Giant Many-Body Enhancement of Low Temperature Thermal-Electron–Acoustic-Phonon Coupling in Semiconductor Quantum Wires

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We show that in a semiconductor quantum wire the energy relaxation of the excited one-dimensional electron gas due to acoustic phonon emission is strongly enhanced at low temperatures by a subtle many-body effect arising from the phonon broadening caused by the electron–acoustic-phonon interaction. This broadening opens up a "virtual" channel for energy transfer from the electrons to the low energy tail of the phonon spectral function. This peculiarly one-dimensional many-body effect causes a very large enhancement of the power loss over the bare value and restores the Bloch-Grüneisen power law temperature dependence of the energy loss rate at low temperatures.

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Most of the "anomalous" (as compared to systems of higher dimensionality) transport properties of low-dimensional electronic systems arise because of the severe phase-space restrictions of reduced dimensionality. In a one-dimensional electron gas (1DEG) at zero temperature, the only possible low energy excitations have wave vectors of magnitude 0 or $2k_F$ (where k_F is the Fermi wave vector). If one considers the low temperature momentum relaxation of a 1DEG for instance, it has to occur through the transfer of momentum $2\hbar k_F$ from the electrons to the phonons or impurities. If one considers the low temperature energy relaxation of a 1DEG interacting with acoustic phonons, which have a linear dispersion of the form $\omega(q)=cq$, again it can only occur through the emission of $2k_F$ phonons. Therefore one would expect that the low temperature ($k_BT \ll E_F$; k_B is the Boltzmann constant, T the temperature, and E_F the Fermi energy) energy relaxation of an electron gas interacting with acoustic phonons, being mediated entirely by phonons of energy $2\hbar ck_F$, would follow an exponential behavior $\exp(-2\hbar ck_F/k_BT)$ when $k_BT \ll 2\hbar ck_F$, simply because there is no phase space at low temperatures for the scattering of an electron in a 1DEG to excite a phonon of lower wave vector and frequency. Thus, in a 1DEG there is no conventional Bloch-Grüneisen algebraic temperature dependence regime in the momentum or energy relaxation rate at low temperatures—all low temperature bare acoustic phonon scattering processes are exponentially activated in a 1DEG. This is a striking theoretical difference of a 1DEG from higher-dimensional systems.

However, the electron-phonon interaction also renormalizes the phonon dispersion relation. Because the electron gas is polarizable, and can track the deformation of the lattice to take advantage of regions of lower energy of the electron states, it costs less energy to create a lattice deformation in the presence of the electron gas than otherwise. This softening is the cause of the Kohn anomaly [1] in the phonon dispersion around $2k_F$, and can eventually drive $\omega(2k_F)$ to zero (Peierls transition) in a one-dimensional system [2]. In addition to renormalizing the phonon frequency, the interaction also causes a broadening of the phonon dispersion relation, which is usually not of much consequence in higher dimensions. This means that the frequency of the phonons interacting with the electrons is not a well defined quantity, but rather a resonance: There is a nonzero amplitude for phonons of arbitrary frequencies for wave vectors close to $2k_F$. As a consequence, the energy relaxation of the 1DEG interacting with phonons can occur through the emission of $2k_F$ phonons not just with the bare frequency $2\hbar ck_F$, but with all possible frequencies (and, in particular with arbitrarily low frequencies), and the simple exponential behavior arising from the existence of just one phonon energy scale $2\hbar ck_F$ alluded to above is lost. This broadening is unimportant for acoustic phonons interacting with a higher-dimensional electron gas, because there scattering by phonons of arbitrarily small wave vectors is allowed anyway, and some broadening of the phonon spectral function is a negligibly small effect. The integrated rates in higher dimensions can be obtained [3] by substituting for the spectral weight of the phonon the single bare phonon pole. Thus, many-body broadening lifts the severe phase-space restriction of a 1DEG with respect to the thermal electron-acoustic phonon scattering. We emphasize that in higher (i.e., 2 and 3) dimensions this many-body renormalization of electron–acoustic-phonon interaction is quantitatively (and qualitatively) totally insignificant and is usually neglected in theoretical calculations.

An observable consequence of this is that, for k_BT much smaller than $2\hbar ck_F$, the energy relaxation rate in a 1DEG is strongly enhanced over the "bare phonon" result, and the asymptotic low temperature behavior of the energy relaxation of a 1DEG in a semiconductor quantum wire is not the one characterized by an Arrhenius type $\exp(-2\hbar ck_F/kT)$ behavior, but an algebraic temperature dependence, T^n, with a substantially higher power loss than the simple, unrenormalized result. Thus, Bloch-Grüneisen behavior is restored in a 1DEG due to a subtle many-body electron-phonon interaction effect.

We now quantify the above claims by evaluating the expression for the power loss per electron in a GaAs quantum wire of width a and one-dimensional electron density n:

$$P=\frac{1}{n}\sum_q |M(q)|^2\int_{-\infty}^{+\infty}\frac{d\omega}{\pi}\hbar\omega n_B(\hbar\omega/kT)\times \mathrm{Im}\chi(q,\omega)\mathrm{Im}D(q,\omega)\,, \qquad (1)$$

where $n_B(\hbar\omega/kT)$ is the Bose occupation factor at a temperature T (where T is the electron temperature, higher than the lattice temperature which is taken here to be zero), $D(q,\omega)$ is the phonon propagator, with $\mathrm{Im}D(q,\omega)$ defining the spectral weight function of the phonons, $|M(q)|^2$ the squared matrix element for the one-dimensional electron-acoustic phonon deformation potential interaction, given by

$$|M(q)|^2=\frac{\hbar\Xi^2q^2}{2\rho a^2\omega_q}\,, \qquad (2)$$

and $\chi(q,\omega)$ is the reducible polarizability of the electron gas, given in the random phase approximation by

$$\chi(q,\omega)=\chi_0(q,\omega)[1-V(q)\chi_0(q,\omega)]^{-1}\,. \qquad (3)$$

In Eq. (3), $\chi_0(q,\omega)$ is the finite temperature Lindhard polarizability [4] of a 1DEG, and $V(q)$ the matrix element of the electron-electron interaction in the lowest subband of a quantum wire with an effective width a. Using a parabolic well model for the lateral confinement [5] of the quantum wire electrons, $V(q)$ is given by

$$V(q)=\frac{2e^2}{\varepsilon_0}K_0(qa)\,, \qquad (4)$$

where ε_0 is the lattice dielectric constant of GaAs and K_0 is the modified Bessel function of zeroth order. The renormalized phonon propagator is given by

$$D(q,\omega)=\frac{2\hbar cq}{(\hbar\omega+i\eta)^2-(\hbar cq)^2-2\hbar cq|M(q)|^2\chi(q,\omega)}\,. \qquad (5)$$

The last term in the denominator of Eq. (5) corresponds to the phonon renormalization caused by the electron-phonon interaction. Neglecting this renormalization D has poles at $\omega=\pm cq$ corresponding to the bare phonon frequency in the absence of the electron gas. If the bare phonon propagator is substituted into Eq. (1), one gets the simple well-known expression for the power loss P that can be obtained by elementary kinetic reasoning, by considering the difference between the transition rates between electronic eigenstates of the system with emission or absorption of phonons [3,6].

A numerical evaluation of Eq. (1) is shown in Fig. 1 as a function of temperature, and is compared to the result obtained using the bare phonon approximation. Materials parameters for GaAs used in the computations are $\Xi=7.0$ eV (deformation potential), $\varepsilon_0=12.8$, $m/m_0=0.066$ (electron effective mass over free electron mass, $\rho=5.307$ $g\,\mathrm{cm}^{-3}$ (density) and $c=4.73\times10^5$ $\mathrm{cm\,s}^{-1}$.

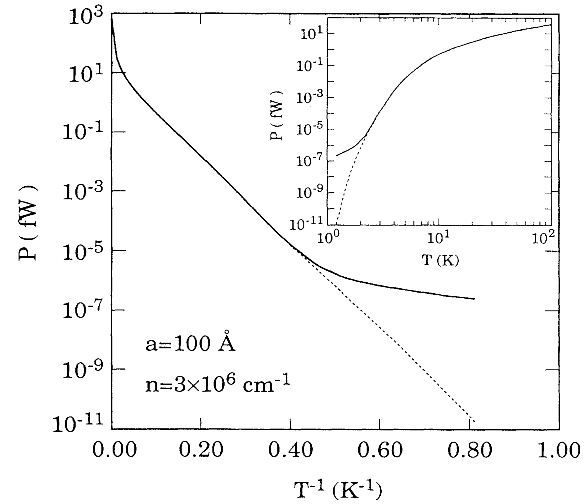

FIG. 1. Power loss per electron in a semiconductor quantum wire, as a function of electron temperature, for GaAs parameters, plotted both as an Arrhenius plot and (inset) as a log-log plot. The dashed line is the result obtained by neglecting the many-body renormalization of the phonons.

All the qualitative behavior as a function of temperature discussed above appears in this result. For $2\hbar ck_F\gg k_BT$, the power loss tends to be linear in T, but an exponential regime sets up quickly as the temperature is lowered, and persists indefinitely for the bare phonon approximation. For even lower temperatures, however, a power law behavior reappears in the many-body calculation with renormalized phonons. In fact, around $T=1$–2 K, the virtual phonon emission causes an enhancement of several orders of magnitude in the power loss over the real phonon emission.

To see more clearly how this result comes about, we rewrite Eq. (1) in terms of dimensionless quantities $x=q/2k_F$, $y=\hbar\omega/E_F$, $t=k_BT/E_F$, $\tilde D=E_FD$, $\tilde\chi=\chi/N_F$, and $\tilde c=mc/4\hbar k_F$, where N_F is the density of states of the 1DEG at the Fermi energy; and, after some algebraic manipulation, obtain

$$P=P_0\int_0^{+\infty}dx\,x\int_{-\infty}^{+\infty}\frac{dy}{\pi}\frac{y}{\tilde c}n_B(y/t)\mathrm{Im}\tilde\chi(x,y,t)\times\mathrm{Im}\tilde D(x,y,t)\,, \qquad (6)$$

$$P_0=4\frac{\Xi^2}{\hbar}\frac{nm}{\rho a^2}\,. \qquad (7)$$

The power loss is a sum of contributions from each momentum x and energy y. For $t\ll1$, $\mathrm{Im}\tilde\chi$ is significantly different from zero only in the region

$$4x|x+x_0|<y<4x(x-x_0)\,, \qquad (8)$$

$$x_0=(\mu/E_F)^{1/2}\,, \qquad (9)$$

where μ is the chemical potential, and x_0 approaches 1 as

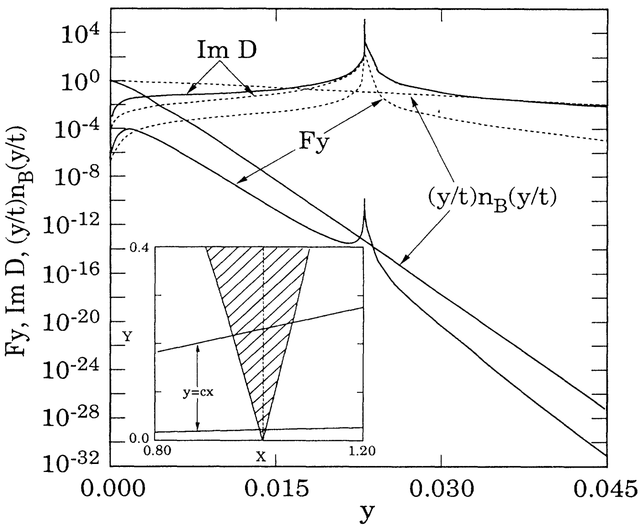

FIG. 2. Im$\tilde{D}$, $(\hbar\omega/k_BT)n_B(\hbar\omega/k_BT)$, and the product of these two factors, giving the integrand Fy of the integral of Eq. (6), for the same parameters as in Fig. 1, and for $T=10$ K (dashed lines) and $T=1$ K (solid lines), showing how the main contribution to the integral shifts from the only peak of Im$\tilde{D}$ (phonon peak) to the low-ω region at low T. The inset shows the region in phase space that contributes to the power loss in one dimension. The upper line for the phonon dispersion $y=\tilde{c}x$ corresponds to $n=3\times10^5$ cm^{-1}, and the lower one to $n=3\times10^6$ cm^{-1}. The dashed line is $x=(\mu/E_F)^{1/2}$, along which the quantities shown in the main body of the figure are evaluated.

t goes to zero. The inequality of Eq. (8) specifies the allowed region for electron-hole pairs in the 1DEG. As illustrated in the inset of Fig. 2, in only a small region around $x=1$ there is a significant overlap of Imχ and ImD. If one neglects the phonon renormalization, a good approximation for P can be obtained by replacing the integrals in Eq. (6) by an integral over the path $y=\tilde{c}x$ from $x=1-\tilde{c}/4$ to $x=1+\tilde{c}/4$. Since $\tilde{c}/4<1$, we can approximate the path length by $\tilde{c}/2$, and the argument of the integrand by its value at $x=1$ and $y=\tilde{c}$. This is the "phase-space" argument, from which one gets the approximate analytical result

$$P_B=\frac{P_0\tilde{c}\pi}{8}\frac{n_B(\tilde{c}/t)}{|\varepsilon(1,\tilde{c},t)|^2}, \tag{10}$$

where we have used the fact that Im$\tilde{\chi}=$Imχ_0/ε and Im$\tilde{\chi}(x,y,t)\approx\pi/4x$ in the integration region. The subscript in P_B indicates that this is the result obtained with bare phonons. This estimate explains the numerical results shown by the dashed line in Fig. 1, and makes it clear that the exponential behavior comes from the fact that at low temperatures energy relaxation in a 1DEG occurs only via the emission of $2k_F$ phonons.

In the main body of Fig. 2 we show Im$\tilde{D}(x,y,t)$, $n_B(y/t)$, and the argument of the y (frequency) integral of Eq. (6), for the same parameters used to obtain the curve in Fig. 1, for two different temperatures. The spectral weight function is weakly dependent on temperature and has only one peak, which defines the renormalized phonon frequency. It also shows how, as the temperature is lowered, this peak contributes less to the integral over y than the low frequency (low y) region. Again we stress that there is no resonant behavior at low y. The collective mode for the electron gas (plasmon) lies far above the acoustic phonon, and the plasmon-phonon coupling effect is not operative here. The result of the integration over y is shown in Fig. 3, and compared to the bare phonon result. For higher temperatures, the phonon resonance still gives the dominant contribution, and there is no

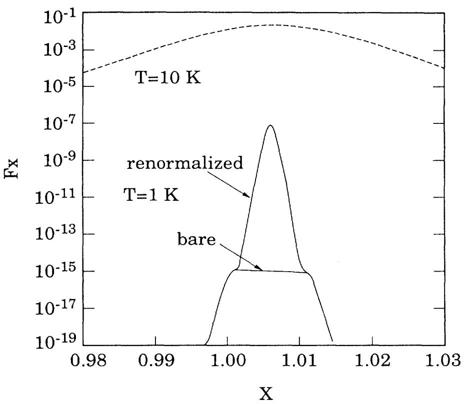

FIG. 3. Result of the integration over y of Fy, as a function of reduced wave vector x. In the scale of this figure, the renormalized and bare results are indistinguishable at $T=10$ K, but there is a strong enhancement due to the phonon renormalization at $T=1$ K.

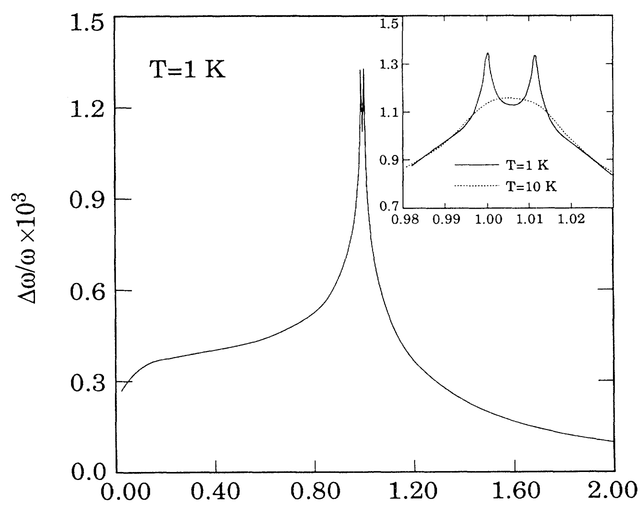

FIG. 4. Relative shift to lower frequencies of the phonon peak in Im$\tilde{D}$, for $T=10$ K (dashed line) and $T=1$ K (solid line). In the scale of the main body of the figure the two are indistinguishable; the inset shows the small difference between them.

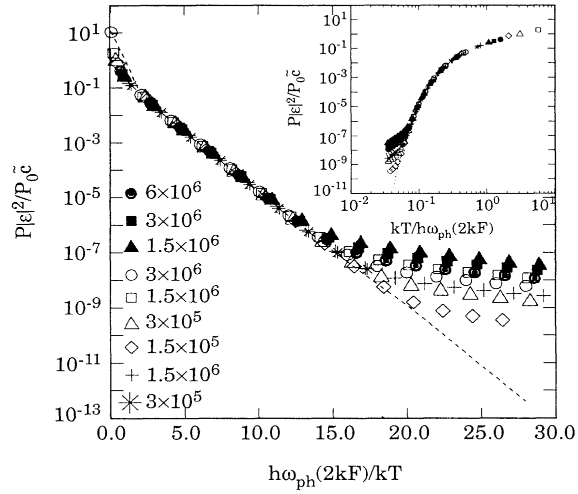

FIG. 5. Power loss per electron as a function of temperature, for several values of well width and electron density, for GaAs parameters, scaled according to the approximate analytical result, Eq. (10). The dashed line shows the single result for all these parameters when the phonon renormalization is neglected. The numbers indicate the electron densities in cm^{-1}; solid symbols: $a=50$ Å; open symbols: $a=100$ Å; cross and star: $a=200$ Å.

difference, in the scale of that figure, between the results with or without phonon renormalization, but for the lower temperature the renormalized result is increased by orders of magnitude over the bare one. One might think that the low temperature behavior could be accounted for by the shift of the resonant peak; we demonstrate this not to be the case by explicitly showing the values of that shift, for the same temperatures, in Fig. 4. The frequency renormalization is essentially the same at both temperatures, and moreover, its magnitude is too small to cause any difference. This again shows that our results cannot be obtained by the resonant coupling approximations which neglect the many-body phonon broadening.

Finally in Fig. 5 we present the integrated results for several choices of quantum wire widths and carrier densities, showing that the scaling implied by Eq. (10) is well-satisfied down to c/t values of the order of 10. For lower temperatures, a power-law behavior appears, and this is the region where emission of virtual phonons is important. Qualitatively, we have a many-body explanation for the restoration at low temperatures of the Bloch-Gruneisen behavior. But in this case, this low temperature behavior is not caused by the excitation of low wave vector phonon modes, which cannot contribute in a 1DEG due to phase-space restrictions, but rather by $2k_F$ low energy virtual phonon modes.

More generally, since the previous argument can, in principle, be applied to momentum relaxation caused by phonon scattering [7], one has an important conceptual guide. When the simple phase-space argument reflects a severe restriction on the scattering rate, the many-body quantum-mechanical (*not thermal*) uncertainty in the modes that couple to electrons can, by itself (and without any extraneous broadening caused by impurities), provide the needed continuum of final states to lift the phase-space restriction. Thus, consistent with several recent experimental observations [8] of various *other* electronic properties of quantum wires, the 1DEG behaves not very differently from higher-dimensional systems with respect to its low temperature transport properties as well, even though this "similarity" with the higher-dimensional behavior is caused by a rather subtle many-body effect in this case.

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