Ministério da Ciência e Tecnologia - Instituto Nacional de Pesquisas Espaciais - Laboratório Associado de Plasma

Macroscopic instabilities in lightning

G.O. Ludwig[1], M.M.F. Saba[2]

[1]Laboratório Associado de Plasma, [2]Divisão de Geofísica Espacial

X Latin American Workshop on Plasma Physics combined with the 7th Brazilian Meeting on Plasma Physics
São Pedro, SP, Brazil, 30 November - 5 December, 2003

Characteristics of lightning discharges

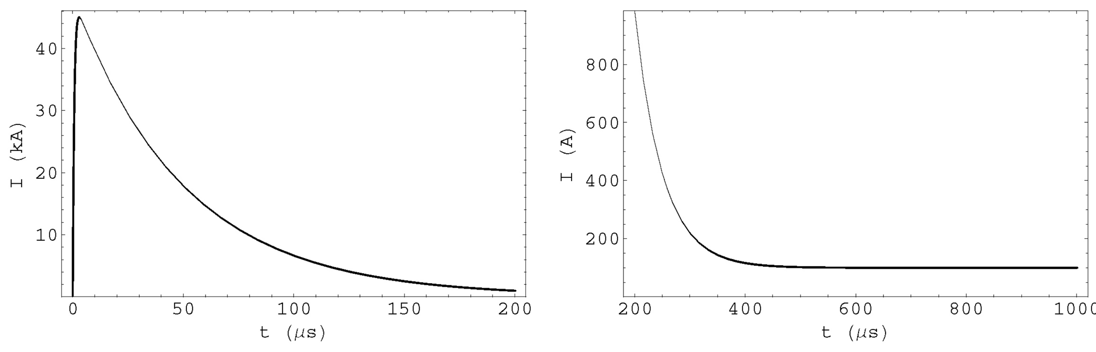
Typically, a lightning stroke lowers negative charge to earth from a thundercloud. The peak current in the **return stroke** is 10~100kA, and the discharge may last for 0.5s extending over a distance of 5km.

The first return stroke in a lightning discharge is preceded by a stepped ionization wave called a stepped leader. Subsequent strokes in a **multiple-stroke discharge** are preceded by a fast moving, continuous ionization wave called a dart leader. About 2/3 of natural lightning discharges present, on the average, 3 to 5 subsequent strokes.

Typical return stroke current pulse

The current in the main return stroke attains the median value of 30-40kA in ~1μs and decays with a time constant 30-60μs to a continuous value ~100A during pauses between successive strokes. The current peaks in subsequent strokes are usually smaller than the main peak, with rise times <1μs. The time interval between successive strokes is ~30-40ms.

The figure shows the model current-time curve for a typical return stroke. The **high current** associated with successive strokes leads to the development of a **shock wave** and the associated **hydrodynamic instability** discussed in the following.



Decay of the lightning channel

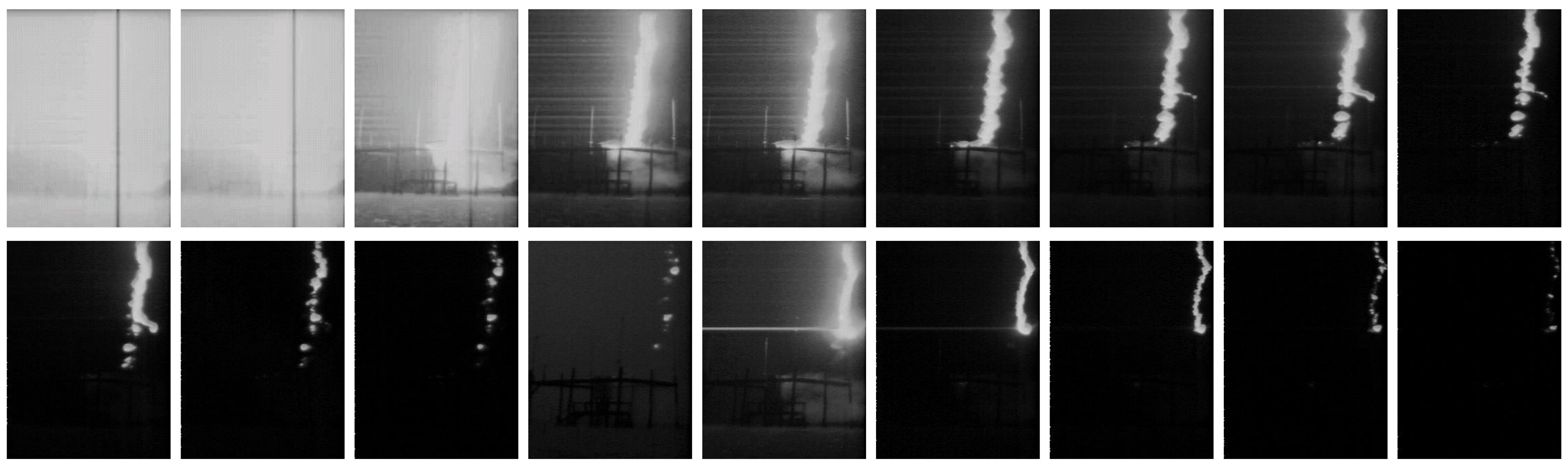
Near each current peak the discharge attains its maximum temperature of 30,000-40,000K, which rapidly decays to 10,000-8,000K during pauses, when the pressure in the channel falls to nearly the ambient pressure.

	t = 3 s	t = 300 s	t = 1ms
I_0 (A)	>30,000	100	100
T (K)	40,000	10,000	8,000
p (atm)	>15	>~1	1
a (cm)	<1	9	8
	expansion	contraction	

Simulations (N.L. Aleksandrov, É.M. Bazelyan and M.N. Shneider, Plasma Phys. Rep. **26**, 893, (2000)) show a **slow contraction** of the channel in the final phase of the current decay. The **pressure and density gradients are oppositely directed** giving rise to the Rayleigh-Taylor instability.

Artificially triggered lightning

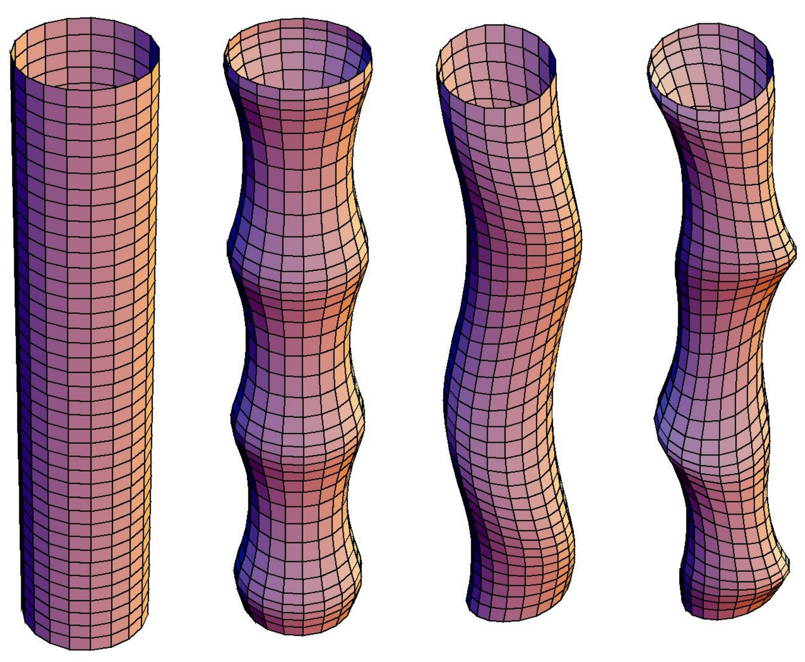
Lightning can be triggered by launching small rockets carrying a thin copper wire connected to the launching platform. In this way, thirteen flashes were artificially produced during the past four years in the International Center for Triggered and Natural Lightning in Cachoeira Paulista, SP, Brazil. A fast digital camera (1,000 frames per second) was used for the first time to obtain detailed images of the slow cooling stage of the discharge channel.



The above images show a sequence of pictures at 1ms time intervals corresponding to the last two strokes of a 10-stroke flash. The peak current was 45kA and the visible channel had a radius of 0.5m estimated from the size of the launching platform. The last return stroke ended on the tip of a Franklin lightning rod that can be seen on the right side of the sequence. The images show the slow evolution of the **beaded structure** usually observed during pauses in triggered lightning experiments..

Macroscopic instabilities

The purpose of this poster is to examine the role of the **hydromagnetic and Rayleigh-Taylor instabilities** in the formation of the **beads**. The figure shows a cylindrical discharge deformed by the m=0, m=1, and combined modes.



Since the magnetic pressure is much smaller than the kinetic pressure (v_A^2/c_i^2<<1), the hydromagnetic instabilities are much weaker than the hydrodynamic instability while this one lasts. Actually, it will be shown that the **beaded structure** can be explained solely in terms of the **hydrodynamic Rayleigh-Taylor instability** in a cylindrical discharge with **anomalous viscosity.**

Viscous fluid equations of motion

Mass conservation $\quad \frac{}{t} \quad \vec{u} \quad 0$

Equation of state $\quad \frac{d}{dt} \frac{p}{} \quad 0$

Faraday's law + Ohm's law (perfectly conductive fluid) $\quad \frac{\vec{B}}{t} \quad \vec{u} \quad \vec{B}$

Ampère's law $\quad \vec{B} \quad {}_0\vec{j}$

Navier-Stokes equation $\quad \frac{d\vec{u}}{dt} \quad p \quad \vec{j} \quad \vec{B} \quad {}^2\vec{u} \quad \frac{1}{3} \quad \vec{u} \quad \vec{g}$

Magnetostatic equilibrium

Equations without time dependence and with vanishing fluid velocity $\quad p \quad \vec{j} \quad \vec{B} \quad \vec{g} \quad \vec{B} \quad {}_0\vec{j} \quad \vec{B} \quad 0 \quad \vec{u} \quad 0$

External azimuthal magnetic field in a cylindrical discharge (surface current) $\quad B \quad r \quad \frac{{}_0 I}{2 \quad r} \quad I \quad 2 \quad aK$

Pressure balance for a narrow acceleration profile at the edge $\quad p_i \quad - \quad a \quad gL_g \quad p_e \quad \frac{{}_0 I^2}{8 \quad {}^2 a^2} \quad L_g \quad a$

Mass inside cylindrical shell of unit height, radius *a* and thickness L_g $\quad 2 \quad aL_g \quad - \quad a$

Stability analysis

Perturbation in the fluid velocity in terms of the Lagrangian displacement $\quad \vec{u} \quad \vec{} / t \quad i \quad \vec{}$

Linearized equation of motion $\quad \vec{g} \quad g \quad \vec{Q} \quad \vec{B}$

$^2\vec{} \quad \vec{} \quad p \quad p \quad \vec{} \quad \vec{} \quad g \quad \frac{1}{0} \quad \vec{Q} \quad \vec{B} \quad \frac{1}{0} \quad \vec{B} \quad \vec{Q} \quad i \quad \frac{4}{3} \quad \vec{} \quad \vec{}$

Perturbation of the normal vector $\quad \hat{n} \quad \vec{} \quad \hat{n} \quad \hat{n} \quad \hat{n} \quad \vec{} \quad \hat{n}$

Boundary conditions

Continuity of the magnetic field in the presence of the surface current $\quad \hat{n} \quad \vec{B}_a \quad 0 \quad for \quad \vec{K} \quad 0$

Fluid continuity and no-slip condition $\quad \hat{n} \quad \langle \vec{u} \rangle \quad 0 \quad \hat{n} \quad \langle \vec{u} \rangle \quad 0$

Stress continuity $\quad \hat{n} \quad \langle \quad \vec{u} \quad \hat{n} \quad \frac{}{2} \quad \vec{u} \quad \hat{n} \rangle \quad 0$

Pressure balance $\quad \langle p \quad - \quad a \quad g \quad \frac{B^2}{2 \quad 0} \quad 2 \quad \hat{n} \quad \vec{u} \quad \hat{n} \quad \frac{2}{3} \quad \vec{u} \rangle \quad 0$

Boundary conditions for the perturbations

Continuity of the magnetic field in the presence of the surface current $\quad \langle \hat{n} \quad \vec{B} \quad \hat{n} \quad \vec{} \quad \vec{B} \rangle \quad 0$

Fluid continuity and no-slip condition $\quad \hat{n} \quad \langle \vec{} \rangle \quad 0 \quad \hat{n} \quad \langle \vec{} \rangle \quad 0$

Stress continuity $\quad \hat{n} \quad \langle \quad \hat{n} \quad \vec{} \quad \vec{} \quad \hat{n} \rangle \quad 0$

Pressure balance $\quad \langle \quad p \quad - \quad a \quad g \quad \hat{n} \quad \frac{2}{3} i \quad \vec{} \quad - \quad a \quad \vec{} \quad \vec{g} \quad \frac{\vec{B} \quad \vec{B}}{0} \quad \vec{} \quad \frac{B^2}{2 \quad 0} \quad 2i \quad \hat{n} \quad \vec{} \quad \hat{n} \rangle \quad 0$

Differential equations for the perturbations

Magnetic scalar potential outside the discharge $\quad \vec{B} \quad 0 \quad m$

Equilibrium magnetic field $\quad B \quad r \quad B \quad a \quad a/r \quad r \quad a$

Differential equations (weak viscosity and low-frequency) $\quad {}^2 {}_m \quad 0$

$e \quad {}^2 \vec{}_e \quad {}_e p_e \quad \vec{}_e \quad {}_e \vec{g} \quad \vec{}_e$

$i \quad {}^2 \vec{}_i \quad {}_i p_i \quad \underbrace{4i \quad {}_i /3}_{compressive\ energy\ 0} \quad \vec{}_i \quad {}_i \vec{g} \quad \vec{}_i \quad \underbrace{i \quad {}_i \quad \vec{}_i}_{vorticity\ 0}$

Cylindrical symmetry $\quad \vec{} \quad {}_r \quad r \quad \hat{r} \quad r \quad \hat{} \quad {}_z \quad r \quad \hat{z} \quad e^{im \quad ikz}$

Dispersion relation

Normal pressure balance $\quad {}_e p_e \quad {}_e \quad - \quad a \quad g \quad {}_e \quad \hat{n} \quad \frac{\vec{B} \quad \vec{B}}{0} \quad \vec{}_e \quad \frac{B^2}{2 \quad 0} \quad {}_i p_i \quad \vec{}_i \quad - \quad a \quad g \quad {}_i \quad \hat{n} \quad \underbrace{2i \quad {}_i \hat{n} \quad \vec{}_i \quad \hat{n}}_{\sim surface\ tension} \quad 0$

Dispersion relation $\quad \frac{{}_i I_m \quad {}_i a}{{}_i I_m \quad {}_i a} \quad \frac{{}_e K_m \quad {}_e a}{{}_e K_m \quad {}_e a} \quad 2 \quad 2 \quad \frac{{}_i I_m \quad {}_i a}{I_m \quad {}_i a} \quad {}_i \quad 1 \quad \frac{m^2 K_m \quad ka}{kaK_m \quad ka} \quad \frac{- \quad a \quad v_A^2}{a} \quad 2 \quad - \quad a \quad g \quad 0$

$c_i \quad \sqrt{{}_i p_i / \quad {}_i} \quad c_e \quad \sqrt{{}_e p_e / \quad {}_e} \quad v_A \quad \sqrt{B^2 \quad a \quad / \quad {}_0 \quad - \quad a} \quad {}_i \quad \sqrt{k^2 \quad {}^2 / c_i^2} \quad {}_e \quad \sqrt{k^2 \quad M^2 \quad {}^2 / c_i^2} \quad M \quad c_i / c_e$

Approximate dispersion relation

Short wavelength limit $\quad {}^2 \quad \frac{i \quad {}_i k^2}{- \quad a} \quad \frac{v_A^2}{2a} \quad g \quad k \quad 0$

Interchange instability for negligible viscosity (large *g* in an expanding discharge stabilizes interchange instabilities) $\quad i \quad \sqrt{\frac{v_A^2}{2a} \quad g \quad k}$

Rayleigh-Taylor instability for negligible magnetic field (abrupt interface in a contracting discharge - last stage of the channel decay) $\quad i \quad \sqrt{|g|k} \quad g \quad 0$

Maximum growth-rate $\quad i \quad {}_{\max} \quad \sqrt{\frac{|g|}{L}} \quad k_{\max} \quad \frac{1}{L} \quad L \quad \frac{\langle \quad \rangle_a}{a} \quad a$

Viscosity effects

Balance between inertial and viscous forces $\quad {}^2 \quad i \quad {}_i \quad k^2$

Viscous forces prevail when $\quad R \quad {}_i k^2 \quad |i| \quad {}_i \quad {}_i / \quad {}_i$

Reynolds number for anomalous viscosity $\quad R \quad u\ell / \quad {}_i$

Estimates of velocity fluctuations and scale length of turbulence $\quad u \quad K_1 c_i \quad \ell \quad K_2 a$

Initially $\quad R \quad 1 \quad \ell \quad L \quad K_1 \quad {}_i / \quad L \quad c_i \quad K_2 \quad L \quad / a \quad (L \quad 0.2mm \quad a \quad 8cm)$

Fully developed turbulence $\quad K_1 \quad 1 \quad L \quad / a \quad K_2 \quad 1/3$

$T_i \quad 10{,}000K \quad {}_i \quad 0.0059 m^2 / s \quad c_i \quad 2.0 km / s \quad R \quad 10{,}000$

Scaling for the Rayleigh-Taylor instability

Maximum growth-rate is $\quad i \quad {}_{\max} \quad g^2 / \quad R \quad {}_i \quad {}^{1/3} \quad k_{\max} \quad |g| / \quad R \quad {}_i \quad {}^2 \quad {}^{1/3}$

Comparing with the previous estimate $\quad i \quad {}_{\max} \quad \sqrt{|g| / L}$

boundary can be assumed abrupt when $\quad L \quad R \quad {}_i \quad {}^2 / |g| \quad {}^{1/3}$

At the onset of the instability the molecular viscous forces define the scale length $\quad R \quad 1$

giving an estimate of the "gravitational" acceleration $\quad |g| \quad {}_i^2 / L^3$

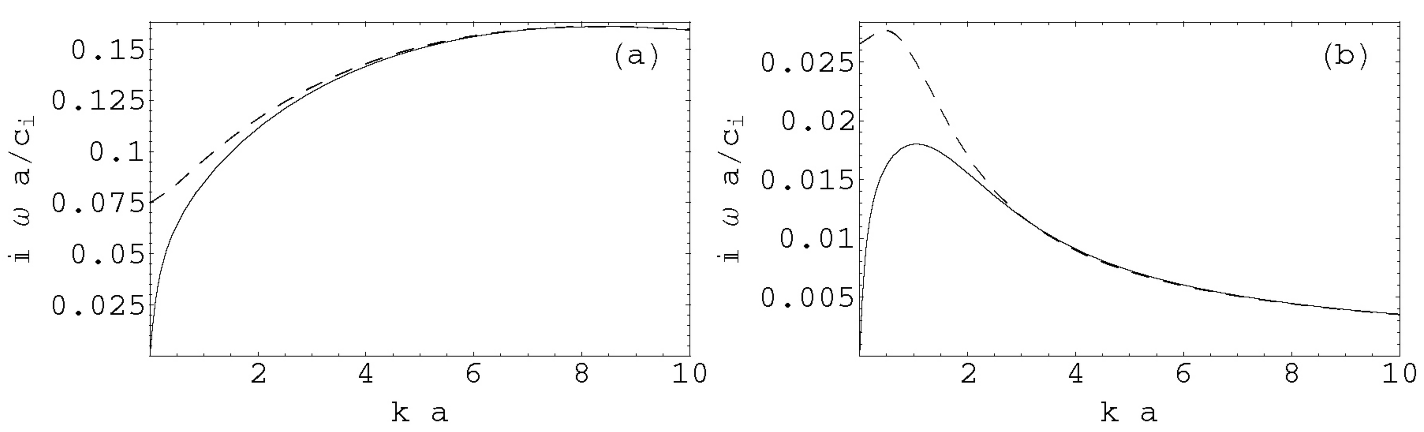
and a scaling for the e-folding time and wavelength at maximum $\quad L^2 R^{1/3} / \quad {}_i \quad 2 \quad L \quad R^{2/3}$

Application to atmospheric discharges

Lightning parameters
continuous line: m=0 (sausage)
dashed line: m=1 (kink)

$a \quad 8cm \quad T_i \quad 10{,}000 \quad L \quad 0.2mm \quad \sim L_g \quad I \quad 1kA \quad \sim 200 \quad s$

$7 \quad s \quad 150 \quad s \quad 1mm \quad 0.6m \quad for \quad ka \quad 1$



(a) $L \quad 0.5mm \quad R \quad 1{,}000$

(b) $L \quad 1.0mm \quad R \quad 10{,}000$

$10 \quad ka \quad 0.5 \quad 5.0cm \quad 1.0m \quad 0.2 \quad i \quad a / c_i \quad 0.03 \quad 0.20ms \quad 1.3ms$

Conclusions

In the beginning of the **contracting stage** the "gravitational" acceleration is relatively strong, the **Rayleigh-Taylor instability** rises very fast and the turbulence sets in on the small length scale L_p. During the contraction the turbulence develops, the large scale fluctuations fill the arc channel and the **viscosity becomes strongly anomalous, shifting the wavelength** of the most unstable modes to values of the order of and larger than the channel radius *a*. This process takes about 1ms, before the instability weakens and the **spatial structure becomes frozen**. From this point on one may conjecture that the discharge channel diffuses for a few milliseconds before the turbulence decays in the absence of a driving energy source. The visible pictures correspond to a diffuse channel showing the frozen spatial structure of the instabilities during the history of their evolution.