, vol. 16, pp. 1451–1458, October 1998.

[2] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[3] V. Tarokh and H. Jafarkhani, "A differential detection scheme for transmit diversity," *IEEE J. Sel. Areas in Commun.*, vol. 18, no. 7, pp. 1169–1174, July 2002.

[4] M. Uysal and C. N. Georghiades, "Non-Orthogonal space-time block codes for 3Tx antennas," *IEEE Electronics Lett.*, vol. 38, no. 25, pp. 1689–1691, December 2002.

[5] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1468–1489, July 1998.

[6] T. M. Duman and A. Ghrayeb, *Coding for MIMO Communication Systems*. New York: John Wiley & Sons, 2007.

[7] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, October 1999.

[8] D. J. Love and R. W. Heath Jr., "Equal gain transmission in multiple-input multiple-output wireless systems," *IEEE Trans. Commun.*, vol. 51, no. 7, pp. 1102–1110, July 2003.

[9] R. Machado and B. F. Uchôa-Filho, "Space-time block coding with hybrid transmit antenna/code selection," in *Proc. IEEE Int. Conf. on Commun. (ICC)*, Paris, France, June 2004, pp. 819–822.

[10] S. Şahin, M. E. Çelebi, and Ü. Aygölü, "Space diversity schemes with feedback for three transmit antennas," *AEÜ-International Journal of Electronics and Communications*, vol. 60, no. 8, pp. 613–617, September 2006.

[11] R. Machado and B. F. Uchôa-Filho, "Extended techniques for transmit antenna selection with STBCs," *IEEE Journal of Communication and Information Systems*, vol. 21, no. 3, pp. 188–195, December 2006.

[12] R. W. Heath Jr. and A. Paulraj, "A simple scheme for transmit diversity using partial channel feedback," in *Proc. 32nd Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, California, November 1998, pp. 2441–2444.

[13] D. J. Love and R. W. Heath Jr., "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2735–2747, October 2003.

[14] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple-antenna systems," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2562–2579, October 2003.

[15] Z. Chen, J. Yuan, and B. Vucetic, "Analysis of transmit antenna selection/maximal-ratio combining in Rayleigh fading channels," *IEEE Trans. Vehic. Tecnology*, vol. 54, no. 4, pp. 1312–1321, July 2005.

[16] C. R. Murthy and D. Rao, "Quantization methods for equal gain transmission with finite rate feedback," *IEEE Trans. Signal Processing*, vol. 55, no. 1, pp. 233–245, January 2007.

[17] J. Akhtar and D. Gesbert, "Extending orthogonal block codes with partial feedback," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1959–1962, November 2004.

[18] F. M. L. Tavares, R. Machado, B. F. Uchôa-Filho, and A. N. Barreto, "Linear Dispersion Codes for Limited Feedback Channels with Feedback Impairments," *IEEE Journal of Communication and Information Systems*, vol. 23, no. 02, pp. 235–247, April 2011.

[19] I. Choi, J.-K. Kim, H. Lee, and I. Lee, "Alamouti-codes based four-antenna transmission schemes with phase feedback," *IEEE Communications Letters*, vol. 13, no. 10, pp. 749–751, October 2009.