Numerical Study of the Stables Orbits Around the Moon Using the Full Four-Body Problem

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Abstract

The objective of this paper is to study a region of direct stable orbits around the Moon, whose stability is related to the Family H2 of periodic orbits and to the quasi-periodic orbits that oscillate around them. The dynamical system considered was the full four-body problem, Sun-Earth-Moon-probe. We also take into account the eccentricity of the Earth's orbit, the eccentricity and inclination of the Moon's orbit and the pressure of solar radiation on the probe. Then, we analyzed the evolution of the region's size by varying the inclination of the probe's initial osculating orbit between 0 and 180° relative to the Moon's orbital plane. The size of the stability region diminishes; nevertheless, it remains significant for inclinations until 40°. The orbit of this region can be useful to keep around the Moon constellation of communication's satellites and remote sensory, and spacecrafts in logistic mission in favor of the future lunar bases.

1 Introduction

The stable orbit around the Moon has been studied by many authors, Brouke [1], Hénon [2], Jefferys [3], Winter [4] and Winter and Vieira Neto [5]. They all made use of the restricted three-body problem (RP3B). In a previous work [6], we investigate, taking as dynamical system the full four-body problem (FP4B) Sun-Earth-Moon-Probe, the evolution of the size of a region of stable orbits around the Moon related to the Family H2 [1] and to the quasi-periodic orbits that oscillate around them. The eccentricity of the Earth's orbit, the eccentricity and the inclination of the Moon's orbit and the pressure of solar radiation on the probe were considered too.

The goal of this paper is to make a more detailed investigation of the region already known using the FP4B and varying the inclination of the probe's initial osculating orbit relative to Moon's orbital plane between 0 and 180°. As we are tempting approximate of Earth-Moon real system we consider the fact of the Moon's equator has an inclination of +6.5° relative to Moon's orbital plane. On other hand, the orbits of Family H2, that give origin the stable region studied, have them perilunes between Earth and Moon, in Synodic coordinates system. So, relative to lunicentric coordinates system we vary the inclination between 173.5 and –6.5°, on clockwise sense, the eccentricity between 0 and 0.8 and semi-major axes between 25000 and 32000km. From these considerations was possible to establish the parameters for that the stable orbits can be used by many kinds of missions of space vehicles around the Moon requiring low maintenance costs. For example, they can be useful to constellations of communication's satellites, remote sensory and mapping of lunar superficies, spacecrafts awaiting lunar landing maneuvers, rendezvous and logistics missions, all in favor of future lunar colonies. The existence of those colonies will are facilitating in case of presence of ice in the lunar poles be confirmed. This fact will make of the our natural satellite a trampoline for future manned interplanetary missions.

This article is structured as follows: in the next section, we will show the stability regions associated to the Family H2 obtained to RP3B and FP4B. In the section 3, we discussed the evolution of the size's stability region by varying the inclination of the probe's initial osculating orbit relative to the Moon's orbital plane between 0 to 180°. In the section 4, we discussed the main features of the orbits of region studied, and we have finished in section 5 with a conclusion on this work.

2 The Stability Region

Winter and Viera Neto [5], considering the RP3B, determined the localization and the size of the stability region associated to the Family H2. This region can be seen in the Figure 2.1.a expressed in terms of the eccentricity versus semi-major axes of the initial osculating orbit relative to the Moon and in terms of the time that probe remains around the Moon, or capture time. The stability criteria adopted to definite the stability region is based on the monitoring of two-body Moon-probe orbital energy, since the distance between them not exceed the limits of the Moon's sphere of influence. This energy does not remain constant in dynamical system considered, due to interaction with the Earth's gravitational field. Nevertheless, through monitoring, it is possible to have a clear idea of the influence of the Moon's gravitational field on the probe's path. A change in the sign of the Moon-probe energy could indicate, for example, that the probe's osculating orbit changed from a closed to an open orbit relative to the Moon, escaping from the latter. Then, one orbit is considered stable if its Moon-probe orbital energy remains negative for, at the least, 1000 days; an amount of time that seems adequate to us for practical purposes.

Recently [6], we considering the full four-body problem (FP4B), Sun-Earth-Moon-probe, and some stability criteria above described, then, we analyzed the effects of the intrinsic properties of Earth-Moon system (the eccentricity of the Earth's orbit, the eccentricity and inclination of the Moon's orbit) in the evolution of the size's stability region. For an inertial coordinates system, the FP4B equations of motion are:

$$\ddot{\vec{r}}_i = \sum_{j=1}^{4} \mu_j \frac{\vec{r}_{ij}}{r_{ij}^3} \qquad j \neq i \qquad , \qquad (2.1)$$

Where $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$, and μ_j is the parameter mass of j-body. So, μ_1 is the Sun's mass parameter, μ_2 is the Earth's, μ_3 is the Moon's, and μ_4 is the probe's. The stability region found considering the FP4B, and the properties of the Earth-Moon system (with inclination zero relative to Moon's orbital plane), can be seen in the Figure 2.1.b. Comparing the Figures 2.1.a and 2.1.b, we can check the reduction of the stability region for ¼ of its original size, but it continues existing with considerable size.

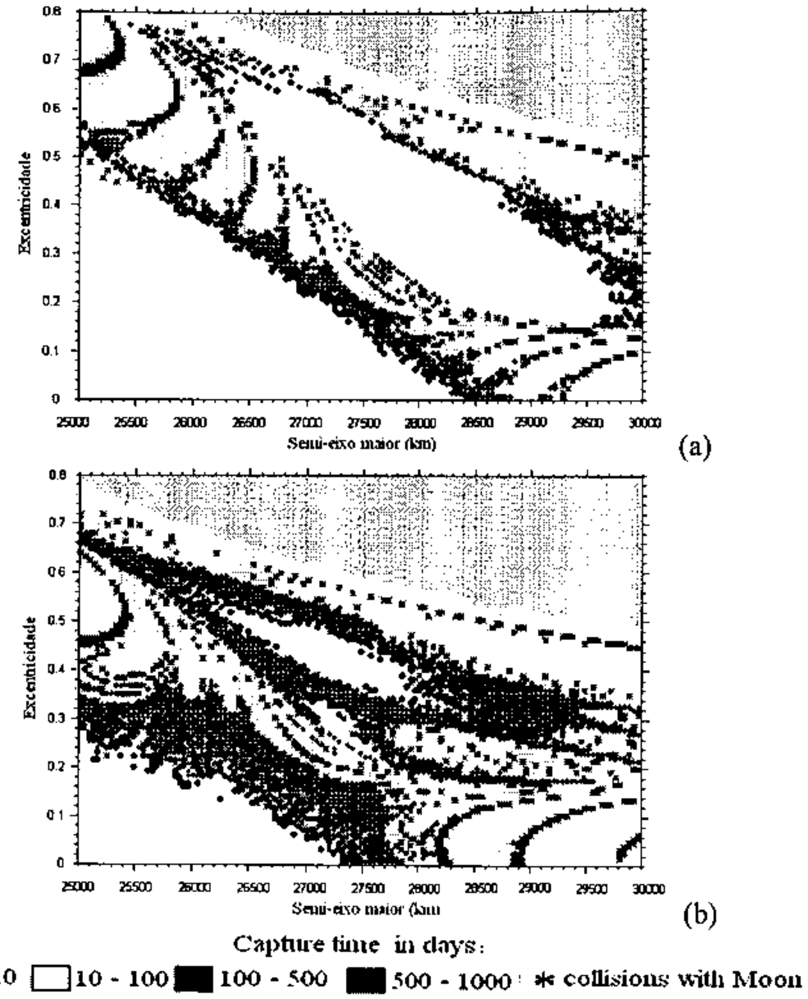

Figure 2.1 - Eccentricity versus semi-major axes diagrams in terms of capture time indicated by color code. The white area in the colorful regions correspond the initial conditions that remains captured for an integrated period of 1000 days and define the region of stable orbits around the Moon: (a) RP3B, (b) FP4B with Earth-Moon properties ($\Delta a = 40km$ e $\Delta e = 0.008$).

For $i = 4$, that is, for the components of probe's acceleration, we add the disturbance due of the solar radiation pressure, $\bar{A}_{PSR}$, give by:

$$\bar{A}_{PSR} = P_o EB\left(\frac{A_{efe}}{M_4}\right)\hat{r}_{14} \tag{2.2}$$

Where E is the eclipse's coefficient and its value can be 0 or 1, if $E = 1$, the probe is receiving solar radiation, if $E = 0$ it is not receiving radiation (darkness). B is the optical reflection constant and its value depends on the material which the probe is made: $B = 0$ when the probe is transparent to the solar radiation, $B = 1$ if it absorb 100% of the incident radiation (blackbody) and $B = 2$ if it reflects 100% of the incident radiation

(Myron), A_{efe} area effectively exposed to solar radiation and M_4 is the probe's mass. In our investigation we considered $E = 1$, $B = 2$, $A_{efe} = 14m^2$ and $M_4 = 307kg$, these last two parameter are similar to those SMART 1 lunar probe, launched by ESA in august 2003, but $E = 1$ and $B = 2$ are hypothesis made for this work. These conditions led us to acceleration in the order $8{,}0\text{x}10^{-7}m/s^2$. However, it was not capable of affecting the size of stability region, as we can be see, in the Figure 2.3.a and 2.3.b. In general, our simulation show that the disturbance due to radiations pressure causes periodic variation in the Moon–probe distance of up to $60km$ with a period of approximately 1 year.

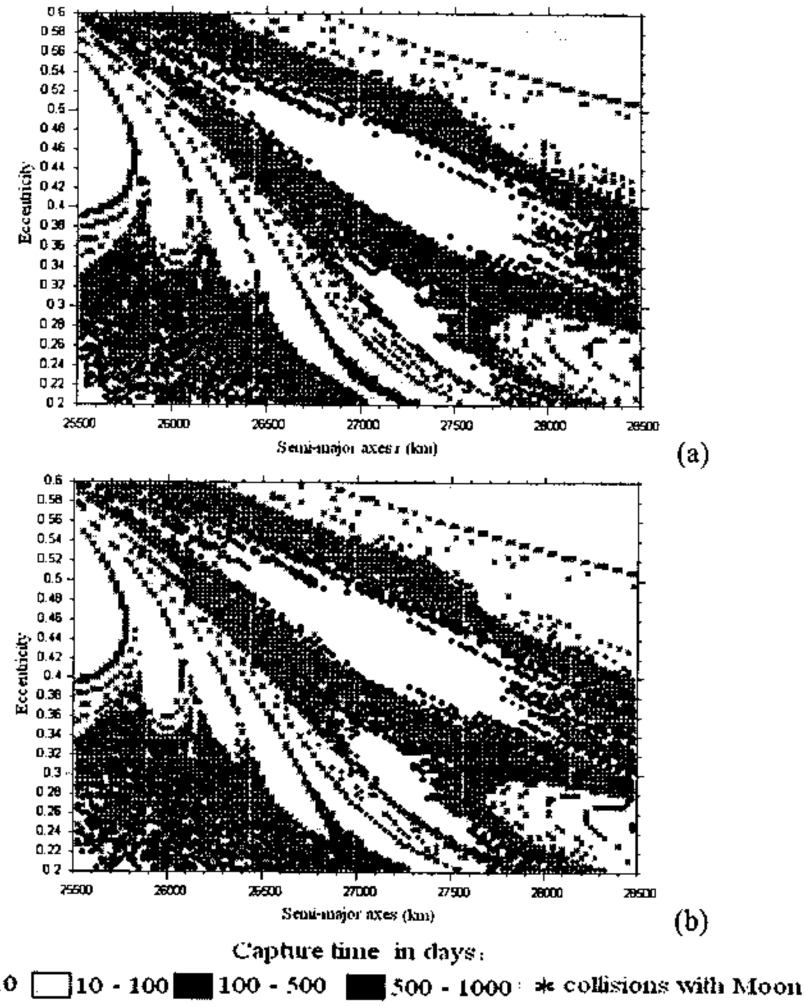

Figure 2.2 - Like the Figure 2.1, with zoom in the stability region to better visualize the effects of the pressure of solar radiation: (a) FP4B without pressure of solar radiation, (b) FP4B more pressure of solar radiation. Only small changes points are observed.

3 The Evolution of the Size of Stability Region with Inclination

The initial conditions of the osculating orbits that define the stability region in the Figures of the section 2 have an inclination relative to the Moon's orbital plane equal zero, we call it of i^* for now on. Thinking about the use of these orbits for practical purposes, we studied the evolution of the stability region varying the inclination of the probe's initial osculating orbit between 0 to 180° relative to the Moon's orbital plane, this is, i^*. The Figure 3, show the evolution of the region to $i^* = 10°$, 20°, 30° and 40°; measured in the clockwise sense from the perilune. Observe of the unexpected form that after reduce its size to i^* between 0° to 30° the stable region returns to grow and has its localization dislocated in relation the original one until 45°. For $i^* \geq 45°$ the region disappears completely, it gives place to a fussy distribution conditions of the initial osculating orbits. To $i^* > 60°$ we have a predominance of collision orbits with the Moon.

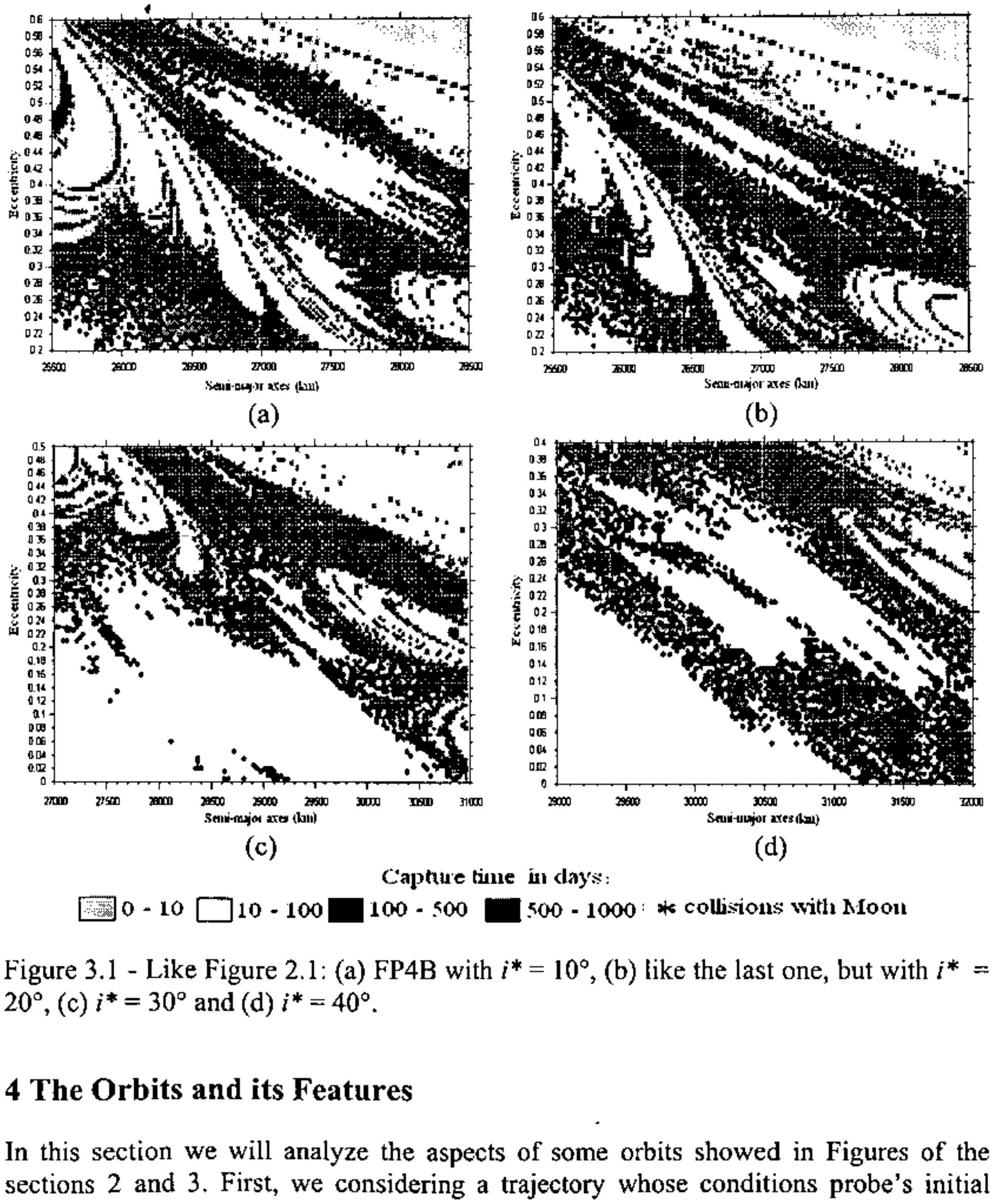

Figure 3.1 - Like Figure 2.1: (a) FP4B with $i^* = 10°$, (b) like the last one, but with $i^* = 20°$, (c) $i^* = 30°$ and (d) $i^* = 40°$.

4 The Orbits and its Features

In this section we will analyze the aspects of some orbits showed in Figures of the sections 2 and 3. First, we considering a trajectory whose conditions probe's initial osculating orbits are $a = 27300km$, $e = 0.4200$ and $i^* = 0$ ($i = 173.5°$). The orbit defined by these values of a and e belongs to the major regions localized in the central part of the diagram of the Figure 2.2.b. This orbit in the Synodic coordinates system can be seen in the picture 4.1.a and b for a period of 1000 days. The variations of the perilune's distance and of the inclination of the probe's instantaneous osculating orbit relative to the Moon's orbital plane are showed in the Figures 4.1c and d respectively. As we can see, the considerations of the dynamical system more complex and realistic provoke an increase in the amplitude of the perilune's distance oscillation and take out the probe of the Moon's orbital plane.

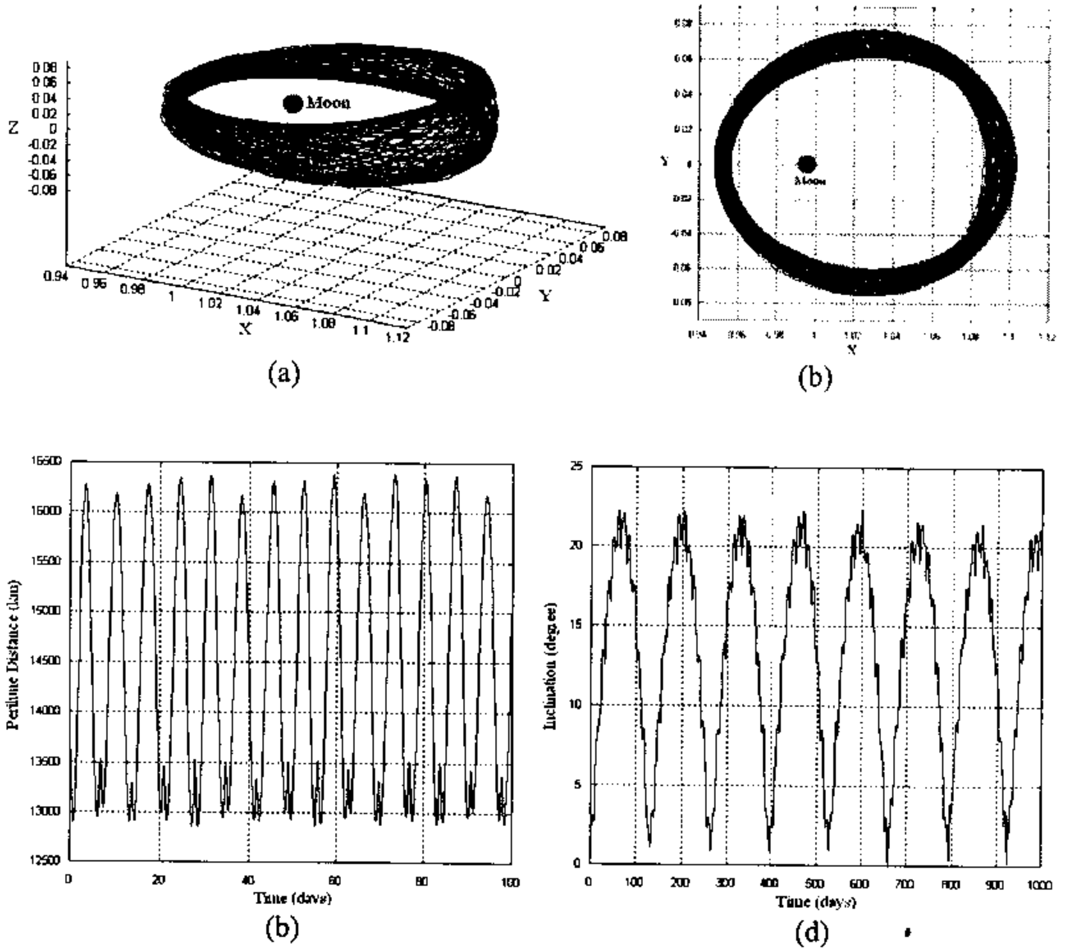

Figure 4.1. The probe's initial osculating orbit conditions: $a = 27300km$, $e = 0{,}4200$ and $i^* = 0°$ ($i = 173.5°$) in the Synodic coordinates system: (a) Space view and (b) projection in plane xy. (c) Variation of the perilune's distance for 100days, but the behavior is the same for 1000 days, and (d) variation in time of the inclination of instantaneous osculating orbit relative to the Moon's orbital plane for 1000 days.

The Figures 4.2 and 4.3 show the trajectory whose the probe's initial osculating orbit conditions were removed of the diagram of the Figure 3.1. In the Figure 4.2, we have a = 27500*km*, e = 0.4500 and i^* = 20°, in the Figure 4.3, a = 30500*km* and e = 0.1800 and i^* = 40°. These Figures show the trajectories in the Synodic coordinates system: (a) in a special view, (b) the projection in plane xy, and in the letters (c) and (d) show the variations in the time of perilune's distance and in the inclination of the probe's instantaneous osculating orbit relative to the Moon's orbital plane.

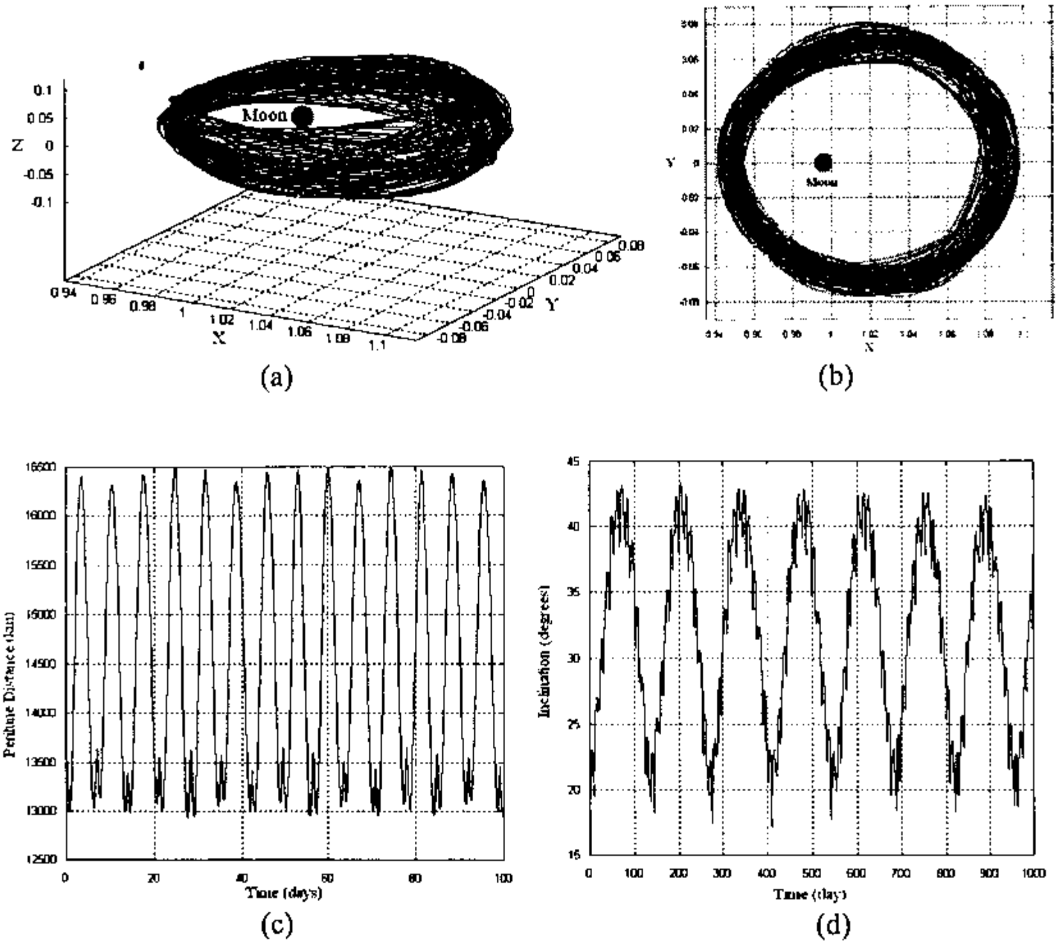

Figure 4.2. The probe's initial osculating orbit conditions a = 27500km, e = 0.4500 and i^* = 20° (i = 153.5°) in the Synodic coordinates system: (a) Space view, (b) projection in plane xy. (c) Variation of the perilune's distance for 100days, but the behavior is the same for 1000 days, and (d) variation in time of the inclination of instantaneous osculating orbit relative to the Moon's orbital plane for 1000 days.

Observing the Figures 4.1 and 4.2, we note that for $i^* \leq 20°$ the oscillation's amplitude of the perilune's distance, consequently, the amplitudes of semi-major axes and eccentricity of the probe's instantaneous osculating orbit are practically equals. The same occur with the oscillation's amplitude of the instantaneous inclination of the probe's osculating orbit. In the Figure 4.3, where the inclination of the probe's initial osculating orbit is $i^* = 40°$, the oscillation's amplitude of perilune's distance is smaller than ones of Figures 4.1 and 4.2. This is due to the fact of the probe's initial osculating orbits with $i^* = 40°$ have eccentricities smaller than ones belong the regions found to $i^* \leq 20°$. The variation of the instantaneous inclination of the probe's osculating orbit also is slightly bigger when $i^* = 40°$. And, the instantaneous inclination and perilune's distance have time evolutions different of the others. This difference can to mean that stability region found to $i^* = 40°$ not represent simply a set of stable orbits derived of stable orbits associated to the Family H2 like the others. However, while more investigations not were made, we prefers associate them with the stable orbits of Family H2.

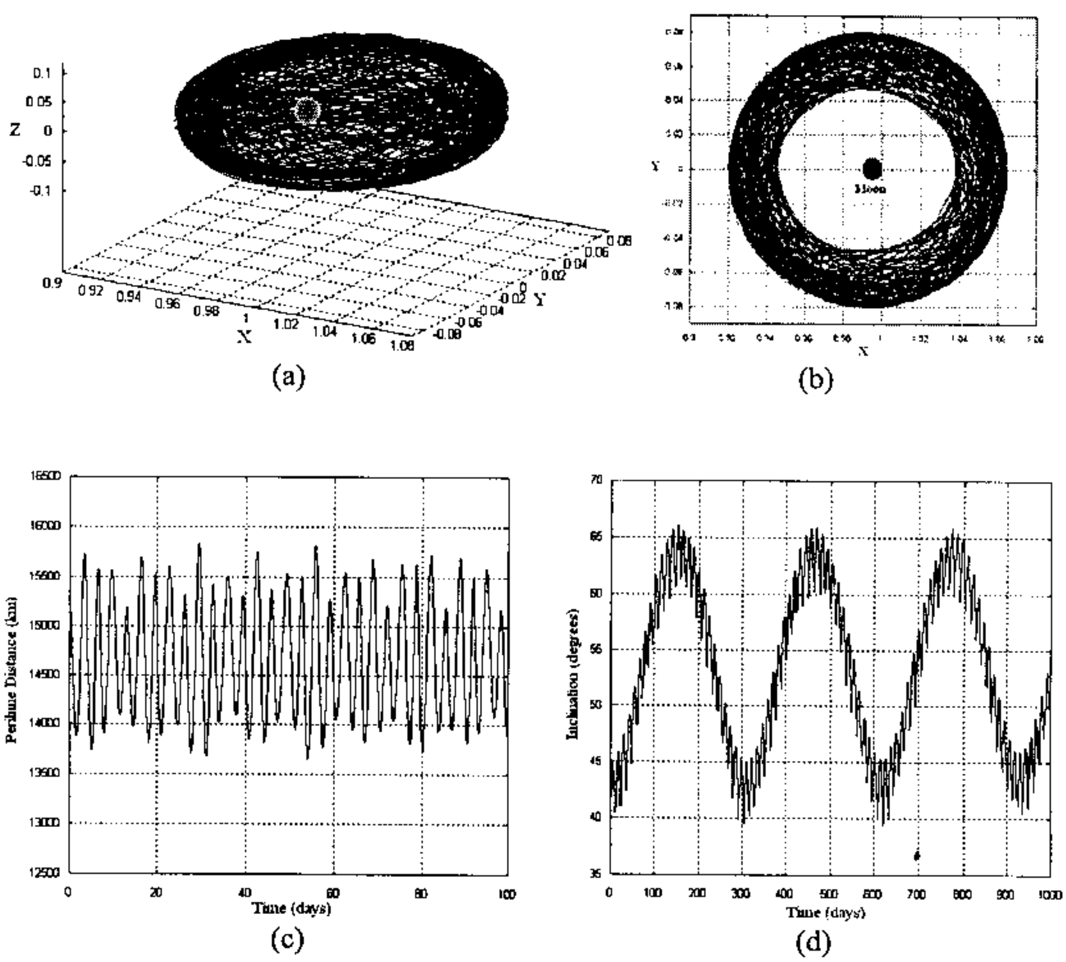

Figure 4.3. The probe's initial osculating orbit conditions a = 30500km, e = 0.1800 and i^* = 40° (i = 133.5°) in the Synodic coordinates system: (a) Space view, (b) projection in plane xy. (c) Variation of the perilune's distance for 100days, but the behavior is the same for 1000 days, and (d) variation in time of the inclination of instantaneous osculating orbit relative to the Moon's orbital plane for 1000 days.

5 Conclusions

Ours investigations show that the stability region obtained to RP3B (Figure 2.1) continues existing for a more complex and realistic dynamical system, the FP4B added the proprieties of Earth-Moon system and the pressure of solar rotation, in spite of stability region reduces its size to 1/4 of the original one to $i^* = 0$. Even so, it still represents a meaningful set of the initial conditions of stable orbits around the Moon. With the inclusion of a inclination different of zero (relative to the Moon's orbital plane) in initial conditions of the probe's osculating orbit, we can note, that still is possible to find stability regions to $i^* \leq 40°$. And, as showed in the Figures 4.1 to 4.3, the variations of the inclination of probe's instantaneous osculating orbit is not brusque, otherwise, it is so slow, for example we can observe in the Figures 4.2, for example, a $\Delta i = 20°$ in 100 days. These features associated with stability of orbits requiring low maintenance costs and can be explored for remote sensory and mapping of lunar superficies, constellations of communication's satellites and more others missions with destiny to Moon.

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