Plasmas Simulations Applied to the Study of Magneto-Aerodynamics Effects

G. N. Marques
A. J. Preto
S. Stephany
Laboratório Associado de Computação e Matemática Aplicada – LAC/INPE, Brasil
gleber@lac.inpe.br
airam@lac.inpe.br
stephan@lac.inpe.br

A. Passaro
Laboratório de Engenharia Virtual
M. A. S. Minucci
Laboratório de Termodinâmica e Hypersônica
Instituto de Estudos Avançados – IEAv/CTA, Brasil
angelo@ieav.cta.b
sala@ieav.cta.br

Abstract

A preliminary review of the literature about magneto-aerodynamics (MAD) is presented, focused on the study of super and hypersonic flow field of air plasmas generated by upstream focused laser energy deposition. The main phenomena which interplay in MAD are briefly discussed as well as some proposed models and difficulties in the related research areas. Alternative approaches and models are pointed-out.

Keywords: *magneto-aerodynamics, finite element method, plasma simulation, particle-in-cell model, computational fluid dynamics.*

1. Introduction

The study of aerodynamic drag reduction is of high interest in Aeronautic and Aerospace research. Many different approaches for magneto-aerodynamics (MAD) have been proposed and investigated both experimentally and computationally [18]. As a rule, the studies on this multidisciplinary research area require the interplay of advanced knowledge in electromagnetism, Aerodynamics, chemical and quantum Physics and Applied Mathematics. MAD typically requires high performance parallel computing.

Essentially, the physical modelling of a MAD problem consists in simulating an electrically conducting fluid flow in the presence of electric and magnetic fields. The electromagnetic field can properly modify the ionized gas flow. This means that the applied electromagnetic field interacts with the charged particles of the fluid and the gas flow can be, in certain way, controlled by that field. At hypersonic flight speeds, the air temperature is very high producing considerable ionization. Also laser energy deposition has been successfully used, as well as plasma injection, for both drag reduction and hypersonic flow control research. Additionally, one can consider externally applied electromagnetic fields interacting with the flow field of the plasma [18], which can be injected [17] or generated [16, 7]. Many difficulties arise when modeling all the physical phenomena involved in the process. "This interdisciplinary endeavor not only presents extremely complex sciences issues, but it demands a significant knowledge not completely available at present." This quote refers to MAD [18].

Experimental research has been developed at the Institute for Advanced Studies (IEAv/CTA). The present work proposes the use of computer simulations for the understanding of some of the physical phenomena involved in the aerodynamic drag reduction in a blunt body by means of air plasma generated by laser energy deposition.

2. Some Challenges and Methodologies

The induced air spike, caused by the laser energy deposition, produces shock waves that can favorably interact with the flow field, while the plasma expands and is dragged over the body. Successive laser pulses are needed in order to continuously generate the plasma. These processes can be didactically associated to some research areas, such as plasma generation, MAD, and laser-plasma interactions. Many potential new contributions can arise in the analysis, modeling and simulation of some of the involved phenomena, as shown in next subsections.

2.1 Plasma generation

One of the main issues on using plasma upstream in a blunt body is that the shock wave envelope in the plasma field exhibits an outward displacement compared to a simple gas dynamic shock [18]. However, the study of the possible effects on the flow field of an ionized fluid (plasma) under the action of external electromagnetic fields requires an accurate assessment of the intrinsic properties of the plasma, which are difficult to determine either theoretically or experimentally [18]. Some models have been proposed for simulating the air breakdown caused by a laser beam [8, 9, 23]. These simulations can provide the input required for simulating the plasma expansion, leading to a more accurate initial configuration of the particle system.

2.2 Plasma expansion

After the air breakdown, the plasma is dragged by the flow field, which might be modeled by MHD equations. However, the required inputs for solving these equations include the physical properties of the fluid (the plasma). For this purpose, a collisional Particle-In-Cell (PIC) model [6, 13, 20] can be adopted to simulate the evolution of the charged particles and to determine the required physical properties of the medium. The PIC methodology indirectly solves the Vlasov equation by separately solving the Maxwell Equations for the fields and the Newton's motion equations [4]. The Maxwell equation is solved for an initial configuration of the particle system, using as inputs the charge and current densities at the nodal points. The computed fields are then interpolated at the particle positions and, by means of the Lorentz force equation, the motion equations are integrated. The Maxwell equations are usually solved by a Finite Difference Method (FDM) and a Spectral Method. However, recent works show the successful coupling of the Finite Element Method (FEM) with the PIC model [14, 15]. In this case, the Newton's equations can be integrated using an Euler method [5] or a leapfrog scheme [14, 15].

2.3 Magneto-Hydrodynamics (MHD)

The modeling of the plasma flow in the presence of electric and magnetic fields combines both the Maxwell and Navier-Stokes equations, which leads to a very complex set of equations. Although characteristic-based methods like approximate-Rieman [18] are often used for solving this set of equations, we intend to adopt a recently reported stabilized FEM approach [21]. Other methods may be considered for solving the full Navier-Stokes equation, such as discontinuous Galerkin methods [1, 2, 3,] or element-free Galerkin methods [10, 21].

2.3.1 Governing Equations

The governing equations can be written in the following flux vector form:

$$\frac{\partial \mathbf{V}}{\partial t}+\frac{\partial \mathbf{F}_x}{\partial x}+\frac{\partial \mathbf{F}_y}{\partial y}+\frac{\partial \mathbf{F}_z}{\partial z}=\frac{\partial \mathbf{F}_{x,r}}{\partial x}+\frac{\partial \mathbf{F}_{y,r}}{\partial y}+\frac{\partial \mathbf{F}_{r,z}}{\partial z} \quad (1)$$

where $\mathbf{V}=(\rho,\rho u,\rho v,\rho w,\rho E,B_x,B_y,B_z)^T$ is the vector of conservative dependent variables. The flux vectors $\mathbf{F}_{x,r}$, $\mathbf{F}_{y\ r}$ and $\mathbf{F}_{z,r}$ are associated with the resistivity, heat transfer and viscous dissipation, thereby, in the case of ideal MHD[1] the right hand side of Eq. (1) is identically null. The ideal MHD equations can be written as:

$$\frac{\partial \mathbf{V}}{\partial t}+\frac{\partial \mathbf{F}_x}{\partial x}+\frac{\partial \mathbf{F}_y}{\partial y}+\frac{\partial \mathbf{F}_z}{\partial z}=0\,. \quad (2)$$

The FEM discretization can be written as:

$$\left(\int_\Omega \mathbf{N}^T\mathbf{N}d\Omega\right)\Delta\overline{\mathbf{V}}=-\Delta t\left\{\int_\Omega \frac{\partial \mathbf{N}^T}{\partial x_i}\left(\mathbf{F}_i^{n+1/2}+\mathbf{G}_i^n\right)d\Omega+\int_\Gamma \mathbf{N}^T\left(\mathbf{F}_i^{n+1/2}+\mathbf{G}_i^n\right)\mathbf{n}_i d\Gamma\right\} \quad (3)$$

where, $\mathbf{N}$ represent the FEM base functions, $\mathbf{F}$ corresponds to the ideal MHD flux and $\mathbf{G}$ to the non ideal model ($\mathbf{G}_i = \mathbf{F}_{ir}$) [21]. However, the standard Galerkin procedure leads to spurious numerical oscillations in the velocity and pressure solutions, for advection dominated flows. These instabilities can be reduced or even eliminated by applying some stabilization method, like the Stream-line upwind Petrov-Galerkin (SUPG) and Galerkin Least Squares. For instance, the SUPG stabilization term is introduced by considering a Petrov-Galerkin weight function of the form:

$$\mathbf{W}+\tau\mathbf{A}_k^T\frac{\partial \mathbf{W}}{\partial x_k}\,, \quad (4)$$

where, $\mathbf{A}_k=\dfrac{\partial \mathbf{F}_k}{\partial \mathbf{V}}$, (5)

and τ is a matrix calculated at the element level and is dependent on a suitable defined element length and the local velocity, in each direction [12]. A well detailed derivation of the flux vector form for the MAD equations, as well as the details of the FEM discretization and the resulting stabilized SUPG weak form can be found in [21].

2.4 Laser – Plasma interaction

Since we are interested in maintaining the drag reduction, which is due to shock waves and electromagnetic interactions, the plasma must be generated continuously. Different models can be employed for this phenomena and the choice of the model depends on the effects that must investigated. In some situations, a gas dynamic based model can be used, while in others, effects such as absorption and scattering effects must be considered in order to find an optimized rate for the laser pulse. Depending on the plasma density, the laser energy can be scattered or

1 The ideal MHD equations consider a physical model where the conductivity is assumed infinite, and viscosity and heat transfer is neglected.

absorbed. The modeling of air plasma generation by electron beams is an open area of research [7, 9, 19, 23].

3. Computational methodologies

The computational cost involved in the described simulations is high, since both the number of particles and of nodal variables required for adequately representing the system are enormous. Parallel computing becomes mandatory.

We obtained an excellent speed-up and efficiency with an object-oriented parallel PIC-FEM code developed for a distributed memory parallel machine [15]. Although in this implementation collisional effects were not modeled, the code can be easily modified in order to model these effects by means of a Monte-Carlo (MC) approach, called PIC-MC or PIC-MCC (PIC-MC Collision) [6, 13, 20].

The FEM formulations for the full MAD equations and for the Maxwell field equations result in large sparse linear systems depending on the size of the mesh. High performance parallel codes are also required for resolving these large systems. Moreover, it is necessary to employ costly adaptive strategies for adequately solving the shock waves interactions for the MHD flow field. Furthermore, discontinuous and *p*-adaptive formulations [2] may be required, as well as alternative methods, such as the meshfree methods [10, 22].

Finally, the generality of the mathematical formulations and models allows to simulate many interesting phenomena in MAD, such as power extraction, flow control and design optimization.

4. References

[1] Bassi, F., Crivellini, A., Rebay, S. and Savini, M. "Discontinuous Galerkin Solution of the Reynolds-averaged Navier Stokes and k-ϖ turbulence model equations", *Computer & Fluids,* 2004, article in press.

[2] Burbeau, A., Sagaut, P. "A dynamic *p*-adptive Discontinuous Galerkin method for viscous flow with shocks", *Computers & Fluids,* 2004, article in press.

[3] Burbeau, A., Sagaut, P. "Simulation of a viscous compressible flow past a circular cylinder with high-order discontinuous Galerkin methods", *Computers & Fluids,* **31**, 2002, 877-889.

[4] Kruer, W. L. *The Physics of Laser Plasma Interactions,* Addison-Wesley Publishing Company, Inc., 1988.

[5] Lean, M. H., "Particle simulation of ion cloud in a magnetic field", *IEEE Trans. Mag.*, **34(5)**, 1998, 3122-3125.

[6] Longo, S. "Monte-Carlo models of and ion transport in non-equilibrium plasmas", *Plasmas Sources Sci. Techn.,* **9**, 2000, 468-476.

[7] Marechet, S. O., Shneider, M. N and Miles, R. B., "Modeling of Plasma Generation in Repetitive Ultra-Short DC, Microwave, and Laser pulses", *AIAA 2001-2940*, 2001.

[8] Marechet, S. O., Shneider, M. N and Miles, R. B., "Energy-efficient generation of nonequilibrium plasmas and their applications to hypersonic MHD systems", *AIAA 2001-2880*, June 2001.

[9] Marechet, S. O., Shneider, M. N and Miles, R. B., "Modeling of Air Plasma generation by electron Beams and High-Voltage Pulses", *AIAA 2000-2569*, June 2000.

[10] Marques, G. N., Machado, J. M. and Verardi, S. L. L., "Interpolating element-free Galerkin method results for electromagnetic field computations in inhomogeneous medim", *XV CILAMCE*, 2004, accepted article.

[11] Minucci, M. A. S., Toro, P. G. P., Oliveira, A. C. and Chanes Jr., J. B., "Multi laser pulse investigation of the DEAS concept in hypersonic flow", *Proc. 2nd. Int. Symp. On Beamed Energy Propulsion,* Sendai, 2003.

[12] Mital, S., "Finite element computation of unsteady viscous compressible flows", *Computer Methods in Applied Mechanics and Engineering,* **157**, 1998, 151-175.

[13] Nanbu, K., Mitsui, K. and Kondo, S. "Self-consistent particle modeling of dc magnetron discharges of an O_2 /*Ar* mixture", *J. Phys. D: Appl. Phys.,* **33**, 2000, 2274-2283.

[14] Paes, A. C. J., Abe, N. M., Serrão, V. A. e Passaro, A. "Simulations of Plasma with Electrostatic PIC Models Using the Finite Element method", *Brazilian Journal of Physics*, **33(2)**, 2003, 411-417.

[15] Passaro, A., Abe, N. M., Paes, A. C. J., Marques, G. N., Preto, A. J. and Stephany, S. "A Parallel and Object-Oriented Plasma Simulation Code Based on Electrostatic PIC model and the Finite Element Method", *XV CILAMCE*, 2004, accepted article.

[16] Riggins, D. W. and Nelson, H. F. "Hypersonic flow control using upstream focused energy deposition", *AIAA 99-0898*, 1999, 1-11.

[17] Shang, J. S., Hayes, J. and Menart, J. "Hypersonic flow over a blunt body with plasma injection", *Journal of Spacecraft and Rockets*, **39**(3), 2002, 367-375.

[18] Shang, J. S. "Recent research in magneto-aerodynamics", *Progress in Aerospace Sci.*, **37,** 2001, 1-20.

[19] Shneider, M. N., Marechet, S. O. and Miles, R. B. "Electric *charge* build-up in hypersonic wind tunnels with electron beam energy addition", *AIAA 2003-4035*, Jun 2003.

[20] Straaten, T. A. van der, Cramer, N. F., Falconer, I. S. and James, P. W. "The cylindrical DC magnetron discharge: I. Particle-In-Cell simulation", *J. Phys. D: Appl. Phys.,* **31**, 1998, 177-190.

[21] Verardi, S. L. L., Passaro, A., Machado, J. M. and Shiyou, Y. "A FEM approach to the Equations of Magneto-Aerodynamics", Proceedings of the 6TH Internationl Conference on Advances in Power System Control, Operation and Management, 2003.

[22] Verardi, S. L. L., Machado, J. M. and Cardoso, J. R.. "The element-free Galerkin method applied to the study of fully developed magnetohydrodynamic duct flows", *IEEE Trans. on Mag.*, **38**(2), 2002, 941-944.

[23] Yan, H., Adelgren, R., Boguzko, M., Elliott, G. and Knight, D. "Laser Energy Deposition in Quiescent Air", *AIAA 2003-1051*, January 2003.