

LETTER TO THE EDITOR

Domain-wall-induced XY disorder in the fully frustrated XY model

Enzo Granato

Ministério da Ciência e Tecnologia, Instituto de Pesquisas Espaciais,
Laboratório Associado de Sensores e Materiais, Caixa Postal 515,
12201-São José dos Campos, SP, Brazil

Received 11 February 1987

Abstract. The defect-mediated phase transition in the fully frustrated XY model is discussed. A Migdal-Kadanoff position-space renormalisation group analysis is employed to investigate the critical behaviour of a similar model in the same universality class as the fully frustrated XY model. The resulting phase diagram shows that XY order cannot coexist with Ising disorder. This is in agreement with recently suggested phase transition scenario in the fully frustrated XY model.

The fully frustrated XY model in two dimensions has received considerable attention recently mainly because of its relevance for Josephson-junction arrays in a transverse magnetic field with half a flux quantum per plaquette (Teitel and Jayaprakash 1983) in addition to its applications to magnetic systems (Villain 1975) as well as $^3\text{He-A}$ films (Halsey 1985). The model is described by the following Hamiltonian:

$$H = \sum_{\langle rr' \rangle} J_{rr'} \cos(\theta_r - \theta_{r'}) \quad (1)$$

where the sum is restricted to nearest-neighbour sites of a two-dimensional lattice. On a square lattice, the bonds can be chosen to be ferromagnetic $J_{rr'} = -J$ ($J > 0$) on horizontal rows and ferromagnetic and antiferromagnetic on alternating columns of the lattice. In the triangular lattice it is sufficient to take all bonds as antiferromagnetic. The model is fully frustrated because the product of the bonds in each plaquette of the lattice is always less than zero (Villain 1975).

The ground state of this model possesses a discrete Z_2 symmetry in addition to the underlying $U(1)$ symmetry. So one expects an Ising-like as well as an XY-like transition in the system. Recent studies of this model have raised some controversy about the nature of the phase transition. While some numerical works (Teitel and Jayaprakash 1983, Berge *et al* 1986, Lee *et al* 1986, van Himbergen 1986) seem to indicate a single transition of Ising type, renormalisation group analyses have suggested the possibility of an Ising-like transition (Yosefin and Domany 1985, Choi and Stroud 1985, Granato and Kosterlitz 1986a,b) or a weak first-order transition (Yosefin and Domany 1985, Granato and Kosterlitz 1986a,b). However, one should note that neither the numerical work nor the analytical work are conclusive.

In order to understand the nature of the phase transition in the system, one has to look into the role of the excitations above the ground state which consist of domain walls and vortices, and study the mechanism by which a phase transition can occur. Such an analysis has been carried out in the Coulomb-gas representation (Halsey 1985, Korshunov and Uimin 1986, Korshunov 1986) and also in a coupled XY model representation (Granato and Kosterlitz 1986a,b). For topological reasons, vortices of fractional charges are localised at domain wall bends and corners. Thus when the free energy of a domain wall goes to zero, free corner vortices are available, which presumably immediately unbind the remaining integer vortices excitations disrupting the XY ordered phase. The important point here is that because of this domain-wall-induced XY disorder, it is impossible for XY order to coexist with Ising disorder although the converse, XY disorder and Ising order, is still allowed (Halsey 1985). Which scenario is realised for a particular system will depend on the relative magnitude of the domain wall melting temperature T_m and the vortex unbinding temperature T_v . For the fully frustrated XY model, simple estimates give $T_m < T_v$ and so one should find a single transition from an Ising and XY ordered phase to an Ising and XY disordered phase. However, since in this system one cannot easily vary T_m or T_v , the above scenarios have not yet been confirmed.

Recently, Berge *et al* (1986) have investigated a generalised fully frustrated XY model on a square lattice, in which the strength of the antiferromagnetic bonds is altered by a factor η . By varying η they show that above a critical value $\eta_c = \frac{1}{2}$, the system exhibits two successive transitions with an Ising-like followed by an XY -like transition for increasing temperature. These two transitions seem to coincide when $\eta = 1$. This behaviour is apparently inconsistent with the scenarios mentioned above. However, due to the anisotropy introduced when $\eta \neq 1$, an additional symmetry in the ground state is removed (Halsey 1985), which will alter the mechanism for the transition. One can also understand this behaviour in terms of the phase diagram of coupled XY models (Granato and Kosterlitz 1986a,b).

In order to verify the picture of domain-wall-induced XY disorder, one should therefore vary the domain wall energy but keeping the original symmetry. In this Letter we study the effect of such modification in a model which is believed to be in the same universality class as the fully frustrated XY model (Choi and Doniach 1985, Yosefin and Domany 1985).

$$H = -J \sum_{\langle rr' \rangle} \cos(\theta_r - \theta_{r'}) (S_r S_{r'} + 1) - L \sum_{\langle rr' \rangle} S_r S_{r'} \quad (2)$$

where $S_r = \pm 1$. When $L = 0$, this corresponds to the original fully frustrated XY model. By varying L the domain wall energy is changed and both situations $T_m < T_v$ and $T_m > T_v$ can be realised.

We use a Migdal-Kadanoff (Kadanoff 1976, Migdal 1976) position-space renormalisation group to obtain the phase diagram of (2) in the parameter space $J \times L$. First we consider a more general form of the Hamiltonian (2):

$$-\beta H = \sum_{\langle rr' \rangle} V(\theta_r - \theta_{r'}) (S_r S_{r'} + 1) + \beta L \sum_{\langle rr' \rangle} S_r S_{r'} \quad (3)$$

where $V(\theta)$ is a periodic function with period 2π and $\beta = 1/kT$. The original expression (2) is recovered upon setting $V(\theta) = \beta J \cos \theta$. To apply the Migdal-Kadanoff transformation one first moves half of the bonds of the lattice such that the sites to be integrated out are linked to their neighbours only in one spatial direction. After decimation

one obtains a new Hamiltonian with renormalised parameters. In terms of $u(\theta) = \exp(V(\theta) - V(0))$ and $Z = \exp(V(0) + \beta L)$, the parameters (primed) of this new Hamiltonian are given by the following recursion relations:

$$(Z')^2 = (Z^4 A_1(0) + Z^{-4})/2A_2 \quad (4a)$$

and

$$[u'(\theta)]^2 = (Z^4 A_1(\theta) + Z^{-4})/(Z^4 A_1(0) + Z^{-4}) \quad (4b)$$

where

$$A_1(\theta) = \int_0^{2\pi} \frac{d\varphi}{2\pi} u^4(\varphi) u^4(\varphi - \theta)$$

and

$$A_2 = \int_0^{2\pi} \frac{d\varphi}{2\pi} u^4(\varphi).$$

A numerical iteration of (4) gives the phase diagram of figure 1. The line APC

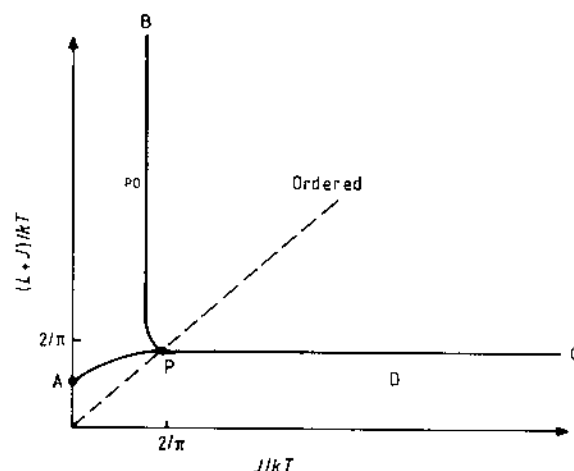


Figure 1. A schematic phase diagram showing an ordered phase with XY and Ising order, a partial ordered phase PO with XY disorder and Ising order and a disordered phase D. The broken line indicates the locus of initial points corresponding to the fully frustrated XY model.

separates the XY and Ising ordered phase and the XY disordered and Ising ordered phase PO from the XY and Ising disordered phase D. The broken line in figure 1 indicates the line of initial points for the original fully frustrated XY model corresponding to $L = 0$. The location of the point P cannot be determined with precision due to the drift to high temperatures in the Migdal-Kadanoff approximation (José *et al* 1977). A detailed analysis of the critical behaviour near this point can be found in Granato and Kosterlitz (1986a,b). Point A is an Ising-type fixed point, while to the right of point B when $L + J \rightarrow \infty$ there is a line of fixed points of XY type. The renormalisation group flows

near the line \overline{APC} are directed towards A except close to the line \overline{PB} , where they are directed towards B. A possible Ising antiferromagnetic ordered phase for large negative values of L is not indicated in the figure.

In the phase diagram of figure 1, one can only have two successive transitions of an XY type followed by Ising type as temperature is increased or a single transition from an ordered state to a completely XY and Ising disordered state without any intervening partial ordered phase. Thus our analysis provides some evidence that XY order cannot coexist with Ising disorder in the fully frustrated XY model in agreement with the possible scenarios that have been suggested for this system.

The author wishes to thank Professor J M Kosterlitz for useful discussions during the initial stages of this work.

References

- Berge B, Diep H T, Chazali A and Lallemand P 1986 *Phys. Rev. B* **34** 3177
- Choi M Y and Doniach S 1985 *Phys. Rev. B* **31** 4516
- Choi M Y and Stroud D 1985 *Phys. Rev. B* **32** 5773
- Granato E and Kosterlitz J M 1986a *J. Phys. C: Solid State Phys.* **19** L59
- 1986b *Phys. Rev. B* **33** 4767
- Halsey T C 1985 *J. Phys. C: Solid State Phys.* **18** 2437
- José J V, Kadanoff L P, Kirpatrick S and Nelson D R 1977 *Phys. Rev. B* **16** 1217
- Kadanoff L P 1976 *Ann. Phys., NY* **100** 359
- Korshunov S E 1986 *J. Stat. Phys.* **43** 17
- Korshunov S E and Uimin G V 1986 *J. Stat. Phys.* **43** 1
- Lee D H, Joannopoulos J D, Negele J W and Landau D P 1986 *Phys. Rev. B* **33** 450
- Migdal A A 1976 *Sov. Phys.-JETP* **42** 743
- Teitel S and Jayaprakash C 1983 *Phys. Rev. B* **27** 598
- van Himbergen J E 1986 *Phys. Rev. B* **33** 7857
- Villain J 1975 *J. Physique* **36** 581
- Yosefin M and Domany E 1985 *Phys. Rev. B* **32** 1778