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Approximation of the K_A distribution by the G_A^0 distribution

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Abstract. In the field of processing and analysis of Synthetic Aperture Radar (SAR) images, the returned signal can be modelled as the product of the inherent *speckle* noise and the terrain backscatter. For amplitude SAR images, the data can be fitted with several distributions depending, among other considerations, on the degree of homogeneity of the areas under study. In this work, unless otherwise stated, linear detection (amplitude) data will be used. In zones where the backscatter can be considered homogeneous, for example: crops, pastures, etc., the $\Gamma^{1/2}$ distribution is a good model for the returned signal. The K_A distribution gives a good fit for homogeneous areas as well as for heterogeneous areas (for example forest on flat terrain) but there are numerical problems caused by the presence of Bessel functions. The $\Gamma^{1/2}$ distribution does not explain data from heterogeneous zones. When the area under study is extremely heterogeneous, as it is the case of cities, or forest on undulated terrain, the $\Gamma^{1/2}$ distribution and the K_A distribution fail to fit these data. In this case, the G_A^0 distribution behaves very well. Taking also into account that this distribution fits equally well homogeneous and heterogeneous areas too, and that its use is more computational and theoretically tractable, it is desirable to substitute the G_A^0 distribution for the K_A distribution. In this work the feasibility of this substitution will be studied. To this end, a correspondence between the parameters of both distributions is proposed in order to approximate, in some sense, the K_A distribution by the G_A^0 distribution. The minimization of a distance between both densities in order to obtain such correspondence will be considered, and the goodness of fit of between K_A distributed data by the G_A^0 distribution model will be measured, using the χ^2 adherence test in Monte Carlo experience.

Keywords: statistical models, G_A^0 distribution, K_A distribution, SAR

1 Introduction

In the field of Synthetic Aperture Radar (SAR) image processing, the multiplicative model is widely used. Within it, the return is modelled as the product of the *speckle* noise and the terrain backscatter. SAR amplitude data can be fitted by several distributions depending, among other factors, on the degree of homogeneity of the areas under study. In this work, unless otherwise stated, linear detection (amplitude) data will be used.

For zones where the backscatter can be considered homogeneous, like crops and pastures, the $\Gamma^{1/2}$ distribution is a good model for the returned data. The K_A distribution models data coming from homogeneous zones (with certain restrictions due to numerical problems arising from the use of Bessel functions) as well as data coming from heterogeneous zones, like forest on flat relief. The $\Gamma^{1/2}$ however, does not fit heterogeneous data appropriately.

When the area under study is extremely heterogeneous , as it is the case for urban areas or forest over undulated relief, the $\Gamma^{1/2}$ distribution as well as the K_A distribution do not model the data adequately. In this case the G_A^0 distribution behaves remarkably well. Taking into account that this distribution models very well data from heterogeneous and homogeneous areas too, and that its use is more computationally and theoretically tractable, it is advisable to substitute the G_A^0 distribution for the K_A distribution.

In this work, the feasibility of this substitution will be studied. To this end, a correspondence between the parameters of both distributions will be considered in order to approximate, in some sense, the G_A^0 distribution to the K_A distribution.

This study is made up of two parts:

- Minimization of the distance in L_2 of the respective densities to obtain a correspondence between distributions.
- The goodness of fit of the G_A^0 distribution to K_A distributed data will be measured using the χ^2 test in a Monte Carlo experiment.

2 Main properties of the K_A and the G_A^0 distributions

The $G_A^0(\alpha_G, \gamma, n)$ and the $K_A(\alpha_K, \lambda, n)$ distributions we will use are characterized by the following densities:

$$f_G = \frac{2n^n \Gamma(n - a_G) z^{2n-1}}{g^{a} \Gamma(n) \Gamma(-a_G) \left(g + nz^2\right)^{n-a_G}} \qquad (1)$$

where $-\alpha_G, \gamma, n, z > 0$

$$f_K = \frac{4\left(\sqrt{ln}\right)^{a_K + n}}{\Gamma(n)\Gamma(a_K)} z^{n-a_K+1} K_{n-a_K}\left(2z\sqrt{ln}\right) \qquad (2)$$

where $\alpha_K, \lambda, n, z > 0$ and K_n is the modified Bessel function of the third kind and order parameter ν.

There are not many computational implementations of this Bessel function (see Gordon et al.(1995)), for a recent algorithm for this function). On the other hand, the only special function in the G_A^0 distribution is the Γ function, for which there are many reliable implementations.

This is the first computational argument in favour of the G_A^0 distribution against the K_A distribution. The second computational argument is aimed in the same direction and requires the definition of the accumulated distribution functions for both distributions.

Let the random variables *V* and *W* be $G_A^0(\alpha_G, \gamma, n)$ and $K_A(\alpha_K, \lambda, n)$ respectively. The accumulated distribution function of the first one is given by

$$\Pr(V \le v) = \frac{n^{n-1}}{g^n} \frac{\Gamma(n - a_G)}{\Gamma(n)\Gamma(-a_G)} v^{2n} H\left(n, n - a_G, n + 1; -\frac{n}{g} v^2 \right) \qquad (3)$$

where $-\alpha_G, \gamma, n, z > 0$ and *H* is the hypergeometric function. This function is easy to evaluate using the Snedecor's F distribution, as can be seen in the appendix of this work.

In order to write the accumulated distribution function of the second random variable it is necessary to impose restrictions on the variation domain of its parameters. Originally, the parametric space of the $K_A(\alpha_K, \lambda, n)$ distribution is R_+^3 but, to be able to write its accumulated distribution function in a recursive form it is necessary to restrict the variation of α_G or the variation of *n* to the integer numbers. For the second case (*n* an integer number), this function is given by

$$\Pr(W \le w) = 1 + \frac{2^{2-a-n}}{\Gamma(a_K)\Gamma(n)} g(v, k, z) \qquad (4)$$

where $z = 2w\sqrt{an}$, $k = 2n - 1$, $v = a - n$, and the function $g(v,k,z)$ is given by the following recursive formula:

$$\begin{array}{ll} -z^{v+1} K_{v+1}(z), & \text{for } k = 1 \\ (k-1)(2v + k - 1) g(v, k-2, z) - z^{v+k} K_{v+1}(z) \ (k-1) z^{v+k-1} K_v(z), & \text{for } k \ge 2 \end{array}$$

More details of this recursive solution for the accumulated distribution function of K_A distributed variables can be found in Yanasse et al.(1995).

From the considerations above on the accumulated distribution functions for G_A^0 and K_A distributions we can deduce the following advantages of the first one over the second one: it is easier to implement, it uses reliable and immediately obtainable implementations, and it does not impose restrictions on the original parameter space.

The importance of the availability of reliable implementations of the accumulated distribution function arises from the need of carrying out goodness of fit tests and from the use of these functions in estimators based on order statistics.

To the stated advantages in the areas of modelling and computational tractability, additional advantages in the fields of inference which favour even more the use of the G_A^0 distribution instead of the K_A distribution, can be added.

For the estimation of the homogeneity parameter of both distributions using the maximum likelihood method, the estimator of α_K is difficult to calculate due to the presence of the derivative of the Bessel function of the third kind with respect to the order parameter. The maximum likelihood estimator of α_G entails the use of the digamma function, which has been widely studied and implemented. These estimators are the solutions of the following equations:

Let us consider the sample $z_1,\ldots, z_k$ of independent observations. The maximum likelihood estimator of the α_K, knowing n and λ is

$$k\Psi(\hat{a}_K)-\sum_{i=0}^{k}\frac{\partial}{\partial \hat{a}_K}\log K_{\hat{a}_K-n}\left(2z_i\sqrt{ln}\right)=\frac{k}{2}\log l+\sum_{i=0}^{k}\log z_i$$

analogously, for the maximum likelihood estimator of α_G knowing n and γ is:

$$\Psi(n-\hat{a}_G)-\Psi(-\hat{a}_G)=-\log g+\frac{1}{k}\sum_{i=0}^{k}\log\left(g+nz_i^2\right)$$

where Ψ is the digamma function and k is the sample size.

3 Minimization of the distance between the G_A^0 distribution and the K_A distribution

In this section, a method by which a G_A^0 distribution approximates a K_A distribution, under the constraint that both must have a mean value equal to one, is described. In other words, given the sets of G_A^0 and K_A distributions with mean value equal to one, a correspondence of elements of the first one to elements of the second one , trough a numerical minimization of an also numerical integration, will be sought. This correspondence will then be established by parameter pairs (see figure 1).

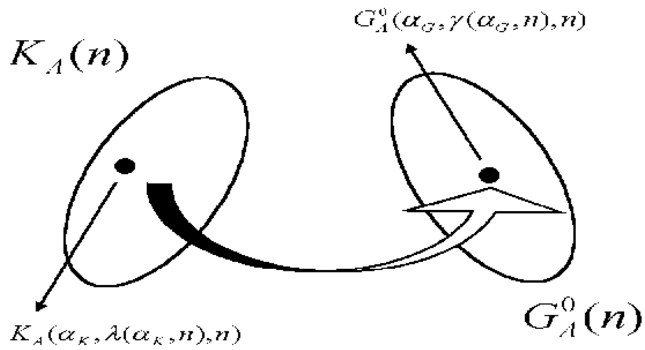


figure 1: correspondence between the K_A and the G_A^0 distributions

In this figure, the set $K_A(n)$ ($G_A^0(n)$ respectively) defined by all the K_A (G_A^0, resp.) with n looks and mean value equal to one. These distributions have only one free parameter: the homogeneity parameter α_K (α_G resp.), because the scale parameter λ (γ resp.) must be chosen in function of the number of looks and the homogeneity parameter in order to guarantee the constraint of mean value equal to one.

The objective of this work is to approximate the K_A distribution by the G_A^0 distribution. To do this, it is necessary to define previously the approximation criterion. Let us consider then, the set of all the distributions that admit a density and call it D. To establish the notion of proximity between distributions in D we will use a distance we will denote d:D×D→[0,∞), through the relation

$$d(\mathrm{D}_1,\mathrm{D}_2) = \int_{-\infty}^{\infty} \left| f_1(z) - f_2(z) \right|^2 dz$$

Where f_1 and f_2 are the densities that characterize the D_1 and the D_2 distributions respectively. This metric has been already used in a similar context (see Joughin et al. (1993)).

Due to reasons that will be explained later, only those distributions $\mathrm{K}_A(\alpha_K, \lambda, n)$ distribution by the G_A^0 (α_G, γ, n) with mean equal to one will be studied. In this way, as the number of looks n and the homogeneity parameter α_K, the scale parameter λ is given by:

$$l = \frac{1}{n_0}\left(\frac{\Gamma(a_K + 1/2)\Gamma(n+1/2)}{\Gamma(a_K)\Gamma(n)}\right)^2 \qquad (5)$$

Analogously, the G_A^0 distributions (with the same number of looks n) will be indexed only by their respective homogeneity parameter α_G because its scale parameter is given by:

$$g = n\left(\frac{\Gamma(-a_G)\Gamma(n)}{\Gamma(-a_G - 1/2)\Gamma(n+1/2)}\right)^2 \qquad (6)$$

We want to find the value of α_G that minimizes the distance

$$d = \int_0^{\infty} \left| \frac{4}{\Gamma(n)} 2(ln)^{(a_K+n)/2} z^{a_K+n-1} K_{a_K-n}\left(2z\sqrt{ln}\right) - \right.$$

$$\left. - \frac{n^n\, g^{-a_G}\Gamma(n-a_G) z^{2n-1}}{\Gamma(-a_G)\left(g + nz^2\right)^{n-a_G}} \right|^2 dz \qquad (7)$$

where the values $\lambda=\lambda(\alpha_K, n)$ and $\gamma=\gamma(\alpha_G, n)$ are the ones that make the mean value equal to one. Then, the value of $\alpha_G < 0$ that minimizes numerically that integral will be sought.

Although α_K varies over all the positive real numbers, for the purposes of this study the search will be done within the interval [4,12]. Very small values of α_K $(0 < \alpha_K < 4)$ correspond to data from extremely heterogeneous areas, which are not modelled by the K_A but by the G_A^0 distribution., as can be seen in Frery et al.(1997); then, for these data, it is not necessary to have an approximation. For values of α_K larger than 15, the observed data can be modelled by the $\Gamma^{1/2}$ distribution, (see Frery et al.(1997), Yanasse et al. (1995) and Yanasse et al.(1993)), which is also a particular case of the

G_A^0 distribution. So, the only region in which it is necessary to approximate the K_A distribution by the G_A^0 distribution is the one that corresponds to values of α_K within the interval [4, 12].

For these values of α_K, the corresponding values of α_G, obtained as a result of the minimization of the integral in formula (7), are shown in the following tables.

$n = 1$				$n = 2$			$n = 4$		
$\alpha_{K.}$	$\alpha_{G.}$	λ	γ	$\alpha_{G.}$	λ	γ	$\alpha_{G.}$	λ	γ
4.	-4.3	2.95	4.15	-4.4	3.32	3.68	-2.9	3.53	3.45
5.	-5.3	3.73	5.42	-5.4	4.20	4.81	-5.5	4.46	4.53
6.	-6.3	4.52	6.09	-6.4	5.08	5.94	-6.4	5.40	5.59
7.	-7.3	5.30	7.96	-7.4	5.96	7.07	-7.4	6.34	6.65
8.	-8.3	6.09	9.23	-8.3	6.85	8.21	-8.4	7.28	7.72
9.	-9.3	6.87	10.50	-9.3	7.73	9.34	-9.4	8.22	8.78
10.	-10.3	7.66	11.78	-10.3	8.61	10.47	-10.4	9.16	9.84

Figure 2 shows the densities of some of the K_A and G_A^0 distributions for a fixed value of α_K and its corresponding value of α_G. In it, it can be noticed that for α_K = 4 and α_G= -4.3, with *n*=1, the difference between both distributions is very small.

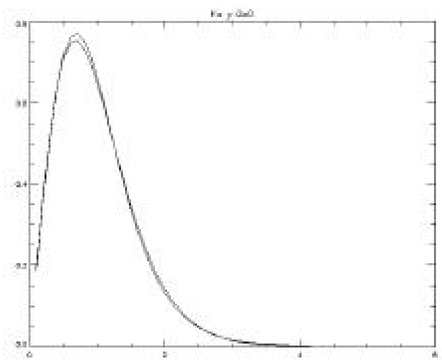


figure 2: densities of K_A and G_A^0 distributions for α_K = 4 and α_G= -4.3, with *n*=1

Figure 3 shows the values of the integral (7) for α_K = 4 and for α_K = 8 in function of $\alpha_G \in$ [4, 10], the minimum is reached for α_G = -4.3 and α_G = -8.3.

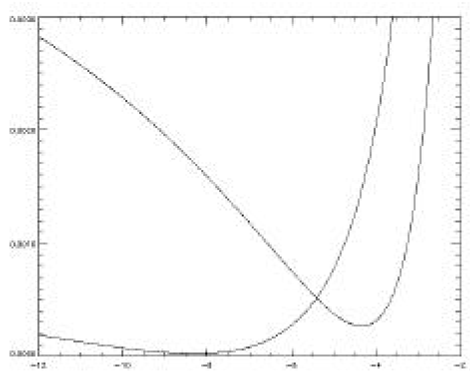


figure 3: distance between both distributions as a function of α_G for α_K = 4.0 and for α_K = 8.0

4 Goodness of fit of K_A distributed data using the G_A^0 distribution

To measure the goodness of the fit of K_A distributed data using the G_A^0 distribution, we will use the χ^2 adherence test in a Monte Carlo experiment.

4.1 χ^2 test

To test if the simulated $K_A(\alpha_K, \lambda, n)$ distributed data can be fitted by the $G_A^0(\alpha_G, \gamma, n)$ distribution, the Pearson´s χ^2 statistic will be used:

$$c^2 = \sum_{i=0}^{k} \frac{(h_i - mp_i)^2}{mp_i} \qquad (8)$$

where

m: total number of K_A distributed data

h_i: number of data in each interval

k: number of intervals

p_i: $F(z_i) - F(z_{i-1})$, (*F* accumulated function)

Let $\{z_A\}$ a sample of length m, of $K_A(\alpha_K, \lambda(\alpha_K, n), n)$ distributed data, taking $\lambda(\alpha_K, n)$ as defined in (5). From this sample $\alpha_{G.}$, the homogeneity parameter of the $G_A^0(\alpha_G, \gamma(\alpha_G, n), n)$, is estimated using the maximum likelihood estimator considering $\gamma(\alpha_G, n)$ as defined in formula (6).

If the random variable *Z* is $G_A^0(\alpha_G, \gamma, n)$ distributed, then its accumulated distribution function is given by:

$$F_{Z_A}(z) = \frac{n^{n-1}\Gamma(n-a)z^{2n}}{g^n \Gamma(n)\Gamma(-a)} H\left(n, n-a; n+1; -nz^2/g\right) \qquad (9)$$

where $H(n,\ n - \alpha;\ n + 1;\ -nz^2/\gamma)$ s the hypergeometric function and can be evaluated using

where $\Upsilon_{\tau,\nu}$ is the accumulated distribution function of a Snedecor distributed random variable.

$$F_{Z_A}(z) = \Upsilon_{2n,-2a}\left(\frac{-a}{g} z^2\right) \qquad (10)$$

4.2 Monte Carlo experiment

A Monte Carlo experience was carried out generating $\mathcal{K}_A(\alpha_K,\ \lambda,\ n)$ distributed data which were fitted with a $\mathcal{G}_A^0$ distribution. The values of the parameters α_K and α_G were estimated by the maximum likelihood method and by the ½ order moment estimator method (see Mejail et al. (1998)). The goodness of fit was evaluated using the p-value of the χ^2 test.

For a number R of replications, the following steps were done:

- For each $\alpha_K \in [4, 12]$ $\mathcal{K}_A$ distributed samples were generated.
- For each of these $\mathcal{K}_A$ distributed samples the roughness parameter α_G and the parameter γ of the $\mathcal{G}_A^0$ distribution were estimated.
- The goodness of fit was evaluated using the p-value of the χ^2 test.

The $\mathcal{K}_A$ distributed samples were generated for values of $\alpha_K \in [4, 12]$ with number of replications R = 100, 1000 and 10000, sample sizes T = 1000 and 10000 and significance level 0.01.

As a particular case, the figures for $\alpha_K = 4$ are shown in the following table, where for each value of number of replications R and each sample size T , the mean value of the estimated α_G, the mean square error *mse* and the rejection percentage $r_{0.01}$ are presented.

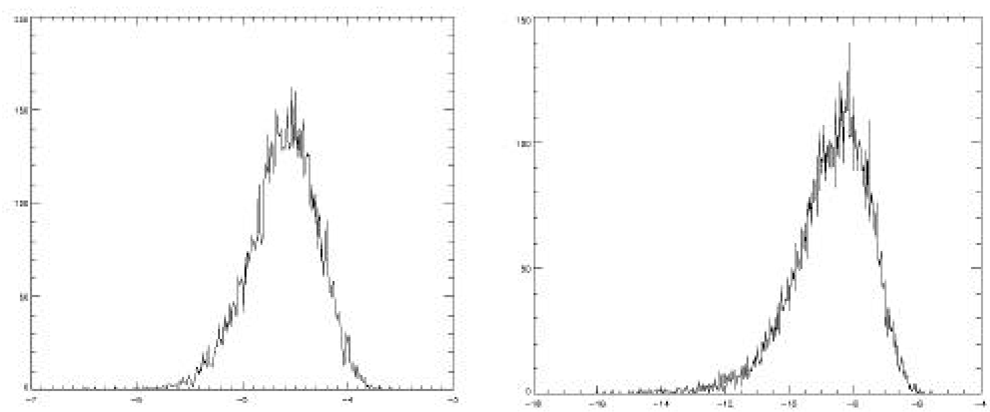
$\alpha_K = 4$, $n = 1$				%
R.	T	α_G.	*mse*	$r_{0.01}$
100	10000	-4.50	0.13	11.0
1000	1000	-4.78	1.80	1.5
1000	10000	-4.53	0.14	10.0
10000	1000	-4.79	2.08	1.3
10000	10000	-4.52	0.13	12.0

This shows that there is no reason to suppose that the $\mathcal{K}_A$ and $\mathcal{G}_A^0$ distributions are different at the proposed significance level.

The next table shows the values coresponding to the mean value of the estimated α_G with $\alpha_K \in [5, 10]$ and $n = 1$

R = 1000, T = 10000 , n = 1			
α_K	α_G	*mse*	$r_{0.01}$
5.	-5.53	0.25	3.4
6.	-6.55	0.50	2.2
7.	-7.61	0.95	1.0
8.	-8.73	1.93	1.1
9.	-9.69	2.448	1.3
10.	-10.83	4.61	0.7
11.	-11.98	6.80	0.7

In figures 4 and 5, the histograms of the estimated α_G for a sample size T = 10000 and number of replications R = 10000, generated with $\alpha_K = 4$ and $\alpha_K = 8$ respectively. Notice that, in each case, the corresponding α_G that minimizes integral (7) are –4.3 and –8.3 respectively.



figures 4 and 5: histograms of estimated α_G with $\alpha_K = 4$ and $\alpha_K = 8$ respectively.

In all the cases, the estimated α_G fall within a small interval around the value of α_G that minimizes integral (7).

5 Conclusions

In this work, the possibility of substituting the $\mathcal{K}_A$ distribution by the $\mathcal{G}_A^0$ distribution has been presented. The importance of this substitution resides in the theoretical and computational tractability of the second one over the first one.

It has been demonstrated that, for a given value of the roughness parameter α_K, the replications of the estimated values of α_G are close enough to the theoretical value found through the minimization of integral (7). Thus, the hypothesis of K_A distributed data can be substituted by the hypothesis of G_A^0 distributed data.

This was demonstrated for the roughness parameter α_K varying within the interval [4, 12]. For $\alpha_K < 4$ it was shown (see Frery et al.(1997)) that the G_A^0 distribution models very well extremely heterogeneous data, and for $\alpha_K > 12$ the $\Gamma^{1/2}$ distribution models well these homogeneous data.

6 Appendix

6.1 G_A^0 distribution function

If the random variable Z_A is $G_A^0(\alpha_G, \gamma, n)$ distributed then the random variable defined by $-(\alpha/\gamma)\ Z_A^2$ has a distribution given by $F_{2n,\,-2\alpha}$ and its accumulated distribution function is given by

$$F_{Z_A}(z) = \Upsilon_{2n,-2a}\left(\frac{-a}{g} z^2\right)$$

In general, let X, Y and Z be random variables such that $z=XY$, with $X \sim \Gamma^{-1/2}(\alpha_G, \gamma)$ and $Y \sim \Gamma^{-1/2}(n, n)$.

If $X^*=1/X$ then $X^* \sim \Gamma^{-1/2}(-\alpha_G, \gamma)$. As $X^{*2} \sim \Gamma(-\alpha_G, \gamma)$ and $Y^2 \sim \Gamma(n, n)$ let us see the distribution of $Z^2 = Y^2 / X^{*2}$.

If W is a $\Gamma(\eta, 1/2)$ distributed random variable, whose density function is

$$f_W(w) = \frac{w^{h-1}\exp(-w/2)}{2^h\,\Gamma(h)} \qquad h, w > 0$$

it coincides with the χ^2 density for $2\eta = v$

$$f_{c_n^2}(t) = \frac{t^{\frac{n}{2}-1}\exp(-t/2)}{2^{n/2}\,\Gamma(n/2)} \qquad t, n > 0. \tag{11}$$

Then we can state that

$$2nY^2 \sim \chi^2_{2n} y^2 \gamma X^{*2} \sim \chi^2_{-2\alpha}$$

taking $X=X_A$, $Y=Y_A$ and $Z=Z_A$ we have that

$$\frac{-a}{g} Z_A^2 = \frac{2nY_A^2 / 2n}{2gX_A^{'2} / (-2a)} \sim F_{2n,-2a}\,, \tag{12}$$

where F is the Snedecor's distribution. As

$$F_{Z_A}(t) = \Pr(Z_A \le t) = \frac{n^{n-1}\Gamma(n-a)t^{2n}}{g^n\Gamma(n)\Gamma(-a)} H\left(n, n-a; n+1; -nt^2/g\right)$$

where H(n, n - α; n + 1; -nt^2/γ) is the hypergeometric function, (see Frery et al.(1997)) and using the result of equation (12) we obtain

$$F_{Z_A}(t) = \Upsilon_{2n,-2a}\left(\frac{-a}{g}t^2\right) \tag{13}$$

where $\Upsilon_{\tau,v}$ is the accumulated distribution function of a Snedecor distributed random variable.

If X is a Snedecor distributed random variable, its density function is given by

$$f_{m.n}(t) = \frac{\Gamma((m+n)/2)}{\Gamma(m/2)\Gamma(n/2)}\left(\frac{m}{n}\right)^{m/2} t^{(m-2)/2}\left(1+\frac{m}{n}t\right)^{-(m+n)/2} \qquad t > 0$$

and its accumulated distribution function is given by

$$\Upsilon_{m,n}(t) = P(X \le x) = \frac{\Gamma((m+n)/2)}{\Gamma(m/2)\Gamma(n/2)}\left(\frac{m}{n}\right)^{m/2}\int_0^\infty t^{(m-2)/2}\left(1+\frac{m}{n}t\right)^{-(m+n)/2} dt$$

6.2 Moments of the K_A and the G_A^0 distributions

Let X_G and Y_K be two random variables such that $X_G \sim G_A^0(\alpha_G, \gamma, n)$ and $Y_K \sim K_A(\alpha_K, \lambda, n)$. Then their rth-order moments will be:

$$E\left(X_G^r\right) = \left(\frac{g}{n}\right)^{r/2}\frac{\Gamma(-a_G - r/2)\Gamma(n+r/2)}{\Gamma(-a_G)\Gamma(n)}$$

for the G_A^0 distribution, and

$$E\left(Y_K^r\right) = (n l_K)^{-r/2}\frac{\Gamma(a_K + r/2)\Gamma(n+r/2)}{\Gamma(a_K)\Gamma(n)}$$

for the K_A distribution.

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