Phase-coherence threshold and vortex-glass state in diluted Josephson-junction arrays in a magnetic field

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We study numerically the interplay of phase coherence and vortex-glass state in two-dimensional Josephson-junction arrays with average rational values of flux quantum per plaquette f and random dilution of junctions. For $f=1/2$, we find evidence of a phase-coherence threshold value x_s, below the percolation concentration of diluted junctions x_p, where the superconducting transition vanishes. For $x_s<x<x_p$ the array behaves as a zero-temperature vortex glass with nonzero linear resistance at finite temperatures. The zero-temperature critical currents are insensitive to variations in f in the vortex glass region while they are strongly f dependent in the phase-coherent region.

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The study of the structure of vortex-lattice states in disordered superconductors in a magnetic field has attracted much recent interest. In three dimensions a true vortex-glass transition[1] at finite temperature is possible for strong disorder in unscreened superconductors,[2] while for weak disorder a Bragg-glass phase with quasi-long-range order has been proposed.[3] In two-dimensions, however, vortex-glass models[4,5] and experiments on superconducting films[6] show that vortex-glass order and phase coherence are destroyed at any finite temperature with a nonzero but exponentially small resistivity in the large disorder limit whereas in the weak disorder limit the situation is less clear and may depend on the particular model of disordered superconductor. Random diluted Josephson-junction arrays have been used to model disordered superconductors[7–11] and can also be fabricated with controlled amount of disorder in two dimensions.[12–14] For a regular array in perpendicular magnetic field with a rational flux quanta per cell f, the ground sate consists of a periodic pinned vortex lattice, with additional discrete symmetries resulting from commensurability effects,[15,16] and phase coherence and vortex order is possible. Thus, diluted arrays in a magnetic field can provide a convenient experimental model system to investigate the effects of *weak* and *strong* disorder on initially pinned vortex lattices and the interplay of phase coherence and vortex glass states in two dimensions. In particular, in order to understand transport properties near percolation threshold in recent experiments[14] on diluted arrays in a magnetic field, it is important to know if disorder and temperature fluctuations can destroy phase coherence at long length scales and the nature of vortex order in this regime. A recent study[17] of a model of random Josephson-junction arrays with a particular type of disorder (positional disorder)[18] in a magnetic field suggests that no transition is possible even for weak disorder in the thermodynamic limit but it is not clear if this scenario would apply in general. In fact, random dilution does not explicitly introduces random phase shifts across the junction unlike positional disorder. In addition, an earlier study of the ground-state stability of a diluted array shows[11] that, in presence of a magnetic field with an average rational value of flux quantum per plaquette f, phase coherence is possible at nonzero temperatures and the transition temperature only vanishes, for increasing dilution of junctions x, at a critical value $x_s(f)$ below the geometric percolation threshold x_p where the transition would vanish in the absence of the magnetic field. A vortex-glass phase with zero-temperature transition should also appear at a critical concentration $x_v \gtrsim x_s$, with nonzero resistivity at finite temperatures but having a diverging short-range correlation length $\xi \propto T^{-\nu}$ which is expected to determine the nonlinear behavior of the current voltage characteristics. An upper bound for the phase-coherence region is set by the behavior at $f=1/2$ since higher order rational values are expected to be much less stable with a corresponding threshold value which may be too small to detect numerically. These results rely on the finite-size behavior of defect energy in the ground state which are inaccessible experimentally. However, experiments often measure transport properties and it is of great interest to know how these effects could show up in the behavior of the current-voltage characteristics.

In this work, we present the results of extensive dynamical simulations of the current-voltage characteristics of resistively shunted Josephson-junction arrays with an average flux quantum per plaquette f and random dilution of junctions. We find evidence of a phase coherence threshold value $x_s<x_p$ as indicated in Fig. 1. For $x<x_s$, the superconducting transition occurs at finite temperatures while for $x_s<x<x_p$ the array behaves as a zero-temperature vortex glass with nonzero and thermally activated linear resistance at finite temperatures and diverging short-range correlation length $\xi \propto T^{-\nu}$. A current-voltage scaling analysis provides an estimate of $\nu \sim 2$. In the vortex-glass region, the zero-temperature critical currents are roughly insensitive to changes in f.

We consider a two dimensional array of superconducting grains coupled to its nearest neighbors by resistively shunted Josephson junctions and with current conservation at each

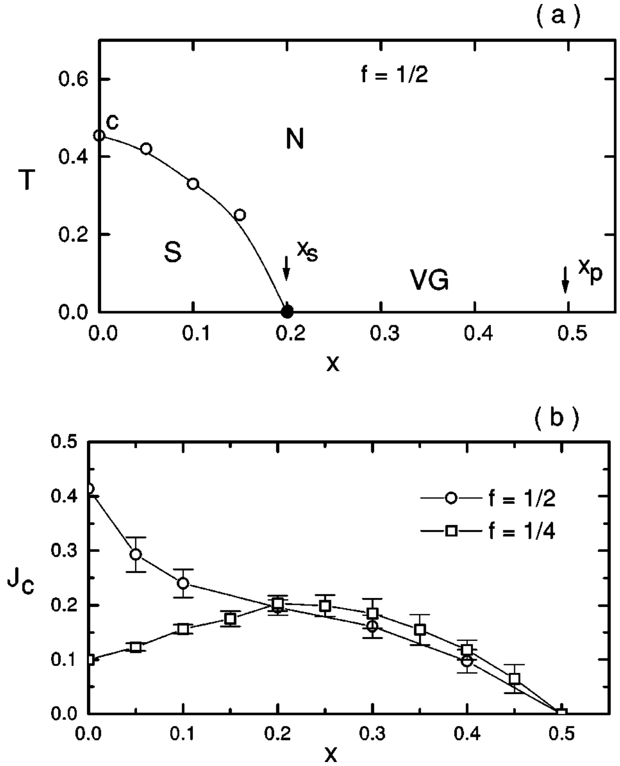

FIG. 1. (a) Phase diagram of a diluted Josephson-junction array as a function of temperature T and concentration x of diluted junctions, for an average rational frustration $f=1/2$. The superconducting phase is denoted by S, the normal phase by N, and the short-range vortex glass state by VG. The geometrical percolation threshold is indicated by x_p and the phase-coherence threshold by x_s. (b) Critical current densities J_c as a function of dilution x for different values of frustration f. Critical temperatures (open circles) in (a) were obtained from current-voltage scaling analysis and the phase coherence threshold x_s (filled circle) was inferred from the change in the behavior of $J_c(f)$ in (b).

site.[19] The equations of motion for the phases θ_i of the superconducting order parameter located at site i of the lattice can be written as[10,20]

$$\frac{\hbar}{2eR_o}\sum_j (\dot{\theta}_i-\dot{\theta}_j) = -\sum_j [I_{ij}\sin(\theta_i-\theta_j-A_{ij})+\eta_{ij}], \tag{1}$$

where R_o is a uniform shunt resistance, $\eta_{ij}(t)$ is a thermal noise with correlations $\langle \eta_{ij}(t)\eta_{kl}(t')\rangle = 2k_BT/R_o\delta_{ij,kl}\delta(t-t')$ and I_{ij} is the junction critical current. The bond variables A_{ij} correspond to the line integral of the vector potential and are constrained to $\Sigma_{ij}A_{ij}=2\pi f$, about each elementary plaquette of the reference (undiluted) lattice. For simplicity we consider a square lattice array and bond dilution of junctions. The qualitative behavior and critical exponents presented below should remain the same for other choices of dilution and for triangular arrays. Dilution of junctions is introduced by taking $I_{ij}=0$ with probability x and $I_{ij}=I_o$, a constant, with probability $1-x$. Dimensionless quantities are used with time in units of $\tau=\hbar/2eR_oJ_o$, current in units of I_o, voltages in units of R_oI_o and temperature in units of $\hbar I_o/2ek_B$. A total current I is imposed uniformly in the array using periodic boundary conditions[20] with current density $J=I/L$, where L is the system size and the average electric field E is obtained from the voltage V across the system as $E=V/L=(\hbar/2e)\langle\dot{\theta}_i-\dot{\theta}_j\rangle$. We use periodic boundary conditions in order to eliminate possible edge contributions to the resistance due to diluted junctions near the boundary which could arise from open boundary conditions.[21] System sizes ranging from $L=8$ to $L=128$ were used in the calculations with a time step $\Delta t=0.07\tau$ and the results averaged over 10 to 500 random diluted configurations of junctions depending on the system size.

Figure 1(b) shows the behavior of the critical current density J_c where a nonzero voltage appears at zero temperature. At low values of x, the behavior of J_c strongly depends on the rational frustration $f=p/q$, as indicated for $f=1/2$ and $f=1/4$ in the figure, but becomes roughly insensitive to f for dilutions larger than a critical value much below the percolation threshold x_p. This is consistent with the proposed vortex glass phase[11] for the range $x_v(f)<x<x_p$ where x_v is a dilution threshold below which vortex-lattice order remains. In the undiluted case and for small enough x the ground-state energy and critical currents correlate with the ordering of $q\times q$ unit cells and so are very sensitive to the q value.[16] However, for $x>x_v$, vortex-lattice order is completely destroyed at long-length scales and its stability and therefore the critical current should be less sensitive to q. Since it is expected that[11] $x_v(1/4)<x_v(1/2)$ and $x_v\gtrsim x_s$, the change in the behavior of J_c in Fig. 1(b) allows for a very rough estimate of the phase-coherence threshold for $f=1/2$ as the value of dilution where the two curves overlap within the estimated errorbars, $x_s(1/2)\sim 0.20(5)$.

We turn now to thermal fluctuation effects. Figure 2(a) shows the temperature dependence of the nonlinear resistivity E/J at a value of dilution $x=0.1$ below the phase-coherence threshold x_s estimated above, for the largest systems sizes $L=64$ and $L=128$. As can be seen from the figure, the linear resistivity $R_L=\lim_{J\to 0}E/J$, estimated from the ratio E/J when $J\to 0$, tends to a finite value at high temperatures but extrapolates to very low values at lower temperatures, independent of system size, consistent with the existence of a finite temperature superconducting transition in the range $T_c=0.3$ to 0.4. This is confirmed by a scaling analysis of the nonlinear resistivity according to which[4] measurable quantities scale with the diverging correlation length ξ near the transition temperature. If the transition occurs at a finite temperature, the relaxation time diverges as ξ^z, where z is the dynamical critical exponent, and the nonlinear resistivity satisfy the scaling form

$$T\frac{E}{J} = \xi^{-z} g_{\pm}\left(\frac{J}{T}\xi\right) \tag{2}$$

in two dimensions, where the $+$ and $-$ correspond to the behavior above and below the transition, respectively. For a transition in the Kosterlitz-Thouless (KT) universality class, the correlation length should diverge exponentially as $\xi \propto \exp(b/|T/T_c-1|^{1/2})$, while for a conventional transition a

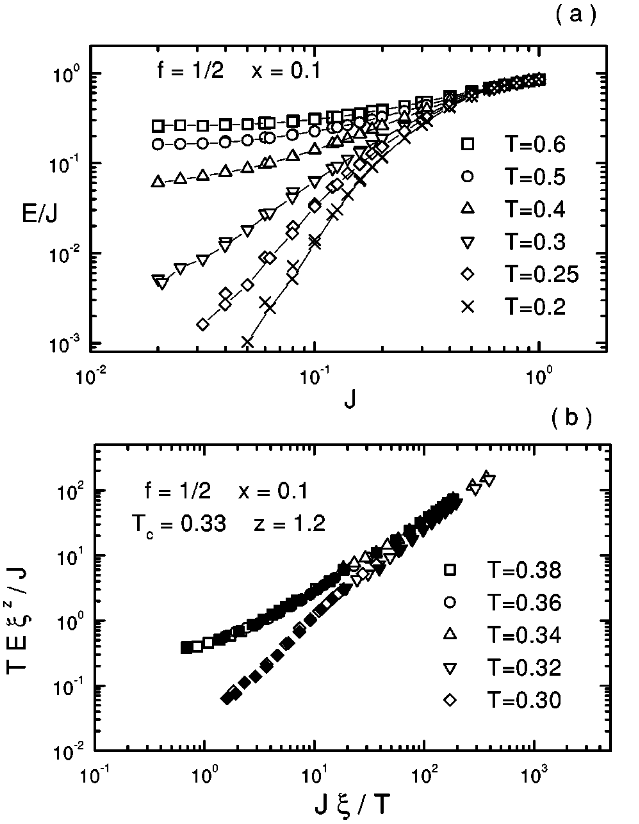

FIG. 2. (a) Nonlinear resistivity E/J as a function of temperature for a dilution $x=0.1$ below the phase-coherence threshold x_s and system sizes $L=64$ and $L=128$ (symbols connected by lines). (b) Scaling plot of the data [not indicated in (a)] for the smallest range near T_c and smallest current densities. Open symbols correspond to $L=64$ and filled ones to $L=128$.

power-law behavior is expected $\xi \propto |T/T_c - 1|^{-\nu}$, with an exponent ν depending on the discrete symmetry of the pinned vortex lattice. A scaling plot according to Eq. (2) can be used to verify the scaling arguments and the assumption of finite-temperature equilibrium transition. This is shown in Fig. 2(b), in the temperature range closest to the apparent T_c and smallest current densities, assuming the correlation length ξ has an exponential divergence as in the KT universality class and using b, T_c, and z as adjustable parameters so that the best data collapse is obtained. As shown in the Fig. 2(b), the two largest system sizes $L=64$ and $L=128$ give the same data collapse and so finite-size effects, ignored in the scaling form of Eq. (2), are not dominant for this range of temperatures and current densities. We estimate a transition temperature $T_c=0.33(2)$ and dynamical exponent $z=1.2(2)$. Although this estimate is based on a scaling analysis of the nonlinear current-voltage characteristics, which is a nonequilibrium property, we find that the finite-size behavior of the linear resistance at T_c is consistent with this analysis. In a finite system the divergent correlation length ξ is cut off by the system size L at the transition. From Eq. (2), the linear resistance at T_c should then scale as $R_L \propto L^{-z}$. The linear resistance can be obtained from the Kubo formula of equilibrium voltage fluctuations as $R_L=(1/2T)\int dt \langle V(t)V(0)\rangle$, without finite current effects, and can

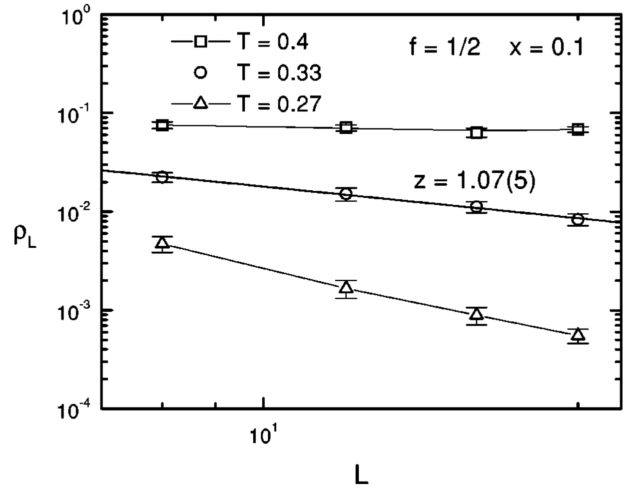

FIG. 3. Linear resistance as a function of system size L for different temperatures at $x=0.1$. A power-law fit at $T_c=0.33$ gives an estimate of the dynamical exponent z.

also be determined from the long-time fluctuations of the phase difference across the system.[5] Figure 3, shows the finite-size behavior of R_L at different temperatures. Above our estimate of T_c it remains finite for increasing L whereas below T_c it appears to extrapolate to zero. Right at T_c, a power-law fit gives $z=1.07(5)$ which is consistent with the estimate from the current-voltage scaling and suggests therefore that the transition corresponds to the underlying equilibrium behavior. It should be noted that for the pure KT transition a dynamical exponent $z=2$ is expected, independent of the particular dynamics. Indeed, for $f=0$ and $x=0$ the same power-law fit gives $z=2.0(1)$ at the critical temperature. However, for $f=1/2$, where an additional Ising order parameter is present, it is found that even for the undiluted system $z<2$ using the present dynamics.[22] We also note that attempting a scaling plot using the conventional power-law correlation length gives a very large value for ν which suggests that the exponential form is the appropriate one. However at $x=0$, both forms of correlation length gives reasonable data collapse as expected for $f=1/2$ from the single transition scenario where the superconducting transition and vortex-lattice disordering transition occurs at the same temperature or else at very close temperatures.[16,23–25] Since the undiluted array at $f=1/2$ is expected to have a transition combining the KT and Ising universalities, our results suggest that for $0 \ll x < x_s$, the superconducting transition is in the KT universality class (statics) while the vortex-lattice disordering transition of Ising symmetry may occur separately in presence of weak disorder.[11] However, our above scaling analysis based on the diverging phase-coherence correlation length does not allow us a determination of the vortex-lattice disordering transition since it is expected to occur within the normal phase.[11] Using the above scaling analysis, the transition temperatures for the different dilutions can be obtained as in Fig. 1(a). For values of $x \neq 0.1$ limited data was used and the results are only rough estimates of the transition temperatures. Nevertheless, the criti-

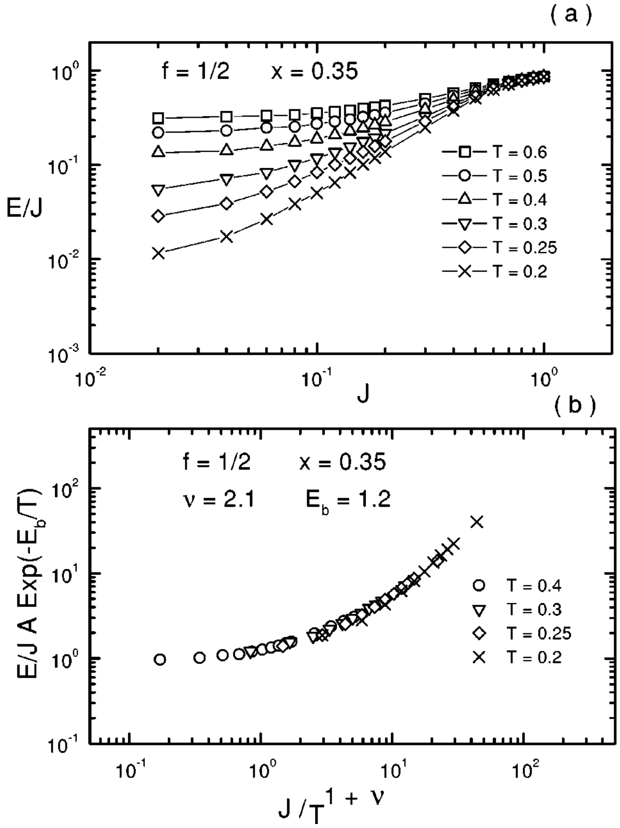

FIG. 4. (a) Nonlinear resistivity E/J as a function of temperature for a dilution $x=0.35$ above the phase-coherence threshold x_s. (b) Scaling plot of the data in (a) for the lowest temperatures and current densities according to a $T=0$ transition.

cal temperature as a function of dilution reasonably extrapolates to the threshold x_s estimated from the behavior of J_c in Fig. 1(b) as discussed above.

In contrast, for a dilution above the phase-coherence threshold $x>x_s$, the linear resistance R_L is finite for all temperatures in the same range as indicated in Fig. 4(a). Although we can not exclude a transition at much lower temperatures based on these data, the behavior is consistent with a superconducting transition and vortex order occurring only at zero temperature as for a vortex glass with a zero-temperature transition.[4,5] This is consistent with defect energy calculations which show that low-energy excitations above the ground state decreases with system size in this range of dilutions.[11] In fact, R_L decreases rapidly with decreasing temperature and for increasing J there is a smooth crossover to nonlinear behavior at a critical current J_{nl} which also decreases with decreasing temperature. From the well-known scaling arguments[4] leading to Eq. (2), if the transition happens only at zero temperature then $\xi \propto T^{-\nu}$ and since the current density scale as $J \propto kT/\xi$, the crossover to nonlinear behavior sets in at $J_{nl} \propto T^{1+\nu}$ which depends strongly on the yet unknown critical exponent ν. Also, the linear resistivity R_L is finite at any nonzero temperature but thermally activated, $R_L \propto \exp(-E_b/kT)$. Thus the relaxation time $\tau \propto 1/R_L$ diverges exponentially for decreasing temperatures. We can then consider the behavior of the dimensionless ratio E/JR_L which must satisfy the scaling form[4]

$$E/JR_L = g(J/T^{1+\nu}) \tag{3}$$

if the assumption of a zero-temperature transition is correct. In Fig. 4(b) we show the scaling plot according to Eq. (3) for the lowest temperatures and current densities which verifies the scaling assumption and provides an estimate of $\nu=2.1(2)$ and an energy barrier $E_b=1.2$. This value of ν is consistent with the estimate ~ 1.9 based on the previous finite-size scaling of defect energy in the ground state.[11] Similar analysis at different dilution $x=0.45$ gives $\nu=2.2(2)$ and $E_b=0.9$ and at different frustration $f=1/4$ gives $\nu=2.3(2)$ and $E_b=1.45$. We note that our estimate of $\nu \sim 2$ is roughly the same as the value obtained for the gauge-glass model of strongly disordered two-dimensional superconductors[4,5] which may suggest a common universality class. However, it should be noted that, for $f=1/2$, the system has a global reflection symmetry ($\theta_i \to -\theta_i$) in addition to the rotational symmetry[26] and one would expect, similarly to the XY (chiral) spin glass[27] which shares the same feature, two different divergent correlation lengths ξ_s and ξ_c with corresponding distinct exponents ν_s and ν_c. In fact, for the chiral glass model a different universality class with $\nu_s \sim 1$ has been found from a current-voltage scaling analysis.[5] On the other hand, an analytic study[28] of the XY spin glass for a particular distribution of disorder find a common exponent. Our estimates suggest that this could also be the case for the present percolative type of disorder or else the exponents are too close to be resolved within the accuracy of our estimate. An apparent common universality class of vortex glass models with clearly distinct symmetries has also been found in three dimensions.[2] In addition, close to the percolation threshold x_p, the above scaling analysis based on a single diverging length scale is not valid, as one must also take into account the percolation correlation length ξ_p and the fractal nature of the system at smaller length scales.[13,14]

In summary, we have studied the interplay of phase coherence and vortex-glass state in two-dimensional diluted Josephson-junction arrays with average rational values of frustration. For $f=1/2$, we found evidence of a phase coherence threshold value x_s much below the geometric percolation threshold x_p. This is in contrast with the conclusions of Gupta and Teitel[17] for a Josephson-junction array with positional disorder where no phase coherence is expected at finite temperatures even for small disorder at length scales much larger than a disorder dependent length. Further work is required to verify whether the present study only reflects the finite-length scale of the system sizes used in the calculation or is a consequence of different type of disorder. In addition, since $f=1/2$ has a particular reflection symmetry, which is preserved in presence of random dilution, the behavior for other values of f could be qualitatively different. On the other hand, experiments[12] are often done on systems sizes comparable to our largest system size and thus the current-voltage scaling behavior discussed here should be observable. In the range $x_s<x<x_p$ the array behaves as a zero-temperature vortex glass with activated nonzero linear resistance at finite temperatures and critical currents much less sensitive to variations in f than in the phase-coherent region. Our results

suggest that the phase coherence threshold can be identified experimentally as the change in the transport properties from the weak to the strong disorder regime. However, the numerical estimates of critical quantities from the current-voltage scaling analysis should be regarded as rough magnitudes which can be measured experimentally and used to verify the prediction of a change in behavior of the transport properties in the different phases. Equilibrium simulations are required to confirm the observed critical behavior and obtain more accurate estimates of the critical exponents.

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