

## FLEXIBLE SPACE SYSTEM STATES AND PARAMETERS ESTIMATION

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Nowadays an increasing number of space missions are using satellites with a rigid hub and long and/or wide flexible appendices, such as solar panels, communication antennas, telescopic structures, and robotics flexible arms. The increasing need for better pointing accuracy of antennas connected to the rigid part of the spacecraft leads to a requirement for more efficient controllers, where more accurate identification process play an important rule into the closed loop system. To meet the requirements for pointing accuracy, flexible parameters as elastic displacement, which are of great importance for control tasks should be continuously identified in the space environment. This paper presents investigation results of elastic displacement identification using the Kalman Filter methodology. A flexible Euler-Bernoulli beam, connected to a rigid core with torques as input and angles and angular velocities as outputs is used as simple mathematical model of a rigid-flexible satellite to apply the Kalman filter identification algorithm proposed. The Kalman filter is tested under several conditions considering cold start with coarse initial knowledge and varied measurement noise levels. At the end comments are drawn about the robustness of the proposed procedure and feasibility of implementation within the control system loop.

### INTRODUCTION

The use of small satellites has been a fast, simple and of low cost way of reaching the space in missions with the most several applications<sup>1, 2</sup>. However, in order to conquer the space it is necessary to launch spacecraft that contain a mix of rigid/flexible structures. These missions are more complex because the satellites have a great number of components like, solar panels, antennas, cameras and mechanical manipulators. As a results, the knowledge of the flexibility influence of such structure play an important role in the dynamics behavior as well as in the Attitude Control System<sup>3,4</sup> (ACS) performance. Others important aspects in the study of the dynamics and control of flexible space structures are: the degree of interaction between the rigid and flexible motion, maintenance of the ACS performance in face the uncertainties of the mathematical model, damping residual vibrations and dynamic parameters identification<sup>5,6</sup> in order to keep pointing precision. This paper presents an identification procedure using the Kalman Filter methodology that may be used to estimate the elastic displacement and identify system parameters in space. Section 2 presents a mathematical model of a simple spacecraft based on a flexible Euler-Bernoulli beam connected to a rigid core. The equations of motion are also derived where the torque is used as input and angles and angular velocities as outputs.

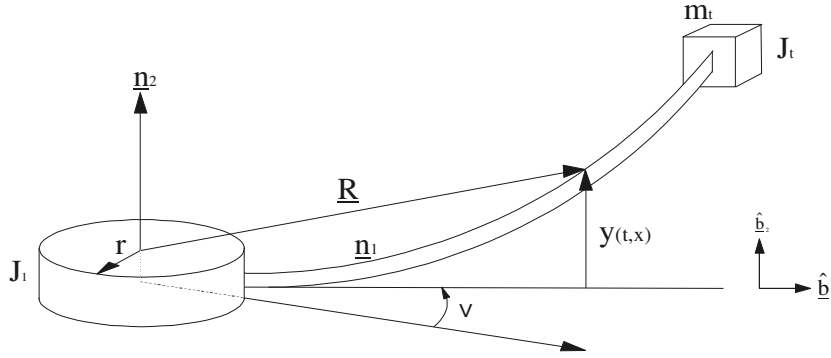
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In section 3 the Kalman filter estimation algorithm is introduced. Section 4 shows the simulation where the Kalman filter is tested under several conditions considering cold start with coarse initial knowledge and varied measurement noise levels. Section 5 concludes the paper.

## SPACECRAFT MATHEMATICAL MODEL

The satellite mathematical model used is composed of a rigid platform with two flexible appendices and masses in the extremities of the appendices. The appendices are identical and opposite, being considered as beam connected to the platform, and subjects to rotational and vibrational motion. In order to derive the equations of motion for this model, one applies the Lagrange methodology, starting from the expression of the kinetics and potential energy of the system. Figure 1 illustrates the system composed by rigid hub, an elastic appendage with a mass in the extremity, which is included in the derivation of the equation of motion, but is disregard in the model used in the Kalman filter application.



**Figure 1** Satellite Model with a Rigid Hub and a Flexible Appendage.

One considers that the inertial reference system coincides with the origin of the fixed reference system in the rigid body, which is represented by the axes  $n_1, n_2, n_3$ . The fixed reference system is coincident with the mass center of the rigid body, which is characterized by the axes  $b_1, b_2, b_3$ . The vector  $r$  represents the radius of the rigid body. The vector  $x$  represents a position of a measured mass element along the axis  $b_1$  direction, in no deformed form with respect to fixed reference system.  $R$  gives the vector position of any point in the appendage relative to the inertial reference system. The vector of elastic displacement (elastic deformation) measured perpendicular to the axis  $b_1$  is represented by  $y(x, t)$ . Therefore, the vector position of any point in the deformed appendage form relative to the inertial reference system is given by

$$\underline{R} = (r + x) \hat{b}_1 + y \hat{b}_2 \quad (1)$$

Considering that  $\dot{\theta}$  is the angular velocity of the satellite the vector velocity is given by

$$\underline{\dot{R}} = \frac{d}{dt}(\underline{R})_i = \frac{d}{dt}(\underline{R})_B + \dot{\theta} \times \underline{R} \quad (2)$$

In order to get the total velocity expression of the satellite hub and appendages, one substitutes Eq. (1) into (2), so as

$$\underline{\dot{R}} = -\dot{\theta} \ y \ \underline{\hat{b}}_1 + [\dot{\theta}(r+x) + \dot{y}] \ \underline{\hat{b}}_2 \quad (3)$$

## EQUATIONS OF MOTIONS

The equations of motion are derived for two generalized coordinates  $x_i$  ( $i=1,2$ ), the angular rotation  $\theta(t)$  and elastic motion  $q(t)$ , using the Lagrange Equation given by

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial V}{\partial x_i} = F_{x_i} \quad (4)$$

where  $T$ ,  $V$  and  $F_{x_i}$  represent the total kinetics energy, the total potential energy and the generalized forces of the system.

The total kinetics energy of the system is sun of the kinetics energy of the hub, the appendage and the tip mass

$$\begin{aligned} T &= T_{hub} + T_{app} + T_{tip} \\ &= \frac{1}{2} J_1 \dot{\theta}^2 + \frac{1}{2} \left[ \int_0^L \rho (r+x)^2 dx + m_t (r+x)^2 + J_t \right] \dot{\theta}^2 + \frac{1}{2} \int_0^L \rho \dot{y}^2 dx + m_t \{\dot{y}(L)\}^2 + J_t \{\dot{y}'(L)\}^2 \\ &\quad + 2\dot{\theta} \left\{ \int_0^L \rho \dot{y} (r+x) dx + m_t (r+L) \dot{y}(L) + J_t \dot{y}'(L) \right\} \end{aligned} \quad (5)$$

where  $J_1$  and  $J_t$  are the hub and the appendage moment of inertia,  $L$  and  $\rho$  are the length and the density of mass of the appendage,  $m_t$  is the mass at the end of the appendage and  $y(x, t)$  represents the elastic displacement.

The total potential energy of the system is only due to the elastic deformations of the appendage and it is given by

$$V = \frac{1}{2} \int_0^L EI \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx + \frac{1}{2} \int_0^L EI \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx = \int_0^L EI \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx \quad (6)$$

where  $E$  represents the module of elasticity and  $I$  the moment of inertia of the beam.

The discretization of the system is done using assumed mode method.<sup>7</sup> Therefore, the elastic displacement  $y(x,t)$  is substituted by

$$y(x, t) = \sum_1^N \phi_j(x) q_{ji}(t) \quad (7)$$

where  $\phi_j(x)$  are the admitted functions and  $q_{ji}(t)$  are the elastic coordinates.

Substituting Eqs.(7), (6) and (5) into Eq.(4) and after some manipulation the equations of motion are given by

$$J_1 \ddot{\theta} + 2\ddot{\theta} \left[ \int_0^L \rho(r+x)^2 dx + m_l(r+L)^2 + J_l \right] + 2 \left[ \int_0^L \rho(r+x)\phi_l(x)dx + m_l(r+L)\phi_l(L) + J_l\phi_l'(L) \right] \ddot{q} = F_x \quad (8)$$

$$2\ddot{\theta} \left\{ \int_0^L \rho(r+x)\phi_l(x)dx + m_l(r+L)\phi_l(L) + J_l\phi_l'(L) \right\} + 2 \left\{ \int_0^L \rho\phi_j(x)\phi_l(x)dx + m_l\phi_j(L)\phi_l(L) + J_l\phi_j'(L)\phi_l'(L) \right\} \ddot{q} + \\ + 2 \int_0^L (EI)\phi_j''(x)\phi_l''(x)q dx = 0 \quad (9)$$

The Eqs.(8) and (9) are associated with the rigid and flexible motion, respectively. They can be put in matrix form

$$\begin{bmatrix} \hat{J} & M_{bq}^T \\ M_{bq} & M_{qq} \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{qq} \end{bmatrix} \begin{bmatrix} \theta \\ q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (10)$$

where  $u$  represents the control input associated with  $F_x$ ,  $\hat{J}$  is the total inertia moment of the system,  $M_{bq}$  represents the sub-matrix associated with the rigid and flexible motion,  $M_{qq}$  represents the sub-matrix associated with the flexible motion and  $K_{qq}$  represents the sub-matrix associated with flexible body.

Considering  $z = [\theta, q]^T$ , Eq. (10) can be written in compact form

$$M \ddot{z} + K z = D u \quad (11)$$

where  $M$  is the mass matrix,  $K$  is the stiffness matrix of the system and  $D$  is the control influence matrix.<sup>5</sup>

Solving the auto-value problem of the Eq.(11) and considering the variable transformation  $z = \Phi \eta$ , where  $\Phi$  are the auto-vectors and  $\eta$  is the new variable. Eq.(11) can be put in modal form

$$\tilde{M} \ddot{\eta}(t) + \tilde{C} \dot{\eta}(t) + \tilde{K} \eta(t) = \tilde{D} u \quad (12)$$

where  $\tilde{M}$ ,  $\tilde{C}$ ,  $\tilde{K}$  and  $\tilde{D}$  represents mass, damping, stiffness and control influence matrices in modal form.

## KALMAN FILTER IDENTIFICATION ALGORITHM

The Kalman filter is a computational algorithm containing a sequence of time and measurement updating of estimates of the system state.<sup>8</sup> The Kalman filter can incorporate dynamic noise in the dynamical model of the state. It is a real time estimator supplying the estimates for the instant that the measurement is available. The filter consists of two cycles:

- Time update
- Measurement update

In short, the Kalman filter processes measurements to produce an estimate of minimum variance of the state of a system using the knowledge of the dynamics of the system, the measurement, the statistics of the noise, and the errors of the measurements, besides the information of the initial condition.

### State Dynamics Model

Let the state dynamical model be represented by

$$\dot{x} = A x + G \omega \quad (13)$$

where  $x = (\eta_1, \eta_2, \dot{\eta}_1, \dot{\eta}_2)$  are the states associated with the angular displacement ( $\eta_1 = \theta$ ) and the elastic displacement ( $\eta_2 = q$ ) continuously variant in the time. The matrix  $G$  define how the noise enter in the system,  $\omega$  is the continuous dynamic noise and  $A$  is named the system matrix which contain the total dynamic information of the system and is given by:

$$A = \begin{bmatrix} 0 & | & I \\ \hline -\tilde{K} & | & -\tilde{C} \end{bmatrix} \quad (14)$$

The system matrix is formed by the identity matrix  $I$ , by  $\tilde{K} = \text{diag} \{ 0, \omega_i^2 \}$  the matrix containing the squared natural frequencies, and the modal damping matrix  $\tilde{C} = \{ 0, 2\xi_i \omega_i \}$ .

### Measurements Model

$$y = H x + v \quad (15)$$

where  $y$  is the measurement vector composed by the angle  $\theta$  and angular velocity  $\dot{\theta}$  measured by the angular position and angular velocity sensor.

The  $H$  matrix relates the measurements to the state by

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (16)$$

and  $v$  represents a white noise vector to model the errors during the measurement process. One assumes for the angular position and velocity a nominally a standard deviation of  $0.1^\circ$  and  $0.01^\circ/\text{s}$ , respectively. Therefore, the white noise vector has the following statistical characteristics

$$v_\theta = N(0, 0.1^\circ) ; \quad v_{\dot{\theta}} = (0, 0.01^\circ/\text{s}) \quad (17)$$

### Time update

In this filter cycle, the time updated state  $\bar{x}$  and the covariance  $\bar{P}$  estimates are computed using the dynamical model of the system given by

$$\dot{\bar{x}} = A \bar{x} \quad (18)$$

with initial condition  $\bar{x}_{k-1} = \hat{x}_{k-1}$ , and

$$\dot{\bar{P}} = A \bar{P} + \bar{P} A^t + G Q G^t \quad (19)$$

with initial condition  $\bar{P}_{k-1} = \hat{P}_{k-1}$ . Eq. (19) is known as the continuous Riccati equation.

### Measurement update

This cycle updates the state and covariance matrix at instant k due to measurement  $y_k$ , by means of the measurement model given by Equation (15). The measurements of instant k provide the information to update the state and covariance. The equations that follow describe the measurement update cycle of Kalman filter

$$K = \bar{P} H^t (H \bar{P} H^t + R)^{-1} \quad (20)$$

$$\hat{P} = (I - K H) \bar{P} \quad (21)$$

$$\hat{x} = \bar{x} + K (y - H \bar{x}) \quad (22)$$

where K represents the Kalman gain,  $\hat{P}$  and  $\hat{x}$  are the covariance and the state updated.

The errors between the actual state and the estimated state will be used to evaluate the algorithm performance for the tests carried out, and is given by

$$\Delta \epsilon_i = x_i - \hat{x}_i \quad (23)$$

The estimated error standard deviation is given by

$$\Delta \hat{\epsilon}_i = P_{ii}^{1/2} \quad (24)$$

### SIMULATIONS

The aim of the simulations is implementing and testing the proposed Kalman filter methodology to estimate the elastic displacement and velocity, assuming that the angular displacement and velocity are measured. The analysis is performed through the utilization of the dynamical model Eq.(13) and Eqs. (15) to (22), which represent the time and measurement update of the state and covariance via the Kalman filter. The investigation philosophy is, first of all, to analysis the nominal case using the initial conditions and parameters shown in Table 1.

**Table 1**

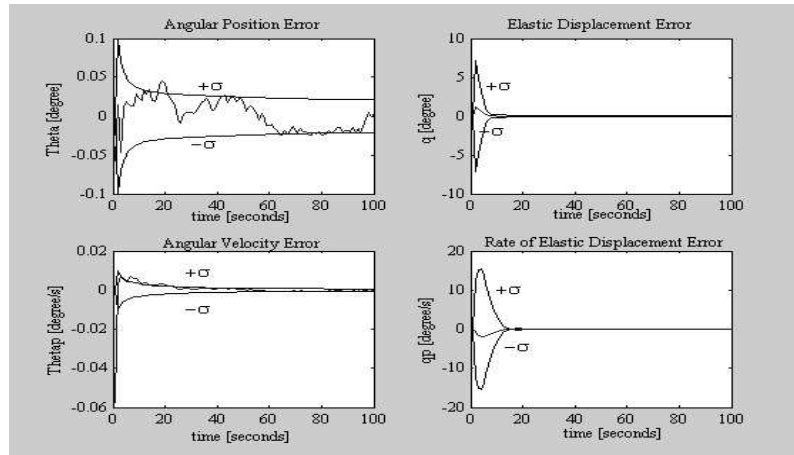
**INITIAL CONDITIONS AND PARAMETERS OF THE KALMAN FILTER**

Symbol	G	$R_{\theta}$ ( $^{\circ}$ )	$R_{\dot{\theta}}$ ( $^{\circ}/s$ )	$Q_{\theta}$ ( $^{\circ}$ )	$P_{\dot{\theta}}$ ( $^{\circ}/s$ )	$\theta_0$ ( $^{\circ}$ )
Values	$I_4$	0.01	0.001	$10^{-6}$	100	0.1
Symbol	$\dot{\theta}_0$ ( $^{\circ}/s$ )	$Q_q$ ( $^{\circ}$ )	$Q_{\dot{q}}$ ( $^{\circ}/s$ )	$P_{\theta}$ ( $^{\circ}$ )	$Q_{\dot{q}}$ ( $^{\circ}/s$ )	$P_{q,\dot{q}}$ ( $^{\circ}/s$ )
Values	0.01	$10^{-8}$	$10^{-6}$	$10^{-2}$	$10^{-8}$	100

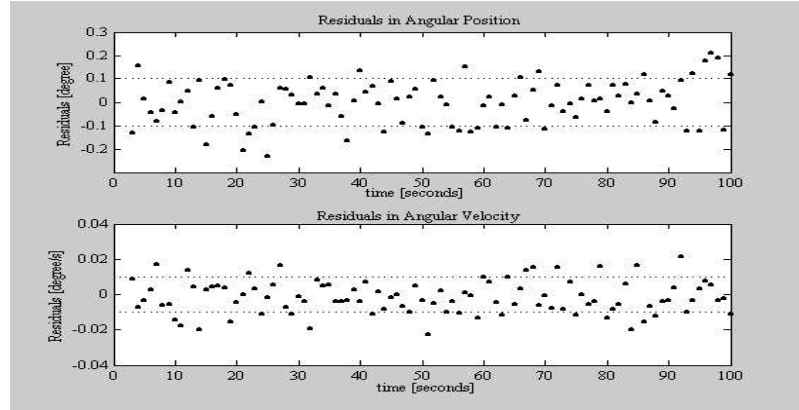
In the sequel, the same conditions used in the nominal case are applied to analyze the behavior of the filter when non-typical measurement errors are imposed to the system. In particular, two cases comprising precise and imprecise measurements are compared to the nominal case. It should be mentioned that in the space area the Kalman filter approach has been used in many application for estimation of states associated with angular position and angular velocity. However, it is not of the authors' knowledge that this approach has been used before for flexible states estimation like elastic displacements and its variation.

### Nominal Case

Figure 2 shows the actual and standard deviations for the angular position, elastic displacement, angular velocity and elastic displacement rate, respectively. Errors were calculated according to Eqs (23)-(24). From Figure 2, one notices that most of the actual errors are within one standard deviation. Convergence was obtained quickly (less than 20s) for all, with exception of the angular position, although it was within  $0.05^{\circ}$ .



**Figure 2 Errors of Angular and Elastic Displacement, Angular and Elastic Displacement Rate.**

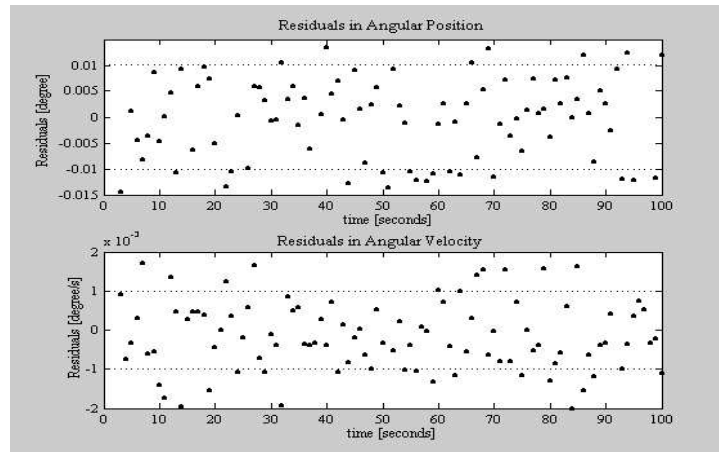


**Figure 3 Residual of Angular Position and Velocity for Nominal Case.**

Figure 3 shows the residual behavior of the angular position and velocity measurements for the nominal case. In this case the residuals are in good shape with one standard deviation around  $0.1^\circ$  and  $0.01^\circ/\text{s}$ , respectively.

### Over-accurate Measurements

In this case measurements are simulated with one order better accuracy. The aim is to verify if there is some relevant gain of accuracy in the identification filter when more accurate sensors are used. It can be observed in Figure 4 an apparent improvement in the residuals profile, one order better than the nominal case of Figure 3. This suggests better state estimates with respect to the nominal case. For this case the angular position and velocity measurements were corrupted with random gaussian noise of  $0.01^\circ$  e  $0.001^\circ/\text{s}$ , respectively.



**Figure 4 Residual of Angular Position Velocity for Over Accurate Measurement Case.**

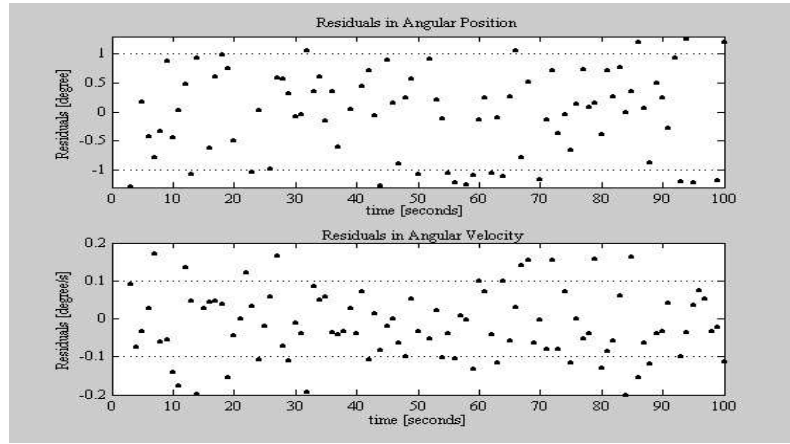


## Under accurate measurements

In the same way, one investigated the effects of less accurate (than nominal) measurements, with the aim of verifying if the accuracy is degraded up to the extent the filter does not estimate the states correctly.

From Figure 5 it can be seen the influence of increasing the measurement errors in the behavior of the estimation scheme. From the simulations it can be seen that even losing sensors accuracy the filter shape did not degrade when compared to the nominal case, in terms of convergence. The measurement errors in this case were random gaussian with  $1^\circ$  and  $0.1^\circ/\text{s}$  of standard deviation, respectively. The behavior of the residuals in this under accurate measurement case is shown in Figure 5.

Consistently the residuals RMS also increased one order of magnitude with respect to the nominal case. Moreover, the states were estimates without sign of divergence of the Kalman filter.



**Figure 5 Residual of Angular Position and Velocity for Under Accurate Measurements Case.**

Table 2 lists the mean and the standard deviation of the states corresponding to the rigid body and the flexible part of the satellite. Through the table it can be observed that for the three cases, namely nominal, over-accurate, and under-accurate measurements, the state components regarding the flexible body,  $q$  (elastic displacement) and  $\dot{q}$  (elastic displacement rate), did not suffer any meaningful change in the mean and in the standard deviation error. On the other hands, the state components of the rigid body,  $\theta$  (angular position) and  $\dot{\theta}$  (angular velocity), have suffered (as expected) with the variation of the noise in the measurement, which means that the use of the Kalman filter approach is quite adequate to obtain information of the elastic displacement and its rate. As a result, the controller can use that information in order to improve control system performance in term of time of response.

As for the robustness of the technique, it can be verified if one consider that in relation to the nominal case, all the cases tested converged to the expected level of errors, showing the robustness of the filter under several levels of measurement accuracy.

**Table 2**  
**MEAN AND STANDARD DEVIATION OF STATE ERRORS**

	<b>Nominal</b>	<b>Over accurate</b>	<b>Under accurate</b>
$\theta (^{\circ})$	$(10^{-3}) \pm (2 \times 10^{-1})$	$(-3.6 \times 10^{-4}) \pm (6 \times 10^{-2})$	$(1.6 \times 10^{-2}) \pm (1 \times 10^0)$
$q (^{\circ})$	$(3 \times 10^{-2}) \pm (1.6 \times 10^{-1})$	$(3 \times 10^{-2}) \pm (1.6 \times 10^{-1})$	$(3 \times 10^{-2}) \pm (1.6 \times 10^{-1})$
$\dot{\theta} (^{\circ}/s)$	$(2.7 \times 10^{-4}) \pm (6 \times 10^{-2})$	$(-5 \times 10^{-3}) \pm (6 \times 10^{-3})$	$(7 \times 10^{-2}) \pm (1.9 \times 10^{-1})$
$\dot{q} (^{\circ}/s)$	$(-1.23 \times 10^{-1}) \pm (4 \times 10^{-1})$	$(-1.2 \times 10^{-1}) \pm (4 \times 10^{-1})$	$(-1.2 \times 10^{-1}) \pm (4 \times 10^{-1})$

## CONCLUSION

In this work a satellite model composed of a central rigid body and two flexible appendages was used to apply an estimation methodology based on Kalman filter approach. The Lagrange formulation was used to derive the equations of motion of the satellite, and the discretization of the elastic motion was performed by the assumed mode method.

The Kalman filter methodology was implemented in order to estimate the satellite rigid and flexible mode (states) composed of the rigid and elastic displacement and their variation in time, considering that only the angular position and velocity measurements are available. Throughout several simulations it was possible to investigate the behavior of the state estimation errors for three distinct cases. In the first one, called nominal case, typical data were considered as the initial conditions of the filter. In this case, it was verified that the satellite position and angular velocity error estimates are within the errors allowed by the filter, being observed a great time for the convergence of the filter in the angular position component. For the elastic displacement and rate the convergence has occur in less than 20 seconds. Afterwards, two simulations considering non-typical conditions, that is, over accurate measurements and under accurate measurements have been investigated. For the over accurate case (as expected) it was detected a remarkable improvement in the real and estimated states with respect to the nominal case.

That results, shows that even in the presence of the no measured elastic deformation the procedure improve when one uses more accurate sensors. In the under-accurate case, it was detected that even with less accuracy of the sensors, the estimated state errors were not so degraded with respect to the nominal case, keeping the filter convergence in acceptable level. Therefore, having in mind the complexity of putting a sensor on the elastic parts of the satellite, the application of the Kalman filter methodology has been showed a good approach to estimate indirectly the flexible parameters of a rigid-flexible satellite. That approach becomes more promising when it is necessary to feedback the elastic measurements into the control system in order to assure better pointing conditions and/or better system performance. The Kalman filter has also shown to be a robust methodology since in the under-accurate case tested, it has maintained a good performance. A next step in that work is to investigate that happens with the performance of the Kalman filter approach used when one increases the numbers of flexible modes in the model.

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