INTEGRATION MODEL COMPARISON FOR COUPLED NAVIGATION USING AN IMU AND THE GPS THROUGH THE EKF

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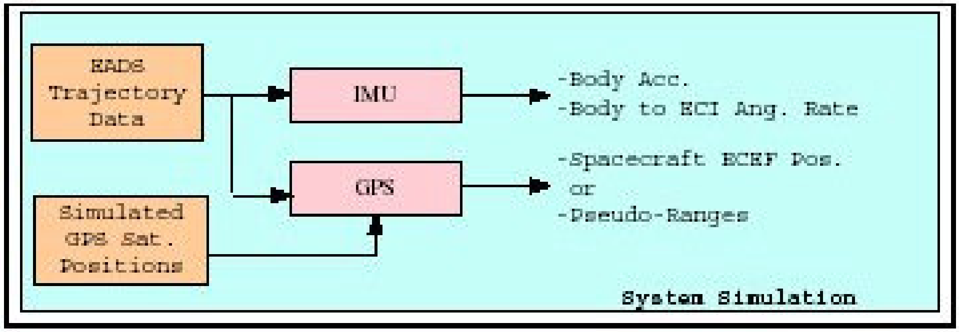
ABSTRACT

In this work a comparison among 5 different implementation cases of a EKF (Extended Kalman Filter) for Spacecraft navigation system using the GPS (Global Positioning System) and the IMU (Inertial Measurement Unity) is presented.

Among them, one is developed with a more realistic sensor simulation provided by EADS Space Transportation-Bremen, where the simulation data provided is related to the **PHOENIX** prototype.



- Case 1: Position measurement in ECEF reference frame
- Case 2: Pseudoranges measurement
- Case 3: Pseudoranges measurement affected by a constant clock bias
- Case 4: Pseudoranges measurement affected by a linear drifted clock bias
- Case 5: Pseudoranges measurement affected by a linear drifted clock bias + EADS sensor simulation

SIMULATION DESCRIPTION – SYSTEM MODEL

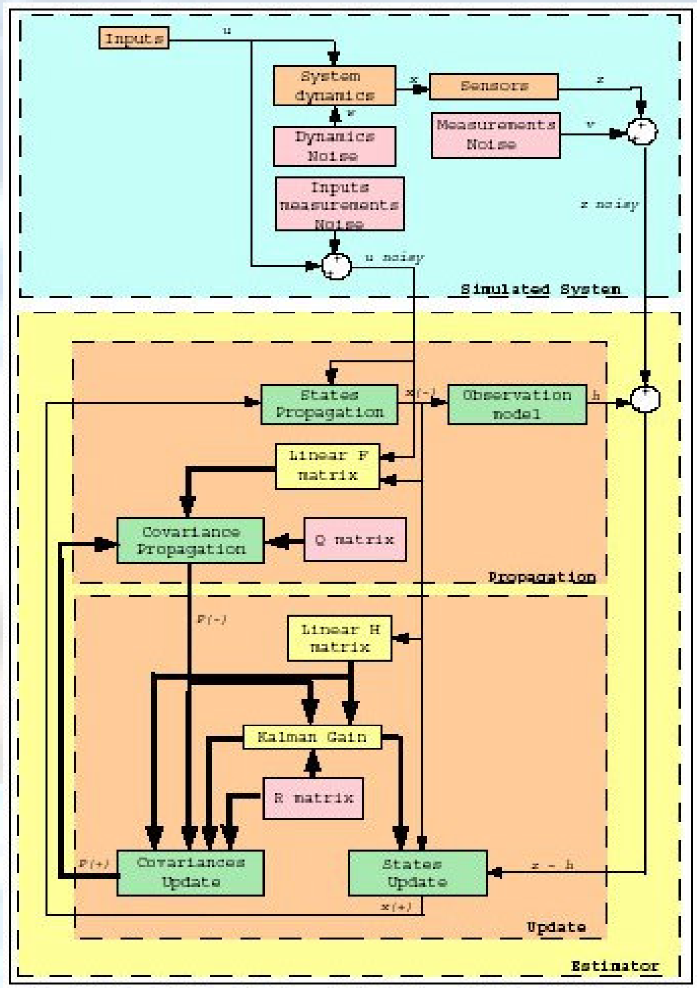
$$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), \underline{u}(t), t) + \underline{\underline{G}} \cdot \underline{w}(t)$$

$$\underline{z}_k = \underline{h}_k(\underline{x}(t_k)) + \underline{v}_k$$

$$\underline{u} = \begin{bmatrix} \underline{a}^{b\,T} & \underline{\omega}^{b\,T}_{i,b} \end{bmatrix}^T$$

Case	State vector	States
1,2	$\underline{x} = \begin{bmatrix} \underline{v}^{i\,T} & \underline{r}^{i\,T} & \underline{q}^{b\,T}_{i} \end{bmatrix}^T$	10
3,4,5	$\underline{x} = \begin{bmatrix} \underline{v}^{i\,T} & \underline{r}^{i\,T} & \underline{q}^{b\,T}_{i} & \delta_{Rec\_clock} \end{bmatrix}^T$	11

Case	Measurements vector	Measur.
1	$\underline{z} = [\underline{r}^{e}_{GPS}]^T$	3
2,3,4,5	$\underline{z} = [PR_1 \quad PR_2 \quad PR_3 \quad PR_4]^T$	4



SIMULATION DESCRIPTION – SYSTEM DYNAMICS

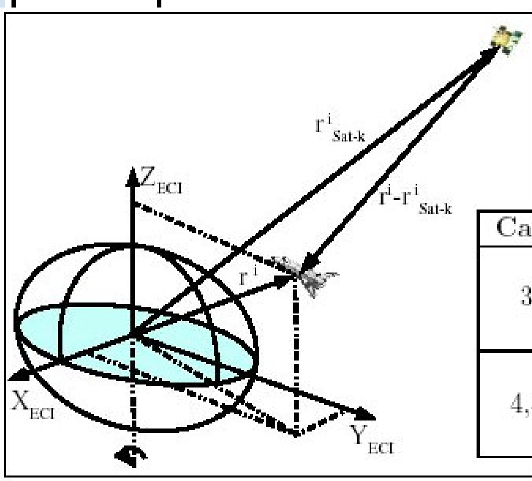
Case	$\underline{\underline{F}}$ matrix
1,2	$\underline{\underline{F}}(\hat{\underline{x}}(t), \underline{u}(t), t) = \left.\frac{\partial \underline{f}(\underline{x}(t),\underline{u}(t),t)}{\partial \underline{x}(t)}\right\vert_{\underline{x}(t)=\hat{\underline{x}}(t)} = \begin{bmatrix} \underline{\underline{0}}_{3\times 3} & \frac{\partial \dot{\underline{v}}^t}{\partial \underline{r}^t} & \frac{\partial \dot{\underline{v}}^t}{\partial \underline{q}_t^b} \\ \underline{\underline{I}}_{3\times 3} & \underline{\underline{0}}_{3\times 3} & \underline{\underline{0}}_{3\times 4} \\ \underline{\underline{0}}_{4\times 3} & \underline{\underline{0}}_{4\times 3} & \frac{\partial \dot{\underline{q}}_t^b}{\partial \underline{q}_t^b} \end{bmatrix}$
3,4,5	$\underline{\underline{F}}(\hat{\underline{x}}(t), \underline{u}(t), t) = \left.\frac{\partial \underline{f}(\underline{x}(t),\underline{u}(t),t)}{\partial \underline{x}(t)}\right\vert_{\underline{x}(t)=\hat{\underline{x}}(t)} = \begin{bmatrix} \underline{\underline{0}}_{3\times 3} & \frac{\partial \dot{\underline{v}}^t}{\partial \underline{r}^t} & \frac{\partial \dot{\underline{v}}^t}{\partial \underline{q}_t^b} & \underline{\underline{0}}_{3\times 1} \\ \underline{\underline{I}}_{3\times 3} & \underline{\underline{0}}_{3\times 3} & \underline{\underline{0}}_{3\times 4} & \underline{\underline{0}}_{3\times 1} \\ \underline{\underline{0}}_{4\times 3} & \underline{\underline{0}}_{4\times 3} & \frac{\partial \dot{\underline{q}}_t^b}{\partial \underline{q}_t^b} & \underline{\underline{0}}_{4\times 1} \\ \underline{\underline{0}}_{1\times 3} & \underline{\underline{0}}_{1\times 3} & \underline{\underline{0}}_{1\times 4} & \underline{\underline{0}}_{1\times 1} \end{bmatrix}$

Case	States function
1,2	$\underline{f}(\underline{x}, \underline{u}, t) = \begin{bmatrix} \dot{\underline{v}}^{t\,T} & \dot{\underline{r}}^{t\,T} & \dot{\underline{q}}_t^{b\,T} \end{bmatrix}^T$
3,4,5	$\underline{f}(\underline{x}, \underline{u}, t) = \begin{bmatrix} \dot{\underline{v}}^{t\,T} & \dot{\underline{r}}^{t\,T} & \dot{\underline{q}}_t^{b\,T} & \dot{\delta}_{Rec\_clock} \end{bmatrix}^T$

SIMULATION DESCRIPTION – SYSTEM OBSERVATION

Case	$\underline{\underline{H}}$ matrix
1	$\underline{\underline{H}}_k(\hat{\underline{x}}_k(-)) = \frac{\partial \underline{h}_k(\underline{x}(t_k))}{\partial \underline{x}(t_k)}\Big\vert_{\underline{x}(t_k)=\hat{\underline{x}}_k(-)} = \left[\underline{\underline{0}}_{3\times 3} \quad \frac{\partial \underline{r}^e_{GPS}}{\partial \underline{r}^i} \quad \underline{\underline{0}}_{3\times 4}\right]$
2,3,4,5	$\underline{\underline{H}}_k(\hat{\underline{x}}_k(-)) = \frac{\partial \underline{h}_k(\underline{x}(t_k))}{\partial \underline{x}(t_k)}\Big\vert_{\underline{x}(t_k)=\hat{\underline{x}}_k(-)} = \begin{bmatrix} \underline{\underline{0}}_{1\times 3} & \frac{\partial PR_1}{\partial \underline{r}^i}{}_{1\times 3} & \underline{\underline{0}}_{1\times 4} & \frac{\partial PR_1}{\partial \delta_{Rec\_clock}} \\ \underline{\underline{0}}_{1\times 3} & \frac{\partial PR_2}{\partial \underline{r}^i}{}_{1\times 3} & \underline{\underline{0}}_{1\times 4} & \frac{\partial PR_2}{\partial \delta_{Rec\_clock}} \\ \underline{\underline{0}}_{1\times 3} & \frac{\partial PR_3}{\partial \underline{r}^i}{}_{1\times 3} & \underline{\underline{0}}_{1\times 4} & \frac{\partial PR_3}{\partial \delta_{Rec\_clock}} \\ \underline{\underline{0}}_{1\times 3} & \frac{\partial PR_4}{\partial \underline{r}^i}{}_{1\times 3} & \underline{\underline{0}}_{1\times 4} & \frac{\partial PR_4}{\partial \delta_{Rec\_clock}} \end{bmatrix}$

Case	Clock bias rate
3	$\dot{\delta}_{Rec\_clock} = 0$
4,5	$\dot{\delta}_{Rec\_clock} = 10^{-7}[s]$

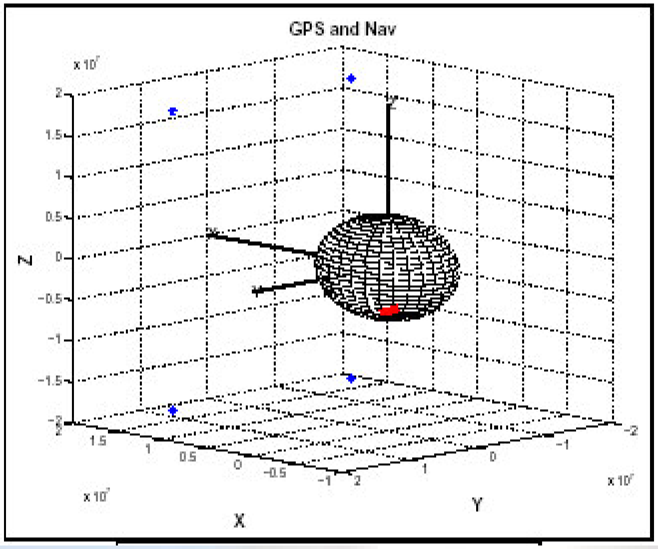


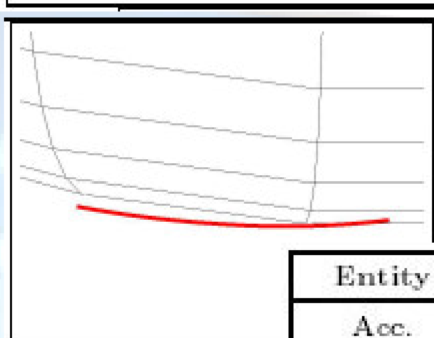
$$PR_k = \left|\underline{r}^i - \underline{r}^i_{Satk}\right| + \Delta r_{clock\_bias}, \quad k = 1, .., 4$$

Case	Clock bias
3	$\delta_{Rec\_clock} = \delta_{Rec\_clock\_0}$
4,5	$\delta_{Rec\_clock} = \delta_{Rec\_clock\_0} + \dot{\delta}_{Rec\_clock} \cdot t$

Case	Measurements function
1	$\underline{h}_k(\underline{x}(t_k)) = \underline{r}^e_{GPS}$
2,3,4,5	$\underline{h}_k(\underline{x}(t_k)) = \left[PR_1 \quad PR_2 \quad PR_3 \quad PR_4\right]^T$

SIMULATION CONDITIONS





System Requirements

Entity	3σ	Reference
Acc.	$0.1\ m/s^2$	Body
Ang. rate	$0.2^\circ/s$	Body
Altit.	$2\ m$	NED
Ground Pos.	$10\ m$	NED
Vel.	$0.2\ m/s$	ECI
Att.	0.3°	ECI

- Inputs noise standard deviations for $\underline{N}_U$ diagonal covariance matrix:
  - $\sigma_{\underline{a}^b} = [\sigma_{Acc1} \quad \sigma_{Acc2} \quad \sigma_{Acc3}]^T \ [m/s^2]$
  - $\sigma_{\underline{\omega}^b_{i,b}} = [\sigma_{Gyro1} \quad \sigma_{Gyro2} \quad \sigma_{Gyro3}]^T \ [rad/s]$
- Process noise standard deviations for $\underline{N}_P$ diagonal covariance matrix:
  - $\sigma_{\underline{\dot{v}}^i} = [0 \quad 0 \quad 0]^T \ [m/s^2]$
  - $\sigma_{\underline{\dot{r}}^i} = [0 \quad 0 \quad 0]^T \ [m/s]$
  - $\sigma_{\underline{\dot{q}}^b_i} = [0 \quad 0 \quad 0 \quad 0]^T$
  - $\sigma_{\dot{\delta}_{Rec\_clock}} = 0$

Case	Measurements σ
1	$\sigma_{\underline{r}^e_{GPS}} = [\sigma_{GPS} \quad \sigma_{GPS} \quad \sigma_{GPS}]^T \ [m]$
2,3,4,5	$\sigma_{\underline{PR}} = \begin{bmatrix} \sigma_{GPS} \\ \sigma_{GPS} \\ \sigma_{GPS} \\ \sigma_{GPS} \end{bmatrix} [m]$

Sensor Noises

Entity	Sensor	Noise σ	Hz	Frame
Acc.	IMU	$5 \cdot 10^{-3} m/s^2$	100	Body
Ang. rate	IMU	$5 \cdot 10^{-8} rad/s$	100	Body
Pos.	GPS	$3m$	10	ECEF

ESTIMATOR DESCRIPTION

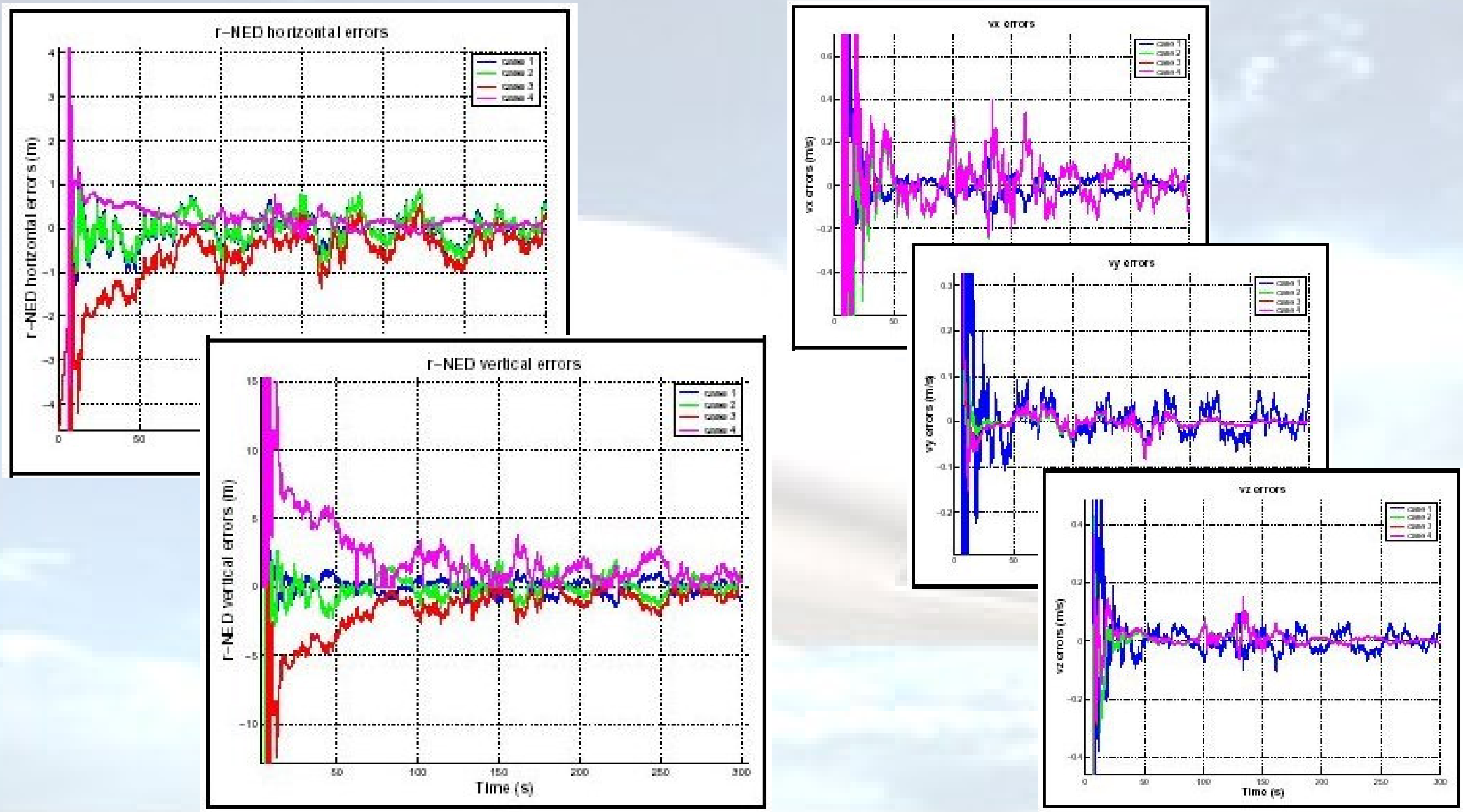
System Model Measurement Model Assumptions	$\dot{\underline{x}}(t) = \underline{f}(\underline{x}(t), \underline{u}(t), t) + \underline{\underline{G}} \cdot \underline{w}(t)$ $\underline{z}_k = \underline{h}_k(\underline{x}(t_k)) + \underline{v}_k, \quad k = 1, 2, \ldots$ $\underline{w}(t) \sim N(\underline{0}, \underline{\underline{Q}}(t)), \quad \underline{v}_k \sim N(\underline{0}, \underline{\underline{R}}_k), \quad E[\underline{w}(t), \underline{v}_k^T] = 0$
Initial Conditions	$\hat{\underline{x}}(0) \sim N(\hat{\underline{x}}_0, \underline{\underline{P}}_0)$
State Estimate Propagation State Covariance Propagation	$\dot{\hat{\underline{x}}}(t) = \underline{f}(\hat{\underline{x}}(t), \underline{u}(t), t)$ $\dot{\underline{\underline{P}}}(t) = \underline{\underline{F}}(\hat{\underline{x}}(t), \underline{u}(t), t)\underline{\underline{P}}(t) + \underline{\underline{P}}(t)\underline{\underline{F}}^T(\hat{\underline{x}}(t), \underline{u}(t), t) + \underline{\underline{G}}.\underline{\underline{Q}}(t) \cdot \underline{\underline{G}}^T$
State Estimate Update State Covariance Update (Joseph form) Gain Matrix	$\hat{\underline{x}}_k^{+} = \hat{\underline{x}}_k^{-} + \underline{\underline{K}}_k \left[\underline{z}_k - \underline{h}_k(\hat{\underline{x}}_k^{-})\right]$ $\underline{\underline{P}}_k^{+} = \left[\underline{\underline{I}} - \underline{\underline{K}}_k \cdot \underline{\underline{H}}_k(\hat{\underline{x}}_k^{-})\right] \underline{\underline{P}}_k^{-} \cdot \left[\underline{\underline{I}} - \underline{\underline{K}}_k \cdot \underline{\underline{H}}_k(\hat{\underline{x}}_k^{-})\right]^T + \underline{\underline{K}}_k \cdot \underline{\underline{R}} \cdot \underline{\underline{K}}_k^T$ $\underline{\underline{K}}_k = \underline{\underline{P}}_k^{-} \cdot \underline{\underline{H}}_k^T(\hat{\underline{x}}_k^{-}) \cdot \left[\underline{\underline{H}}_k(\hat{\underline{x}}_k^{-}) \cdot \underline{\underline{P}}_k^{-} \cdot \underline{\underline{H}}_k^T(\hat{\underline{x}}_k^{-}) + \underline{\underline{R}}_k\right]^{-1}$
Linearization Definitions	$\underline{\underline{H}}_k(\hat{\underline{x}}_k^{-}) \equiv \left.\frac{\partial \underline{h}_k(\underline{x}(t_k))}{\partial \underline{x}(t_k)}\right\vert_{\underline{x}(t)=\hat{\underline{x}}_k^{-}}$ $\underline{\underline{F}}(\hat{\underline{x}}^{-}, \underline{u}(t), t) \equiv \left.\frac{\partial \underline{f}(\underline{x}(t), \underline{u}(t), t)}{\partial \underline{x}(t)}\right\vert_{\underline{x}(t)=\hat{\underline{x}}^{-}}$

ESTIMATION CONDITIONS

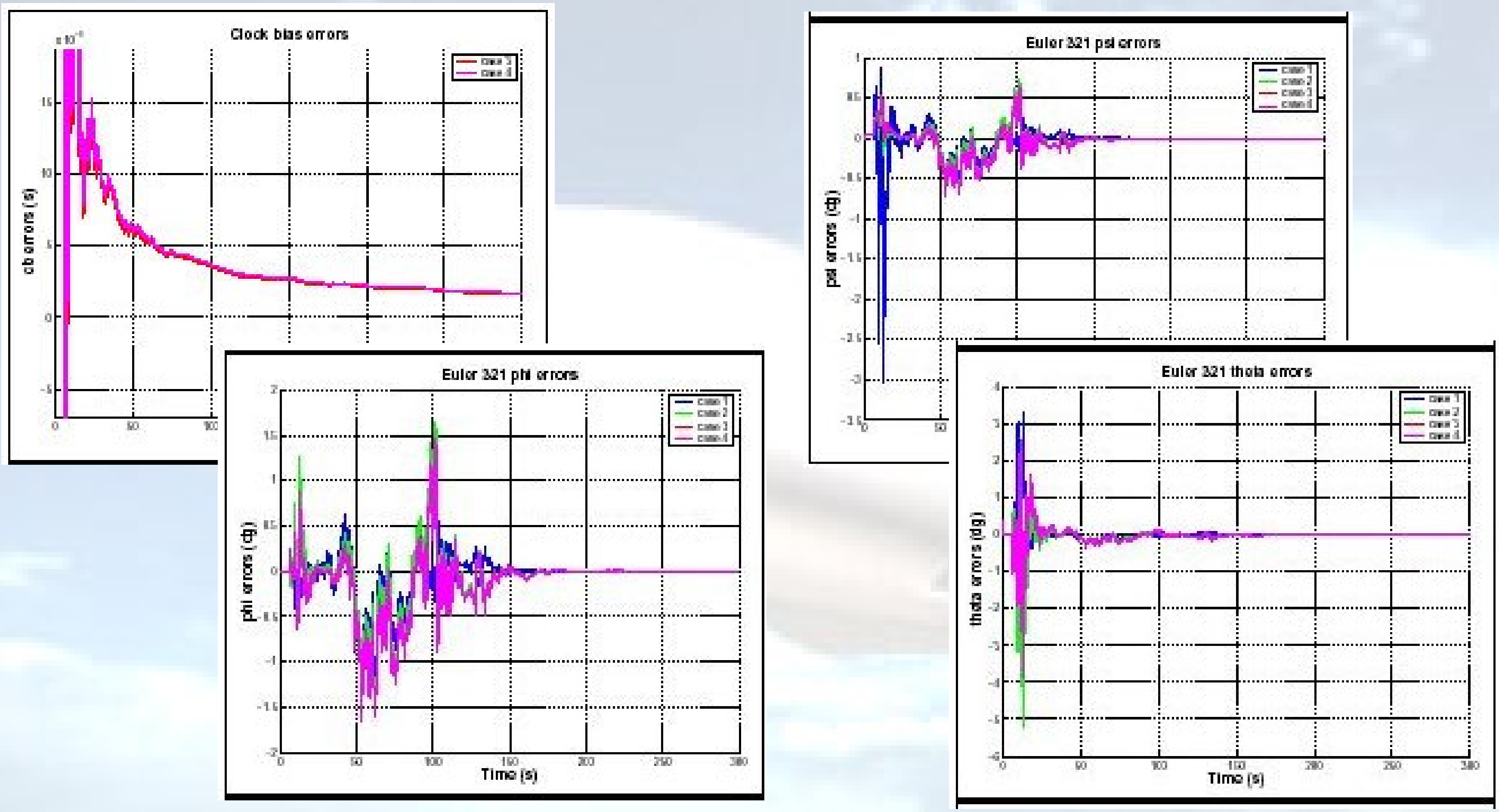
- $\sigma_{\underline{\hat{v}}^i} = \begin{bmatrix} \sigma_{Acc1} & \sigma_{Acc2} & \sigma_{Acc3} \end{bmatrix}^T \quad [m/s^2]$
- $\sigma_{\underline{\dot{\hat{r}}}^i} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad [m/s]$
- $\sigma_{\underline{\dot{\hat{q}}}_i^b} = \begin{bmatrix} \sigma_{Gyros}/2 \\ \sigma_{Gyros}/2 \\ \sigma_{Gyros}/2 \\ \sigma_{Gyros}/2 \end{bmatrix}$
- $\sigma_{\dot{\hat{\delta}}_{Rec\_clock}} = 0$

Case	Measurements σ
1	$\sigma_{\underline{\hat{r}}^e_{GPS}} = \begin{bmatrix} \sigma_{GPS}/3 \\ \sigma_{GPS}/3 \\ \sigma_{GPS}/3 \end{bmatrix} \quad [m]$
2,3,4,5	$\sigma_{\underline{\hat{PR}}} = \begin{bmatrix} \sigma_{GPS}/3 \\ \sigma_{GPS}/3 \\ \sigma_{GPS}/3 \\ \sigma_{GPS}/3 \end{bmatrix} \quad [m]$

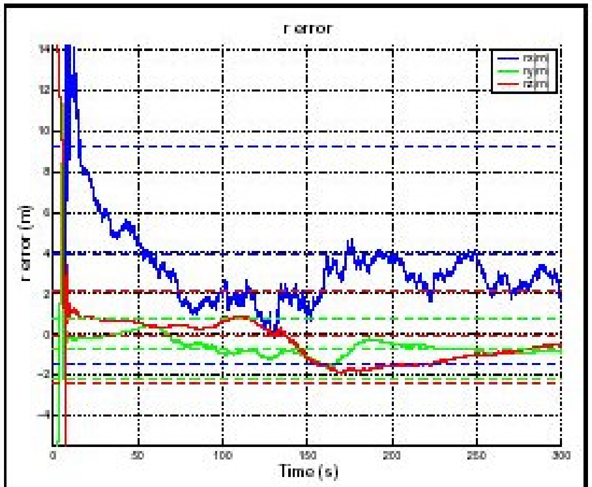
ESTIMATION RESULTS – CASES 1 – 4 – Position and Velocity

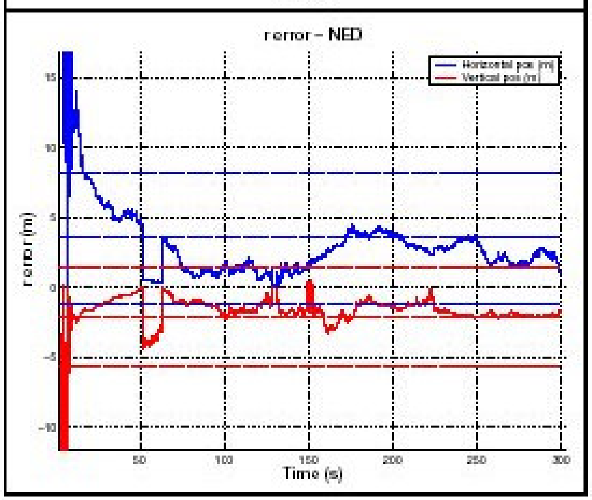


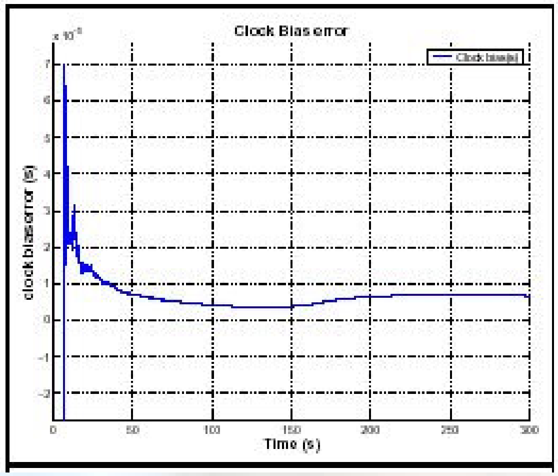
ESTIMATION RESULTS – CASES 1 – 4 – Clock bias and Attitude

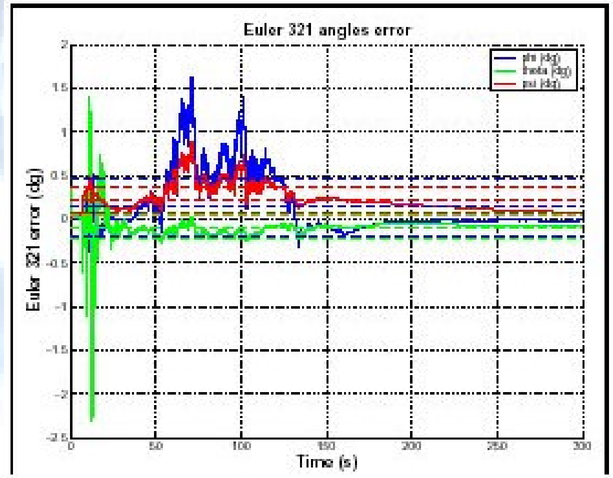


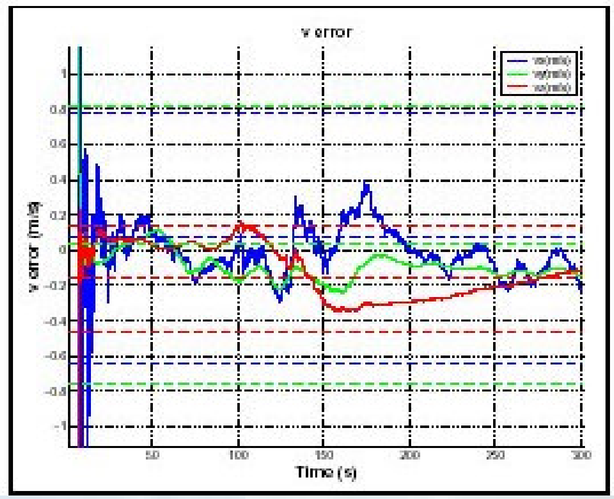
ESTIMATION RESULTS – CASE 5 – Position, Velocity, Clock bias and Attitude











CONCLUSIONS

- Case 1 (Position) X Cases 2-5 (Pseudoranges) – Case 1 worst
  - ✓ Use of Pseudoranges improves Estimation for poor satellite constellation geometry
- Cases 2-5 (Pseudoranges) - Similar behavior
- Case 4 (only clock bias) X Case 5 (various biases) - Similar behavior
- Case 4
  - ✓ Position requirements accomplished
  - ✓ Vertical errors better than Horizontal ones.
- Case 5
  - ✓ Biases estimation could solve the problem.
  - ✓ Sensibility to attitude maneuvers.
- Poor geometry of GPS constellation
  - ✓ Disadvantage in one specific direction
  - ✓ Sensibility to attitude maneuvers.
- Estimation in Attitude maneuvers - better geometry of GPS constellation configurations required.

Thank you!!...