Neoclassical Resistivity Modified by the Driven Electric Field in Tokamak Plasmas

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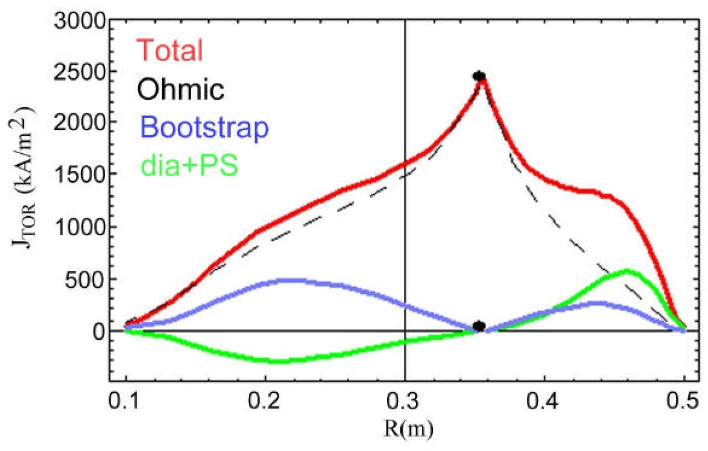
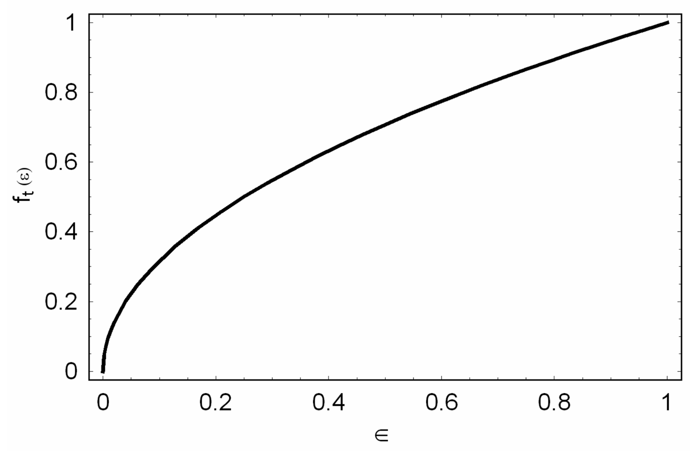
Abstract: The action of the driven electric field upon trapped particles in toroidal devices of tokamak ordering and the possible modification that this may cause on the neoclassical resistivity are investigated in this paper. The toroidal electric field accelerates the parallel component of trapped particles velocity reducing their pitch angle such that they become untrapped. According to the usual neoclassical theory, the trapped particle fraction goes to zero in the magnetic axis as $f_t \approx \sqrt{\varepsilon}$, leading to an undesirable singular behaviour of the current density profile in this region. The objective of this paper is to analyse the overall effect of particle detrapping due to the presence of the toroidal electric field over the whole plasma cross section and in which extension this detrapping may eliminate the "catastrophe" of infinite current density gradient at the magnetic axis. A convolution between the electric field and collisionality effects is developed in order to illustrate the modified profile of the current density in the region near the magnetic axis of a typical tokamak.

Objective

To study how the electric field modifies the singular behaviour of the current density profile in the magnetic axis predicted by the usual neoclassical theory.





The driven electric field accelerates the parallel component of trapped particles velocity reducing their pitch angle → particles become untrapped

Question: In which form the trapped particle fraction goes to zero in the magnetic axis in the presence of the toroidal electric field?

Potential Energy Curve

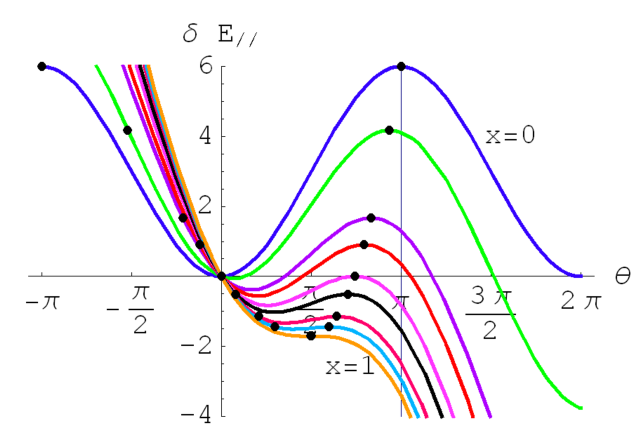
The potential energy curve is given by the combination of the magnetic mirror force and the parallel electric field in terms of the poloidal launch point θ.

$$\delta E_{//}(\theta) = E_{\perp}\varepsilon\,[1-\cos(\theta)] - qeE_{\phi}R_0\theta$$

- E_ϕ → toroidal electric field
- $E_\perp$ → perpendicular energy of a particle at the launch point
- q → safety factor
- R_0 → tokamak major radius

Normalised Electric field: $x(r) = \dfrac{q(r)\, e\, R_0 E_\phi}{\varepsilon\, E_{\perp}(r)}$

x = {0, 0.2, 0.5, 0.6, 0.7246, 0.8, 0.9, 0.95, 1}



Launch Point θ=0

Critical x, x_c=0.7246

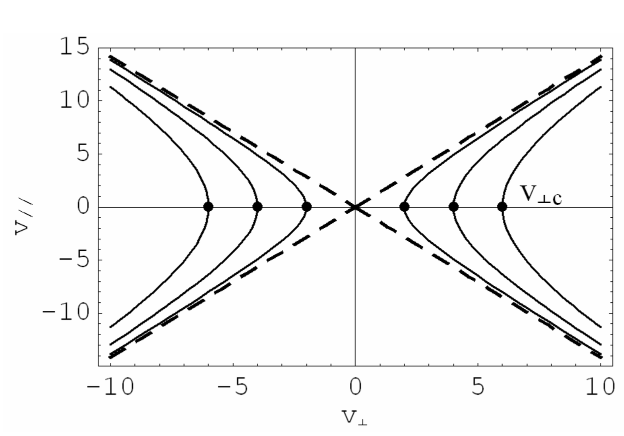
Critical Parallel Energy

$$E_{//c} = E_{\perp}\varepsilon\left[1+\sqrt{1-x^2} - x\,(\pi - \arcsin(x))\right]$$

Critical Electric Field for detrapping: $E_\phi \geq 0.7246\,\varepsilon(r)\,E_\perp/(e\,q(r)R_0)$

Critical Perpendicular Energy: $E_{\perp c} \leq E_\phi\, e\, q(r)\, R_0/(0.7246\,\varepsilon(r))$

Trapped Boundary Modification due to E_f



Trapped particle fraction obtained from the integration of the velocity distribution over the new boundary (full curves):

$$f_t(r) = \int_{v_{\perp c}}^{\infty}\int_{-v_{//c}}^{v_{//c}} 2\pi\, f(v)\; v_\perp\, dv_{//}\, dv_\perp$$

$$f_t(r) = \left(\frac{m}{T}\right)\int_{v_{\perp c}}^{\infty} v_\perp\, \exp[-(v_\perp^2/v_{th}^2)]\;\mathrm{Erf}\left(\frac{v_{//c}(r,v_\perp)}{v_{th}}\right) dv_\perp$$

Trapped Particle Fraction for q=0

Approximated form for $E_{//c}$: $E_{//c}/E_\perp = 2\varepsilon\,(1 - E_{\perp c}/E_\perp)$

Analytical form for f_t: $f_t(r) = \exp(-E_{\perp c}(r)/T_e(r))\sqrt{\dfrac{2\varepsilon(r)}{1+2\varepsilon(r)}}$

When E_ϕ = 0, then $E_{\perp c} = 0$ and the trapped particle fraction reduces to the large aspect ratio expression

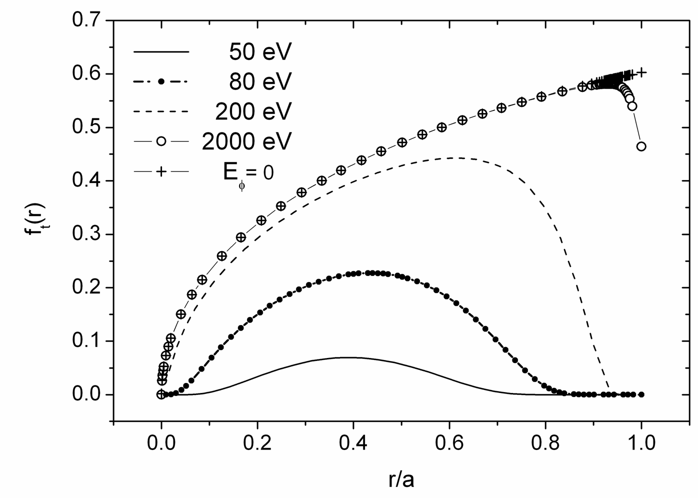
Iterative Calculation for f_t: Calculation of the trapped particle fraction consistent with a given current density profile and for various central electron temperatures. A central current density j_0 is chosen in order to support q_0 =1 and this sets the value of E_ϕ through $E_\phi = j_0/\sigma_{Spitzer}(0)$. The central current density and the toroidal electric field are kept constant for all iterations.

Electron Temperature Profile: $T_e(r) = (T_{e0} - T_{eedge})\left[1-(r/a)^2\right]^{\alpha_T} + T_{eedge}$

f_t for Different Central Electron Temperatures

Teedge	$1.0\times10^{-3}\,T_{e0}$
a_T	2
Z_{eff}	1
B_t (T)	1
q_0	1
R_0 (m)	0.56
R_0/a	3.5

T_{e0} (eV)	50	80	200	2000
E_f (V/m)	7.1	3.5	0.89	0.03



The detrapping effect of the electric field is more evident in low temperature plasmas and takes place in the whole plasma cross section

Launch Point Averaging of Trapped Particle Fraction

Critical Parallel Energy for detrapping as a function of q: $E_{//c}(\theta) = PE_{max} - E_{//0}(\theta)$

- PE_{max} → maximum potential energy achieved by a particle.
- $E_{//0}(\theta)$ → initial potential energy of a particle at the launch point.

$$E_{//c}(\theta) = E_\perp\varepsilon\left(1+\sqrt{1-x^2} - x\,(\pi-\arcsin(x)) - 1 + \cos(\theta) + x\theta\right)$$

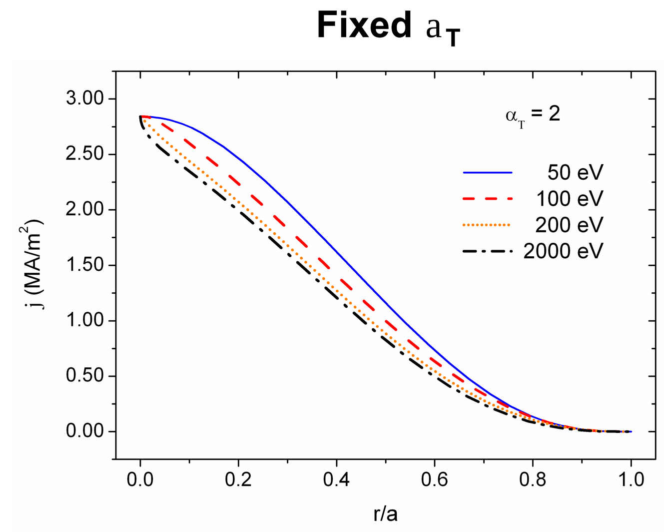
Approximated form: $E_{//c}(\theta) = E_\perp\varepsilon\,(1+\cos\theta)\left[1 - \dfrac{E_{\perp c}(r,\theta)}{E_\perp}\right]$

Analytical form for trapped particle fraction: $f_t(r,\theta) = \exp(-E_{\perp c}(r,\theta)/T_e(r))\sqrt{\dfrac{\varepsilon(r)\,(1+\cos\theta)}{1+\varepsilon(r)\,(1+\cos\theta)}}$

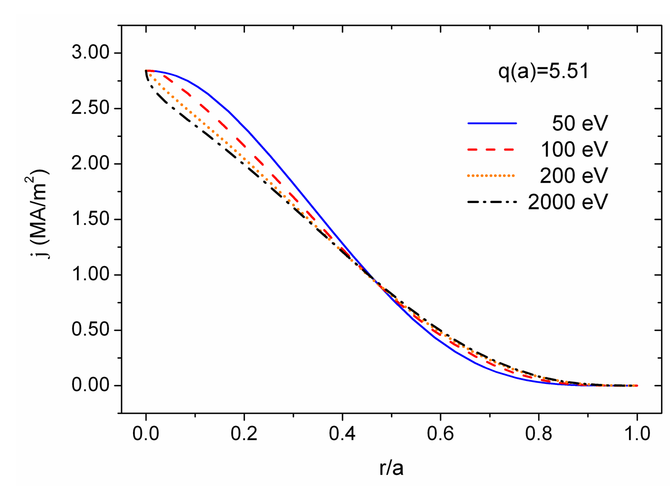
Averaged trapped particle fraction: $f_t(r) = 1/2\pi\int_{-\pi}^{\pi} f_t(r,\theta)\, d\theta$

Current Density Profiles

Fixed a_T

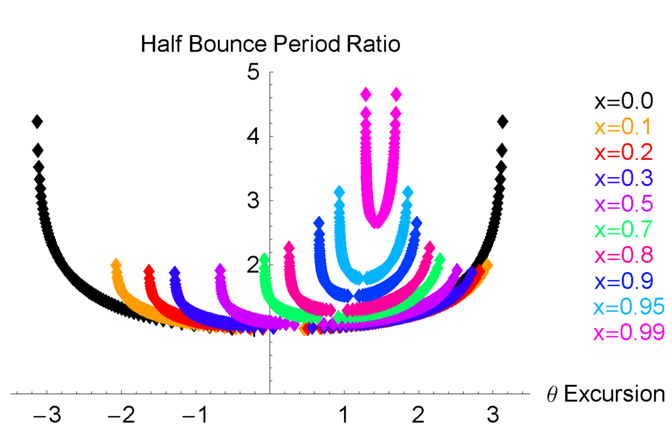


Fixed q(a)



Collisionality Effects

E_ϕ modifies the particle bounce frequency → exacerbates the non linearity of the bounce oscillations and reduces the bounce frequency at small amplitudes.



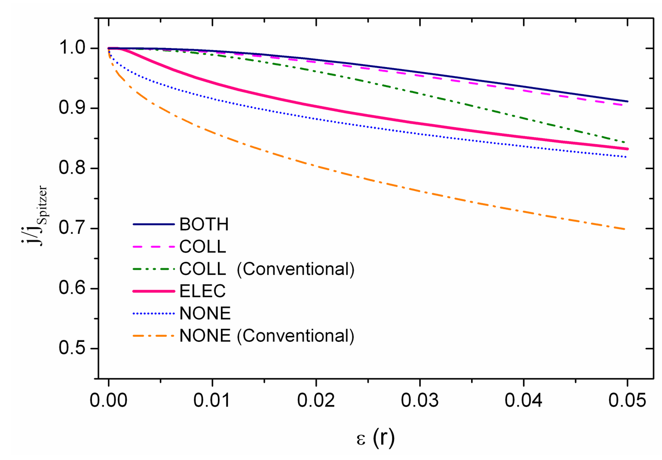
Collisionality parameter

Without Electric Field: $\nu_{*e} = \dfrac{\nu_{eff}}{\omega_b} = \dfrac{\sqrt{2}\,R_0 q}{\varepsilon^{3/2}\,\tau_{ee}\,v_{the}}$,

$$\tau_{ee} = \frac{3}{16\sqrt{\pi}}(4\pi\varepsilon_0)^2\frac{m_e^2\, v_{the}^3}{n_e\, e^4\,\ln\Lambda}$$

Modified by the Electric Field: $\nu_{*e} \approx 1.2\,\dfrac{\sqrt{2}\,R_0\, q(r)}{\varepsilon^{3/2}\, v_{the}\,\tau_{ee}}\,(1-x(r)^2)^{-1/4}$

Current density profile when collisionality effects are taken into account (analytical form):



$$j(r) = \sigma_{Spitzer}\,E_\phi\left(1-\frac{f_t(r)}{1+\xi(Z_{eff})\nu_{*e}}\right)\left(1-\frac{c_R(Z_{eff})f_t(r)}{1+\xi(Z_{eff})\nu_{*e}}\right)$$

$$f_t(r) \approx 0.6\exp\left(-\frac{E_{\perp c}}{T_e}\right)\sqrt{\frac{2\varepsilon(r)}{1+2\varepsilon(r)}}$$

None conventional → f_t is calculated for launch points at the outer midplane

Coll conventional → without the correction on the bounce frequency

Conclusions

An analytic form for the electric field detrapping effect has been developed and new consideration has been given to averaging both the bounce frequency of the trapped particles and the trapping fraction with respect to their poloidal birth position. This effect is enhanced by lower electron temperatures, but it is always sufficient to eliminate the cusp in the central current density, otherwise present when neither the electric field nor collisionality terms are included in the model. The collisionality term dominates the detrapping effect except in start-up conditions with high electric field and low plasma density, but even in normal discharge conditions in some tokamaks, the electric field corrections could be expected to modify some neoclassical phenomena as bootstrap current, potato orbits or neoclassical tearing modes.