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ABSTRACT

The main aim of this paper is to verify the influence of the terms up to J5 on frozen orbits. For that, first of all, it will be developed the long period expressions of Brouwer theory (Brouwer, 1959) clearly, what provide the perturbations due to geopotencial up to J5 term. The odd terms, what are responsible for long period effects, origin the frozen orbits which theory is being applied on space missions, especially on CBERS-1. This development will allow getting more precise previsions for the evolution of CBERS-1 and similar satellites (SPOT, Landsat, ERS and IRS) orbits.

INTRODUCTION

If perturbations are not take into account the orbital motion of an the artificial satellite would be ellipses with constant sizes and eccentricities, in permanent planes and the satellites would stay on these orbits indefinitely. The principal effects that make the orbit changes are the none homogeneity of Earth's mass distribution, the solar-moon attraction, the solar radiation pressure (direct and albedo effects), the atmospheric drag, forces due to Earth's tides, Poynting Robertson drag, Yarkovsky effect and others. For orbital control purposes, can be important that some elements stay "frozen" in order to make easy some maneuvers adjustment. Specially, for maneuvers that have being carried out INPE with the satellite CBERS-1 (China Brazil Earth Resources Satellite), is important that the perigee stay "frozen", that is, with a constant value.

On contrary of natural bodies cases, which distances themselves, are very large in comparison with their sizes, the artificial satellites are so close that geopotencial secondary terms cannot be neglected. The observations of some natural bodies motion were so imprecise that an approximated theory was enough to describe the diversion noticed. However, the precision on artificial satellites observations is very high and, by this reason, the study of satellites motion in an almost spherical body potential field has developed especially strong since the first satellite launch.

The main aim of this paper is to verify the influence of the terms up to J5 on frozen orbits. For that, first of all, it will be developed the long period expressions of Brouwer theory (Brouwer, 1959) clearly, what provide the perturbations due to geopotencial up to J5 term. The odd terms, what are responsible for long period effects, origin the frozen orbits which theory is being applied on space missions, especially on CBERS-1. This development will allow getting more precise previsions for the evolution of CBERS-1 and similar satellites (SPOT, Landsat, ERS and IRS) orbits.

Brouwer solution (Brouwer, 1959) presents clear analytic expressions, as time function, to the variation of the classic orbital elements (a, e, i, ω, Ω, τ) due to the non-uniform Earth's mass distribution. These analytical formulas allow carrying out the analysis of the temporal behavior of these keplerian elements, as well as the magnitude of the perturbations due to geopotencial. Explicit formulas for secular, short and long period terms due to J2 and J4 are presented in Brouwer solution. Expressions containing terms due to J3 and J5 can also be gotten, but through a supplementary work, repeating the method and including such perturbations.

In this paper, the terms of long period perturbations for eccentricity and argument of perigee up to J5 will be developed analytically, through Brouwer theory. Using Fortran language, these equations have been coded. The program has been tested to several situations and a test for validation was performed. Aiming orbital and maneuvers to be conducted by the INPE Satellite Tracking and Control Center, and after several tests, the model was made "functionally" valid for the CBERS-1 satellite.

BROUWER SOLUTION

If the forces that act on a satellite have gravitational origin, depending on the planet potential *U* exclusively, the motion equations in an inertial system Oxyz will be given as follow (Kovalevsky, 1967):

$$\frac{d^2x}{dt^2} = \frac{\partial U}{\partial x} \;;\; \frac{d^2y}{dt^2} = \frac{\partial U}{\partial y} \;;\; \frac{d^2z}{dt^2} = \frac{\partial U}{\partial z} \qquad \textbf{(1)}$$

where $U = \frac{kM}{r}\left[1 - \sum_{n=1}^{\infty} \frac{1}{r^2} J_n P_n(\operatorname{sen}\varphi)\right]$ and J_n is the oblateness coefficient associate with Legendre polynomial P_n.

Using Delaunay variables given by:

$$L = \sqrt{\mu a} \qquad G = \sqrt{\mu a(1-e^2)} \qquad H = \sqrt{\mu a(1-e^2)}\cos i$$

$$l = M = n(t - t_0) \qquad g = \omega \qquad\qquad h = \Omega \tag{2}$$

the system (1) can be written as:

$$\begin{aligned} &\frac{dL}{dt} = \frac{\partial \Phi}{\partial l}\,;\ \frac{dG}{dt} = \frac{\partial \Phi}{\partial g}\,;\ \frac{dH}{dt} = \frac{\partial \Phi}{\partial h} \\ &\frac{dl}{dt} = -\frac{\partial \Phi}{\partial L}\,;\ \frac{dg}{dt} = -\frac{\partial \Phi}{\partial G}\,;\ \frac{dh}{dt} = -\frac{\partial \Phi}{\partial H} \end{aligned} \tag{3}$$

where

$$\Phi = \frac{\mu}{2a} + R \qquad \text{and} \qquad R = U - \frac{kM}{r}$$

The equations (3) that were solved by Brouwer using canonical transformations were suggested by Von Zeipel (Von Zeipel, 1916). Thus, after two convenient canonical transformations, an analytical solution was obtained in form of series evolving powers of eccentricity and J_n for the variables L, G, h, l, g, h. Clear expressions of secular, short and long period terms factored by J2 and J4 are given in Brouwer paper (Brouwer, 1959).

FROZEN ORBITS

A frozen orbit is characterized by keeping (or trying keeping) constant the argument of perigee and eccentricity of the orbit, in way that to a given latitude the satellite always passes the same altitude, benefiting the users by this regularity. That is, this type of orbit maintains almost constant altitude over any point on Earth's surface.

The design of frozen orbits involves selecting the correct values of eccentricity and argument of perigee, for a given semi major axis and orbital inclination. Up to J3 terms, the following system of nonlinear perturbation equations are satisfied (Cutting, Born and Frautnick, 1978):

$$\begin{aligned} &\frac{d\omega}{dt} = \frac{3nJ_2R_T^{\,2}}{a^2(1-e^2)^2}\left(1-\frac{5}{4}\operatorname{sen}^2 i\right)\left\{1+\frac{J_3R_T}{2J_2a(1-e^2)}\left(\frac{\operatorname{sen}^2 i - e^2\cos^2 i}{e\operatorname{sen} i}\right)\operatorname{sen}\omega\right\} = 0 \\ &\frac{de}{dt} = -\frac{3nJ_3R_T^{\,3}\, seni}{2a^3(1-e^2)^2}\left(1-\frac{5}{4}\operatorname{sen}^2 i\right)\cos\omega = 0 \end{aligned} \tag{4}$$

where

a: semi major axis
e: eccentricity
i: orbital inclination

ω: argument of perigee
R_T: Earth equatorial radius
$n = \sqrt{\mu/a^3}$: mean motion
μ: Earth gravitational constant
J_2: second gravity coefficient
J_3: third gravity coefficient

For argument of perigee values equal to 90º and 270º, the eccentricity perturbations vanish.

LONG PERIOD TERMS DUE TO J3 AND J5

Following the development that Brouwer suggested, the next expressions are obtained from the equations that provide secular and long period terms. For the eccentricity and the argument of perigee variations we have:

$$\dot{\omega} = \frac{3nJ_2R_T^{\,2}}{a^2(1-e^2)^2}\left(1-\frac{5}{4}\mathrm{sen}^2\, i\right)\left\{\begin{array}{l} 1+\dfrac{J_3R_T}{2J_2a(1-e^2)}\times\left(\dfrac{\mathrm{sen}^2\, i-e^2\cos^2 i}{e\,\mathrm{sen}\, i}\right)\mathrm{sen}\,\omega+ \\ \dfrac{J_5R_T^{\,3}}{J_2a^3(1-e^2)^3}\left\{\begin{array}{l} \dfrac{5}{64}\left[\left(\dfrac{(1-e^2)seni}{e}-\dfrac{e\cos^2 i}{seni}\right)\times \\ \qquad (4+3e^2)+eseni(26+9e^2)\right]\times \\ \left(1-9\cos^2 i-\dfrac{24\cos^4 i}{1-5\cos^2 i}\right)- \\ -\dfrac{15}{32}e\cos^2 i\times seni(4+3e^2)\times \\ \left(3+\dfrac{16\cos^2 i}{1-5\cos^2 i}+\dfrac{40\cos^4 i}{(1-5\cos^2 i)^2}\right) \end{array}\right\} sen\omega \end{array}\right\}$$

(5)

$$\dot{e} = -\frac{3nJ_2R_T^{\,2}}{a^2(1-e^2)^2}\left(1-\frac{5}{4}\mathrm{sen}^2\, i\right)\left[\begin{array}{l} \dfrac{J_3R_T}{2J_2a}seni+\dfrac{5}{32}\dfrac{J_5R_T^{\,3}}{J_2a^3(1-e^2)^3}\times \\ seni(4+3e^2)\left(\dfrac{1-9\cos^2 i-24\cos^4 i}{1-5\cos^2 i}\right) \end{array}\right]\cos\omega$$

The expressions (5) has already been adapted from Brouwer paper, replacing:

$$k_2 = \frac{J_2R_T^{\,2}}{2} \qquad A_{5.0} = -J_5R_T^{\,5}$$

$$A_{3.0} = -J_3R_T^{\,3}$$

$$\gamma_2' = \frac{k_2}{a''^2\,\eta^4}$$

$$\gamma_3' = \frac{A_{3.0}}{a''^3\,\eta^6}$$

$$\gamma_5' = \frac{A_{5.0}}{a''^5\,\eta^{10}}$$

$$n_\omega = \frac{3nJ_2R_T^{\,2}}{a^2(1-e^2)^2}\left(1-\frac{5}{4}\mathrm{sen}^2\, i\right)$$

$$\eta = (1-e''^2)^{1/2} = (1-e^2)^{1/2}$$

a” = a

e” = e

I” = i

g” = ω

θ = cosI” = cosi

n_0 = n

USED METHOD

Through a program coded in Fortran language and using orbital data of the CBRS-1 satellite, the solution for equations (4) was previously analyzed. The behavior of the solution was compared with the behavior of a known result for other satellite (Cutting, Born and Frautnick, 1978). Results have been plotted with MS-Excel graphic editor aid, so that the nature of frozen orbits could understand. This feature is extremely important for the users of the images obtained for CCD chamber on board of this type of satellite. In fact, the images from different days can be compared for the same latitude, being used to preview harvests, fire on forests, to locate underground airports and others utilities.

CBERS-1 has as nominal eccentricity e = 0.00016. In this paper, we have worked with the orbit freezing that occurs when argument of perigee is equal to 90º and we have realized, by graphics, that the orbit starts escaping from its initial path to argument of perigee values far from 90º and has the tendency of standing limited when these values are closer to 90º.

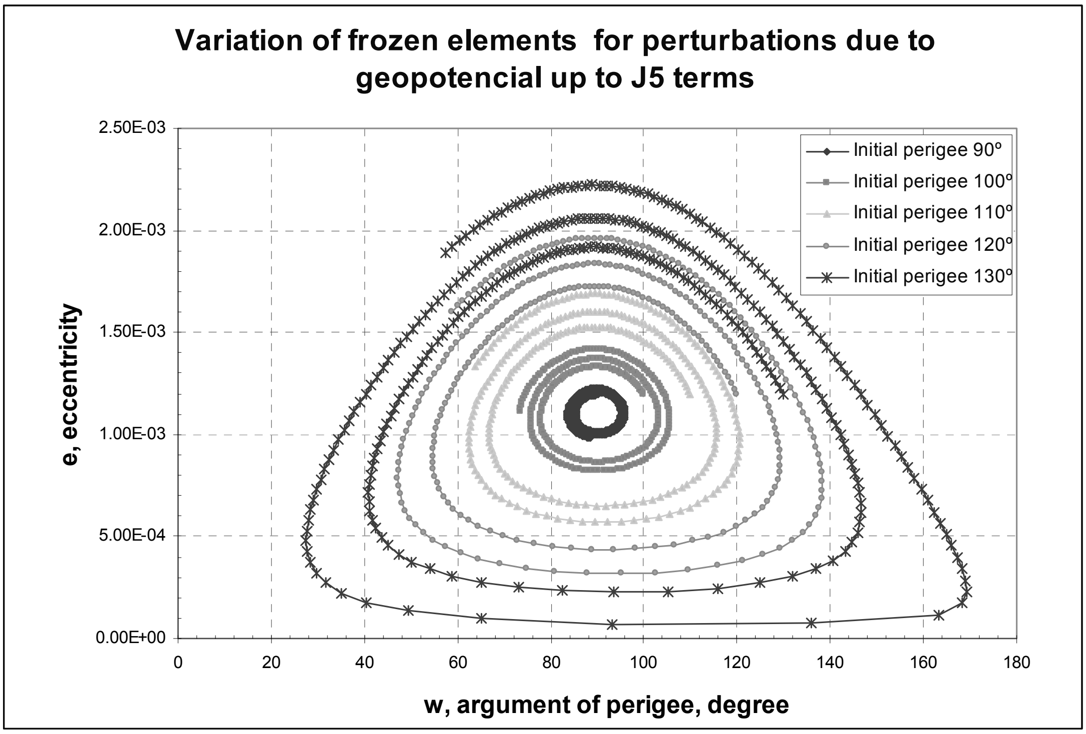
After analyzing the terms which perturbation take into account J3 terms (already existing), terms of the order of J5 were included (see equations (5)). The results were compared with the results for the same equations, but including terms up to J3 only. The results will be shown next.

RESULTS AND ANALYSIS

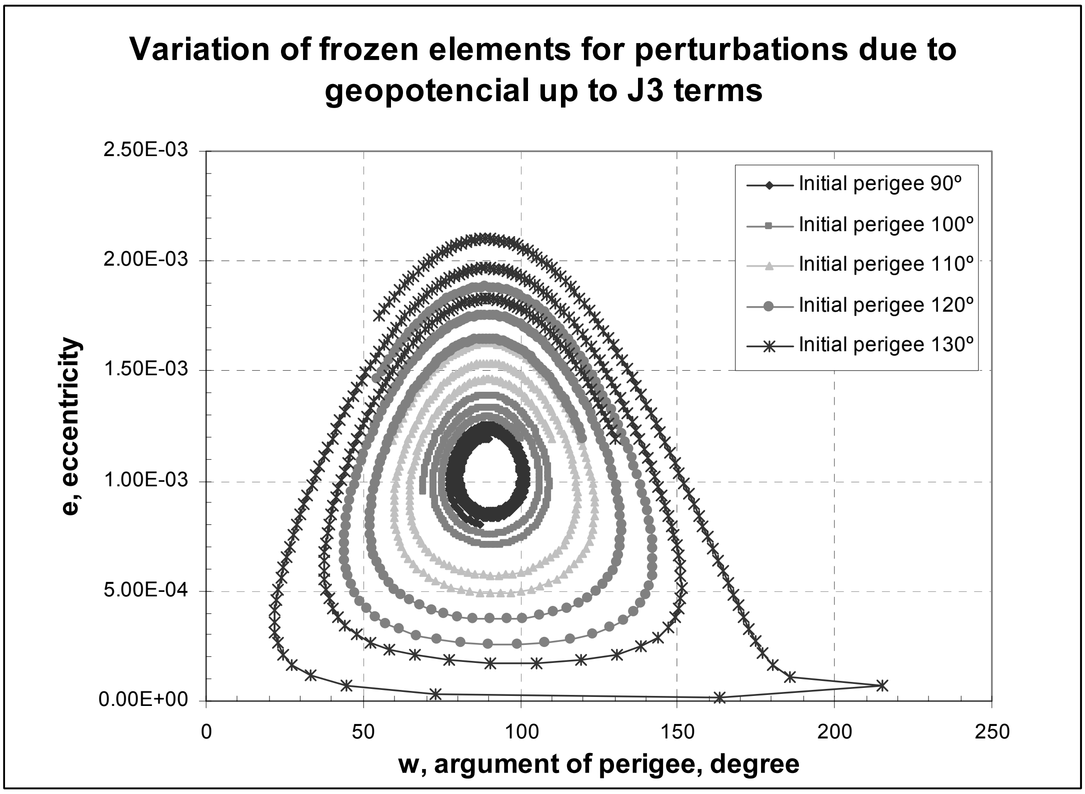
When we include J5 terms on the perturbations due to geopotencial, we expect to get greater precision for the frozen elements (argument of perigee and eccentricity), that is, we expect, before anything, a greater prevision capacity of these elements variations, when compared to the variation including only terms up to J3.

Figures (1) and (2) show curves for several initial conditions for the argument of perigee. These graphics were based on data obtained as the result from equations (5). In Figure (2)

J5 terms were not included, such terms were included only in the program to getting data relative Figure (1).



Graphic 1 – Shows the behavior of eccentricity and argument of perigee to several initial conditions and includes the perturbations up to J5.



Graphic 2 – Shows the behavior of eccentricity and argument of perigee to several initial conditions and includes the perturbations up to J3 only.

Graphics 1 and 2 show CBERS-1 evolution and were obtained for the following values of these satellite orbital elements, to a given date:

- semimajor axis (a) = 7148.763 km
- eccentricity (e) = 0.0011934
- orbital inclination (i) = 98.4896º
- argument of perigee (ω): starting from 90º and variating up to 130º

From the graphics analysis is reasonably perceptible that the curves of Figure 1 have less amplitude of variation for frozen elements argument of perigee and eccentricity than the curves of Figure 2. Just a graphic analysis it is not enough to ensure that the inclusion of J5 actually improves the precision. Thus, we construct a Table considering several initial conditions for the argument of perigee. This Table shows the amplitude of the variations of the argument of the perigee and of the eccentricity. From this Table, we can see the decreasing of the amplitudes (in numerical values) when the perturbations due to geopotencial up to J5 are included.

Initial Conditions		Perturbations due to J3 (Graphic 2)				Perturbations due to J3+J5 (Graphic 1)			
e_0	ω_0	Min. Δe	Max. Δe	Min. Δω	Max. Δω	Min. Δe	Max. Δe	Min. Δω	Max. Δω
0.0011934	90.0º	-2.43E-04	2.08E-04	-11.9387	13.2042	-1.30E-04	-1.16E-04	-5.7368	6.6625
0.0011934	100.0º	-3.83E-04	2.98E-04	-19.8636	20.1881	-3.36E-04	2.61E-04	-16.332	15.9948
0.0011934	110.0º	-3.56E-03	4.86E-04	-29.2081	35.1237	-6.55E-04	4.64E-04	-28.0073	30.0739
0.0011934	120.0º	-9.96E-04	6.30E-04	-47.0009	51.3552	-9.98E-04	6.47E-04	-44.2104	46.9441
0.0011934	130.0º	-1.33E-03	7.50E-04	-71.2391	122.1172	-1.35E-03	7.99E-04	-65.5166	76.2672

Table 1 – Comparative table between the results obtained to J3 and J3+J5

From Table 1 we realized that the amplitude of variation for the argument of perigee, when terms of order of J5 are included, both maximum and minimum amplitudes decreases. This is true for all considered initial conditions. For values of argument of perigee far from the frozen conditions, the values including J5 still complete the cycle, while the cycle with J3 start demeaning (see Figure 2). That is, the inclusion of the effect due to J5 improves the precision for the prevision of the argument of perigee. With respect to the eccentricity, the reduction is subtler, but it still occurs.

In practice, the theory using only term factored up to J3 can persuade to wrong needs of orbital maneuvers correction. If we suppose, for instance, that the mission requires a perigee value between $90^o \pm 10^o$, the J3 theory would foresee a corrective maneuver, while the J5 theory excludes the need of a maneuver, as can be seen on first line of Table 1. So, the conclusion is that it is imperative the inclusion of term up to J5 to improve the precision on planning of maneuvers carry out by the INPE Satellite Tracking and Control Center.

CONCLUSIONS

Long period terms of Brouwer theory, that provide the perturbations due to geopotencial up to J5, were obtained explicitely and coded at computer, using Fortran language. The program has been tested for several situations, and when analysed comparatively with an existent data, the gotten results agree quite well with the reality.

This development allows obtaining more precise previsions for the orbital evolution of CBERS-1 and similar satellite (SPOT, Landsat, ERS e IRS) orbits. By including perturbations due to J5 geopotencial coefficients, we got improvements for precise previsions of frozen orbital elements (argument of perigee and eccentricity). This can was verified by tests and graphics as shown by Table. Thus, we got up to J5 explicit expressions for long period terms of Brouwer theory for argument of perigee and eccentricity. The results are according to what we expected when the precision improves, that is, the reduction of the errors.

Small amplitudes, when J5 is included, improve the precision for prevision of frozen orbital elements. This means to enhance precision not only on maneuvers calculus, but even on maneuvers prevision and so contributing to a better performance in the orbital operations conducted at the INPE Satellite Tracking and Control Center.

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