LOW THRUST ORBITAL MANEUVERS TO INSERT A SATELLITE IN A CONSTELATION

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ABSTRACT

This paper considers the problem of low thrust optimal maneuvers to insert a satellite in a constellation. The main idea is to assume that a satellite constellation is given, with all the keplerian elements of the satellite members having known values. Then, it is necessary to maneuver a new satellite from a parking orbit to its position in the constellation. The control available to perform this maneuver is the application of a low thrust to the satellite and the objective is to perform this maneuver with minimum fuel consumption.

1 - INTRODUCTION

From the analysis of the alternatives available to perform the orbital maneuvers required (Prado, 1989; Prado and Rios-Neto, 1993) the Sub-optimal parametrization is selected to be implemented and used to simulate the maneuvers. The reason for this choice is the simple implementation, in terms of hardware for the satellite, togheter with the fact that the results are very close to the optimal approach, as shown in the references cited above.

The spacecraft is supposed to be in Keplerian motion controlled only by the thrusts, whenever they are active. This means that there are two types of motion:
i) A Keplerian orbit, that is an orbit obtained by assuming that the Earth's gravity (assumed to be a point of mass) is the only force acting on the spacecraft. This motion occurs when the thrusts are not firing;
ii) The motion governed by two forces: the Earth's gravity field (also assumed to be a point of mass) and the force delivered by the thrusts. This motion occurs during the time the thrusts are firing.

The thrusts are assumed to have the following characteristics:
i) Fixed magnitude: The force generated by them is always of constant magnitude during the maneuver. The value of this constant is a free parameter (an input for the algorithm developed here), that can be high or low;
ii) Constant Ejection Velocity: Meaning that the velocity of the gases ejected from the thrusts is constant;
iii) Constrained angular motion: This means that the direction of the force given by the thrusts can be modified during the transfer. This direction can be specified by the angles α and β, called pitch (the angle between the direction of the thrust and the perpendicular to the line Earth-spacecraft) and yaw (the angle with the orbital plane), respectively. The motion of those angles are constrained (constant, linear variations, forbidden regions for firing the thrusts, etc.);

iv) Operation in on-off mode: It means that intermediate states are not allowed. The thrusts are either at zero or maximum level all the time.

The solution is given in terms of the constants that specify the control to be applied and the fuel consumed. Several numbers of "thrusting arcs" (arcs with the thrusts active) can be used for each maneuver.

Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the orbital plane) is used as the independent variable.

2 - DEFINITION OF THE PROBLEM

The basic problem discussed in this paper is the problem of orbit transfer maneuvers to include a satellite in a constellation. The objective of this problem is to modify the orbit of a given spacecraft, from an initial parking orbit to a specific position in a final orbit. In the case considered in this paper, an initial and a final orbit around the Earth is completely specified. The problem is to find how to transfer the spacecraft between the first of those two orbits to a specific position in a final orbit, in such way that the fuel consumed is minimum. There is no time restriction involved here and the spacecraft can leave from any point in the initial orbit. The maneuver is performed with the use of an engine that is able to deliver a thrust with constant magnitude and linearly variable direction. The mechanism, time and fuel consumption to change the direction of the thrust is not considered in this paper.

3 - FORMULATION OF THE OPTIMAL CONTROL PROBLEM

This is a typical optimal control problem, and it is formulated as follows:

Objective Function: M_f,

where M_f is the final mass of the vehicle and it has to be maximized with respect to the control $\boldsymbol{u}(.)$;

Subject to: Equations of motion, constraints in the state (initial and final orbit) and control (limits in the angles of "pitch" and "yaw", forbidden region of thrusting and others);

And given: All parameters (gravitational force field, initial values of the satellite and others).

The equantions of motion are (Biggs, 1978):

$$dX_1/ds = f_1 = SiX_1F_1 \qquad (1)$$

$$dX_2/ds = f_2 = Si\{[(Ga+1)\cos(s)+X_2]F_1+vF_2\sin(s)\} \qquad (2)$$

$$dX_3/ds = f_3 = Si\{[(Ga+1)\sin(s)+X_3]F_1-vF_2\cos(s)\} \qquad (3)$$

$$dX_4/ds = f_4 = SivF(1-X_4)/(X_1W) \qquad (4)$$

$$dX_5/ds = f_5 = Siv(1-X_4)m_0/X_1 \qquad (5)$$

$$dX_6/ds=f_6 = -\, SiF_3[X_7\cos(s)+X_8\sin(s)]/2 \tag{6}$$

$$dX_7/ds = f_7 = SiF_3[X_6\cos(s)-X_9\sin(s)]/2 \tag{7}$$

$$dX_8/ds = f_8 = SiF_3[X_9\cos(s)+X_6\sin(s)]/2 \tag{8}$$

$$dX_9/ds = f_9 = SiF_3[X_7\sin(s)-X_8\cos(s)]/2 \tag{9}$$

where:

$$Ga = 1 + X_2\cos(s) + X_3\sin(s) \tag{10}$$

$$Si = (\mu X_1^4)/[Ga^3 m_0(1-X_4)] \tag{11}$$

$$F_1 = F\cos(\alpha)\cos(\beta) \tag{12}$$

$$F_2 = F\sin(\alpha)\cos(\beta) \tag{13}$$

$$F_3 = F\sin(\beta) \tag{14}$$

and F is the magnitude of the thrust, W is the velocity of the gases when leaving the engine, ν is the true anomaly of the spacecraft.

In those equations the state was transformed from the Keplerian elements (a = semi-major axis, e = eccentricity, i = inclination, Ω = argument of the ascending node, ω = argument of periapsis, ν = true anomaly of the spacecraft), in the variables X_i, to avoid singularities, by the relations:

$$X_1 = [a(1-e^2)/\mu]^{1/2} \tag{15}$$

$$X_2 = e\cos(\omega-\phi) \tag{16}$$

$$X_3 = e\sin(\omega-\phi) \tag{17}$$

$$X_4 = (\text{Fuel consumed})/m_0 \tag{18}$$

$$X_5 = t = \text{time} \tag{19}$$

$$X_6 = \cos(i/2)\cos((\Omega+\phi)/2) \tag{20}$$

$$X_7 = \sin(i/2)\cos((\Omega-\phi)/2) \tag{21}$$

$$X_8 = \sin(i/2)\sin((\Omega-\phi)/2) \tag{22}$$

$$X_9 = \cos(i/2)\sin((\Omega+\phi)/2) \tag{23}$$

$$\phi = \nu + \omega - s \tag{24}$$

and s is the range angle of the spacecraft.

The number of state variables defined above is greater than the minimum required to describe the system, which implies that they are not independent and relations between than exist, like: $X_6^2 + X_7^2 + X_8^2 + X_9^2 = 1$. This system is also subject to the constraints in state and some of the the Keplerian elements of the initial and the final orbit. All the parameters (gravitational force field, initial values of the satellite, etc...) are assumed to be known.

4 - SUBOPTIMAL METHOD

In this approach (Prado, 1989; Biggs, 1978), a linear parametrization is used as an approximation for the control law (angles of pitch (A) and yaw (B)):

$$\alpha = \alpha_0 + \alpha' * (s - s_S) \quad (25)$$

$$\beta = \beta_0 + \beta' * (s - s_S) \quad (26)$$

where α_0, β_0, α', β' are parameters to be found, s is the instantaneous range angle and s_S is the range angle when the motor is turned-on.

These equations are the mathematical representation of the "a priori" hypothesis that α and β vary linearly with the "range angle" s. This is done to explore the possibility of having a model easy to implement in terms of hardware development.

Considering these assumptions, there is a set of six variables to be optimized (start and end of thrusting and the parameters α_0, β_0, α', β') for each "burning arc" in the maneuver. Note that this number of arcs is given "a priori" and it is not an "output" of the algorithm. This is the control available to maneuver the satellite.

By using parametric optimization, this problem is reduced to one of nonlinear programming, which can be solved by several standard methods.

5 - NUMERICAL METHOD

To solve the nonlinear programming problem, the gradient projection method was used (Bazarra & Sheetty, 1979; Luemberger, 1973).

It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{l+1} = \boldsymbol{u}_l - \nabla \mathbf{f}^T . \left[\nabla \mathbf{f} . \nabla \mathbf{f}^T \right]^{-1} \mathbf{f} \quad (27)$$

where f is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$u_{l+1} = u_l + \bar{\alpha}\frac{\mathbf{d}}{|\mathbf{d}|} \quad (28)$$

where:

$$\bar{\alpha} = \gamma \frac{J(u)}{\nabla J(u).\mathbf{d}} \quad (29)$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla \mathbf{f}^T\left[\nabla \mathbf{f}.\nabla \mathbf{f}^T\right]^{-1}\mathbf{f}\right)\nabla J(u) \quad (30)$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and γ is a parameter determined by a trial and error technique. The possible singularities in equations (27) to (30) are avoided by choosing the error margins for tolerance in convergence large enough.

This procedure continues until $|u_{l+1} - u_l| < \varepsilon$ in both equations (27) and (28), where ε is a specified tolerance.

6 - SIMULATIONS

To validate the algorith developed, a transfer maneuver is simulated. It is assumed that the satellite start its motion in a parking orbit. This orbit has the following data:

Semi-major axis of 7000 km, eccentricity of 0.00, inclination of 5 degrees, ascending node of 35 degrees, mean anomaly of 90 degrees.

From this orbit, the satellite has to go to a final orbit, that has the following orbital elements:

Semi-major axis of 26560 km, eccentricity of 0.0131, inclination of 55 degrees, ascending node of 90 degrees, argument of perigee of 180 degrees, mean anomaly of 270 degrees.

It is also assumed that the initial mass of the satellite is 500 kg and the thrust level to perform the maneuver is 20.0 N.

The results are shown in Table 1 for three different situations: a maneuver with two, four and eight thrusting arcs. For each maneuver the table shows, for every thrusting arc, s_s(deg), the range angle of the instant the the engine is turned on; s_e(deg), the range angle of the instant the the engine is turned off; α_0 (deg) the initial value for the pitch angle; β_0 (deg) the initial value for the yaw angle; α' the value of the derivative of the pitch angle with respect to the range angle; β' the value of the derivative of the yaw angle with respect to the range angle; the fuel consumed by the maneuver.

Table 1
Maneuvers with 2, 4 and 8 "thrusting arcs"

Arc	s_s(deg)	s_e(deg)	α_0(deg)	β_0(deg)	α'	β'	Fuel-kg
1	12.9	1732.1	9.6	60.4	0.018	0.300	------
2	1950.3	3720.4	7.0	49.6	-0.210	-0.040	54.25
1	23.1	853.2	0.76	55.7	0.027	0.058	------
2	1003.4	1843.2	11.2	41.5	-0.024	-0.199	------
3	2033.2	2903.1	13.9	31.5	-0.008	0.321	------
4	3073.4	3983.2	8.2	40.8	-0.122	0.163	52.13
1	43.4	443.9	1.7	48.8	0.001	-0.043	------
2	613.3	1063.2	4.7	56.2	-0.121	0.100	------
3	1153.8	1573.8	3.4	49.6	-0.009	0.660	------
4	1713.2	2133.9	4.3	60.2	-0.127	-0.096	------
5	2325.3	2773.4	5.0	55.0	-0.010	-0.086	------
6	2943.5	3363.3	4.7	44.9	0.121	-0.090	------
7	3473.8	3903.8	3.3	39.8	-0.009	-0.463	------
8	4093.4	4513.6	2.8	55.2	-0.138	-0.099	50.03

In a second set of simulations the same maneuvers were performed with the additional constraints that the control angles must be fixed ($\alpha' = \beta' = 0$); and, in a third set, the constraint $\alpha_0 = 0$ was added (only β_0 is a free parameter for the control law). The objective is to know how much more fuel is required to compensate a more simple implementation of the control device and to satisfy the constraints of keeping some equipment (antennas, for example) pointed toward Earth. Table 2 shows the comparison in fuel expenditure for all cases studied.

Table 2
Fuel expenditure (kg) for all maneuvers simulated

Method	2 arcs	4 arcs	8 arcs
Suboptimal	54.25	52.13	50.03
Suboptimal (α'=β'=0)	55.23	53.17	51.17
Suboptimal(α'=β'=α_0=0)	56.78	54.65	52.34

8 – CONCLUSIONS

Suboptimal control was explored to generate an algorithm to obtain solutions for the minimum fuel maneuvers required to insert a satellite in a constellation.

The method have a good numerical behavior. Process time (CPU) is short (around one minute, in a IBM-PC computer) for simple maneuvers, but when several constraints and/or "thrusting arcs" are present the process time can be larger (around 15 minutes in some cases).

The simulations show that an increase in the number of "thrusting arcs" reduce the fuel consumed, a reduction of the order of 5% to 10%.

They also show that the additional restrictions added to the problem generate an increase in the fuel consumed, in the order of 2% to 4%.

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REFERENCES

Bazaraa, M.S. & C.M. Shetty (1979). *Nonlinear Programming-Theory and Algorithms*. John Wiley & Sons, New York, NY.

Biggs, M.C.B. (1978). The Optimization of Spacecraft Orbital Manoeuvres. Part I: Linearly Varying Thrust Angles. The Hatfield Polytechnic, Numerical Optimization Centre, England.

Luemberger, D.G. (1973). *Introduction to Linear and Non-Linear Programming*. Addison-Wesley Publ. Comp., Reading, MA.

Prado, A.F.B.A. (1989), Análise, Seleção e Implementação de Procedimentos que Visem Manobras Ótimas em Órbitas de Satélites Artificiais. Master Thesis, INPE, São José dos Campos, Brazil.

Prado, A.F.B.A. & A. Rios-Neto (1993). Um Estudo Bibliográfico sobre o Problema de Transferências de Órbitas. *Revista Brasileira de Ciências Mecânicas*, Vol. XV, no. 1, pp 65-78.