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DYNAMICS OF COORBITAL SYSTEMS IN THE PLANAR ELLIPTIC CASE

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ABSTRACT

In the restricted problem of three bodies, the motion of the primaries must satisfy the differential equations which describe the dynamics of two bodies problem. Consequently, the primaries might describe elliptic orbits. One purpose in this work is to study when the primaries describe elliptic orbits. The generalization of this case is not trivial: while the restricted circular problem of three bodies still possesses the jacobi integral, the elliptic problem does not. This property of the elliptic problem distinguishes it from the circular problem and indicates the increased degree of difficulty involved in solving it. In this work we show how the two-dimensional elliptic problem can be formulated and numerical simulations in the pulsating coordinates system show the eccentricity effect in the dynamics of coorbital systems.

INTRODUCTION

When the general motion's behavior of a particle near the Lagrangian equilibrium points is studied, more precisely, the horseshoe and tadpole orbits, in a great number of cases it is done using the circular restricted three-body problem. This case, when the primaries move on circles, gives a general definition with the purpose of simplifying its development. However, it is a particular case and to do a more realistic study, elliptical motion of the primaries must be introduced. The case called "elliptic restricted problem", when the primaries move on ellipses is the main subject of this work.

EQUATIONS OF MOTION

The differential equations of motion of the circular restricted three-body problem are deduced using a uniformly rotating Cartesian rectangular coordinates system. In this system, the primaries are fixed and the Hamiltonian does not depend explicitly on the time (Szebehely, 1967, Murray & Dermott, 1999). When the primaries move on elliptic orbits, an uniformly rotating and pulsating system can be introduced, which results again in fixed locations for the primaries. The Hamiltonian, however, does depend explicitly on the independent variable in this case. Such a pulsating coordinate might be introduced by using the variable distance between the primaries as the basic length of the system by which distances are divided. The following dimensionless variables are introduced by using the distance between the primaries

$$r=\frac{a(1-e^2)}{1+e\cos f}\ , \tag{1}$$

where a and e are the semi major axis and the eccentricity of the elliptic orbit of either primary around the other and f is the true anomaly. Now, we introduce a coordinate system, which rotates with the variable angular velocity $\dot{f}$. This angular motion is given by

$$\frac{df}{dt^*}=\frac{k(m_1+m_2)^{1/2}}{a^{3/2}(1-e^2)^{3/2}}(1+e\cos f)^2\ , \tag{2}$$

where t^* is the dimensional time, m_1 and m_2 are the masses of the primaries and k is the gravitacional constant . This equation follows from the principle of the conservation of angular momentum. In the two bodies problem formed by the masses m_1 and m_2 this principle is given by

$$\dot{f}r^2=[a(1-e^2)k^2(m_1+m_2)]^{1/2}$$

Szebehely (1967) shows that the equations of motion in the elliptic restricted problem using the true anomaly as independent variable may be written as

$$\frac{d^2\varsigma}{df^2}+2i\frac{d\varsigma}{df}=grad_{\varsigma}\omega$$

or as

$$\begin{aligned}\frac{d^2\xi}{df^2}-2\frac{d\eta}{df}&=\frac{\partial\omega}{\partial\xi}\\ \frac{d^2\eta}{df^2}+2\frac{d\xi}{df}&=\frac{\partial\omega}{\partial\eta}\end{aligned} \tag{3}$$

where $\varsigma=\xi+i\eta$, $i=\sqrt{-1}$ and ξ, η are the pulsating dimensionless coordinates of the third body in the non-uniformly rotating coordinates system and

$$\omega=\frac{\Omega}{1+e\cos f} \tag{4}$$

Where

$$\Omega = \frac{1}{2}(\xi^2 + \eta^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu)$$

The equations of motion (3) show complete formal identity with the equation describing the restricted problem. The function ω, however, shows that this identity is restricted only to the form but not to the contents of the equations. Therefore, the function ω depends on the coordinates as well as on the independent variable f, while Ω depends only on the coordinates.

ECCENTRICITY EFFECTS ON THE STABILITY OF TADPOLE AND HORSESHOES ORBITS

The object of the numerical simulations is to study the behavior of tadpole and horseshoes orbits, when an asteroid motion is started near the Lagrangian points L3, L4 and L5. For the simulations, a pulsating dimensionless coordinates system (ξ, η) is chosen, with Jupiter's coordinates ς_1 = (μ, 0) and Sun's coordinates ς_2 = ($\mu - 1$,0), where $\mu = m_2/(m_1 + m_2)$. Here, m_2 is the mass of the Sun and m_1 of Jupiter, with $m_1 + m_2 = 1$.

If we take

$$\varsigma = z/r = \xi + i\eta$$

the equations of the motion for (ξ, η) become

$$\frac{d^2\xi}{df^2} - 2\frac{d\eta}{df} = \frac{r}{a(1-e^2)}\left[\xi - \frac{m_1}{m_1+m_2}\frac{\xi-\xi_1}{r_1^3} - \frac{m_2}{m_1+m_2}\frac{\xi-\xi_2}{r_2^3}\right]$$

$$\frac{d^2\eta}{df^2} + 2\frac{d\xi}{df} = \frac{r}{a(1-e^2)}\left[\eta - \frac{m_1}{m_1+m_2}\frac{\eta}{r_1^3} - \frac{m_2}{m_1+m_2}\frac{\eta}{r_2^3}\right]$$

where $r = a(1-e^2)/(1+e\cos f)$ and the positions of the primaries are $\varsigma_1 = \xi_1 = \mu$ for Jupiter and $\varsigma_2 = \xi_2 = \mu - 1$ for the Sun.

We take $m_1 = 0.00095388$ for the simulations and r_1 represents the Jupiter-Asteroid distance and r_2 represents the Sun-Asteroid distance.

The center of mass is the origin of the system.
Therefore, the coordinates of the primaries and the Lagrangian points L3, L4 and L5 are

L3 = (-1, 00039745, 0)
L5 = (0,49904612, -0,86602540)
Jupiter = (0,99904612, 0)
L4 = (0,49904612, 0,86602540)
Sun = (-0,00095388, 0)

The motions of asteroids about the Lagrangian points L5, L4, L3 are shown with the initial conditions specified in Figures 1 to 7.

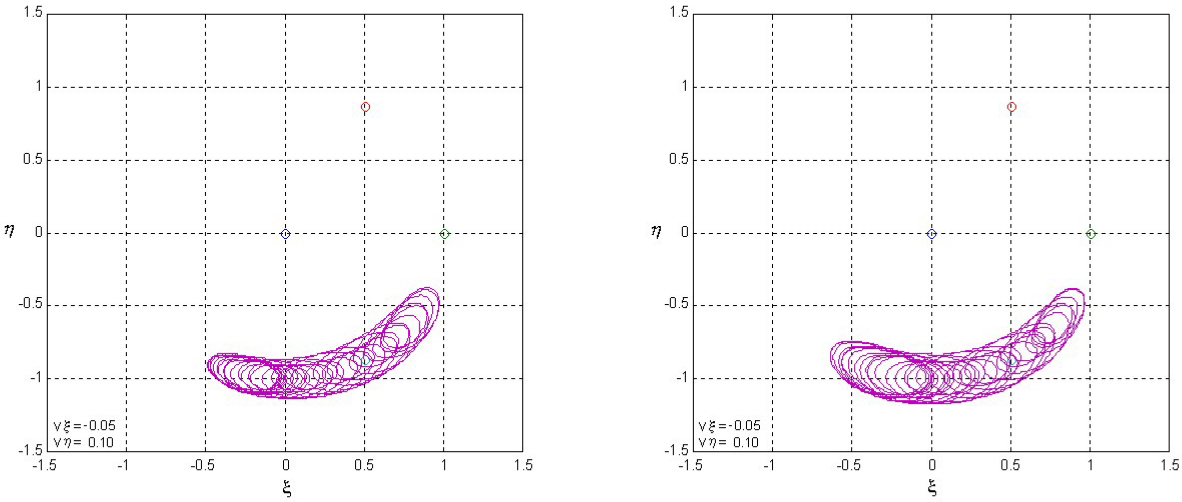

Figure 1: Asteroid's orbit about the L5 equilibrium point with the starting conditions $\dot{\xi} = -0.05$ and $\dot{\eta} = 0.1$. The eccentricity of the left's graphic is e = 0.0 and for the one of the right is e = 0.0489.

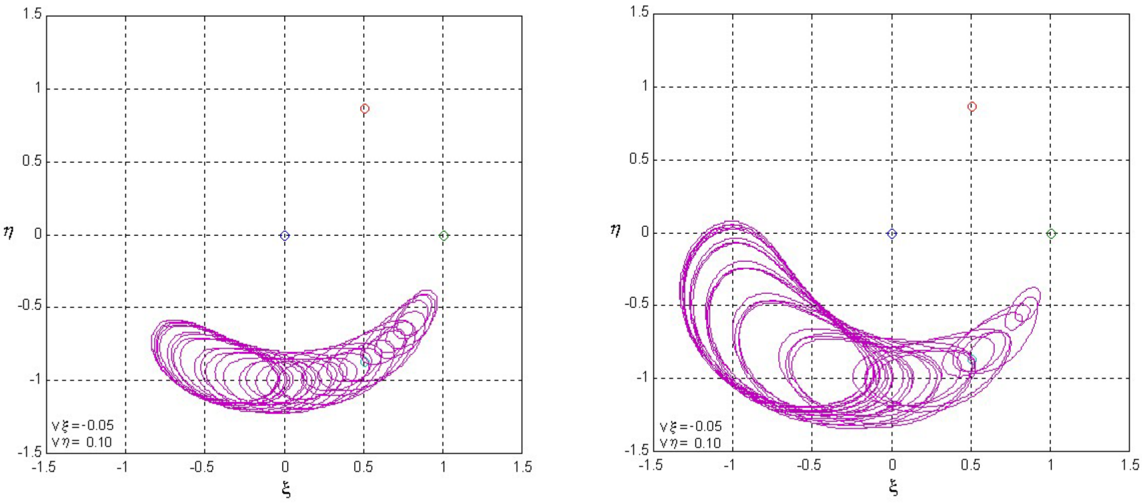

Figure 2: Asteroid's orbit about the L5 equilibrium point with the starting conditions $\dot{\xi} = -0.05$ and $\dot{\eta} = 0.1$. The eccentricity of the left's graphic is e = 0.1 and for the one of the right is e = 0.2.

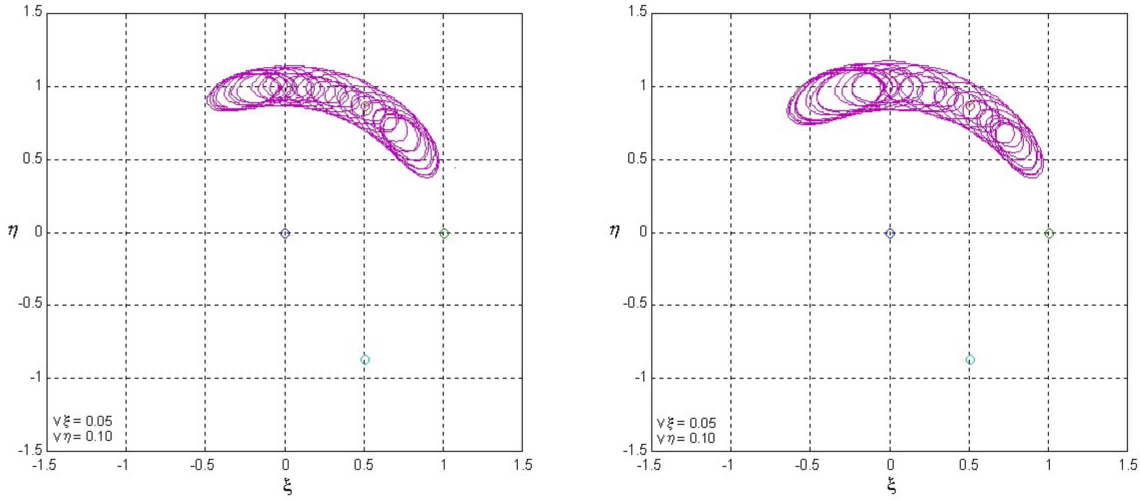

Figure 3: Asteroid's orbit about the L4 equilibrium point with the starting conditions $\dot{\xi} = 0.05$ and $\dot{\eta} = 0.1$. The eccentricity of the left's graphic is e = 0.0 and for the one of the right is e = 0.0489.

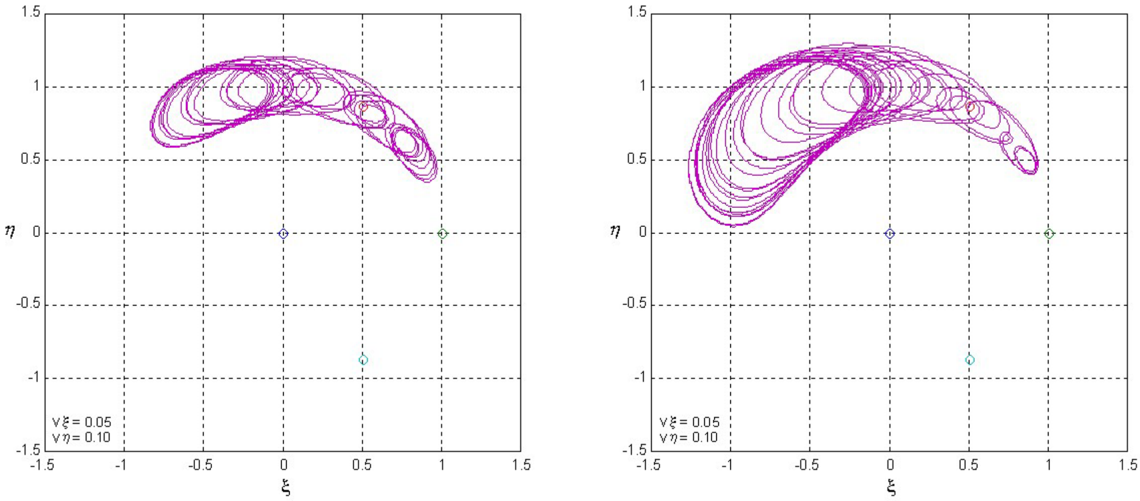

Figure 4: Asteroid's orbit about the L4 equilibrium point with the starting conditions $\dot{\xi} = 0.05$ and $\dot{\eta} = 0.1$. The eccentricity of the left's graphic is e = 0.1 and for the one of the right is e = 0.2.

The trajectories shown in Figure1 have been started with a low eccentricity closer to L5. While in Figure 2 the eccentricity has been increased from 0.1 to 0.2. Note that in each case the path is more elongated toward L3 point . Furthermore, in Figure 1 the orbit extends over 90^{o} ; the orbit which was started with the eccentricity of the order of 0.2 , shown in Figure 2 , extends over 160^{o}. The trajectories of an asteroid , closer to L4, in Figure 3 and Figure 4 have the similar behavior as those shown in Figure 1 and Figure 2 . They seem to have a symmetric distribution along the ξ -axis.

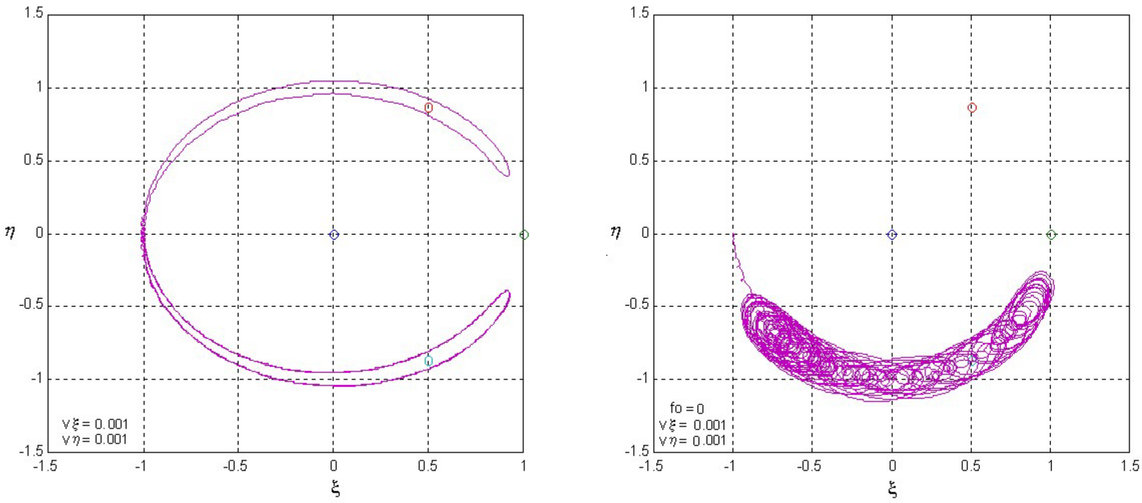

Figure 5: Asteroid's orbit about the L3 equilibrium point with the starting conditions $f = 0$, $\dot{\xi} = 0.05$ and $\dot{\eta} = 0.1$. The eccentricity of the left's graphic is e = 0.0 and for the one of the right is e = 0.0489.

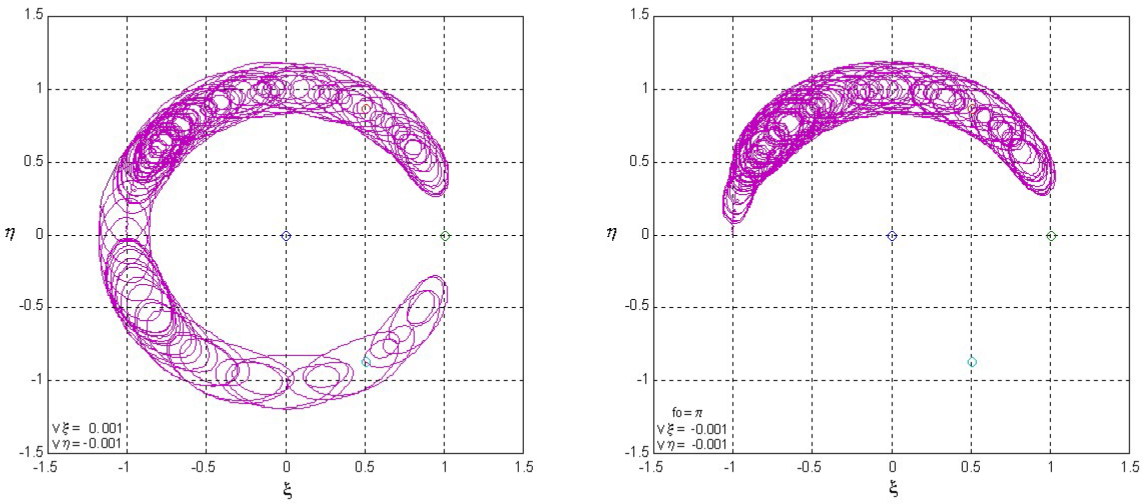

Figure 6: Asteroid's orbit about the L3 equilibrium point with the eccentricity of order of Jupiter. The starting conditions of the left's graphic are $f = 0$, $\dot{\xi} = 0.001$ and $\dot{\eta} = -0.001$ and for the one of the right are $f = \pi$, $\dot{\xi} = -0.001$ and $\dot{\eta} = -0.001$.

The question now as to what kind of orbit would we expect if we increase the initial distance from L4 point or L5 point even more? The resulting orbit will compass both L4 and L5. These are referred to as horseshoe orbits and two examples are shown in Figure 5 when the eccentricity is zero and in Figure 6 with the eccentricity of order of Jupiter. When the eccentricity is non-zero, the libration amplitude of the horseshoe orbit is larger. If we change the initial conditions of velocity and f values, the path is less elongated and the asteroid's orbit librates about L5 point in Figure 5 and about L4 point in Figure 6 . Furthermore, the libration amplitude of the orbit about L5 point is smaller and more narrow than those of the orbit about L4 point.

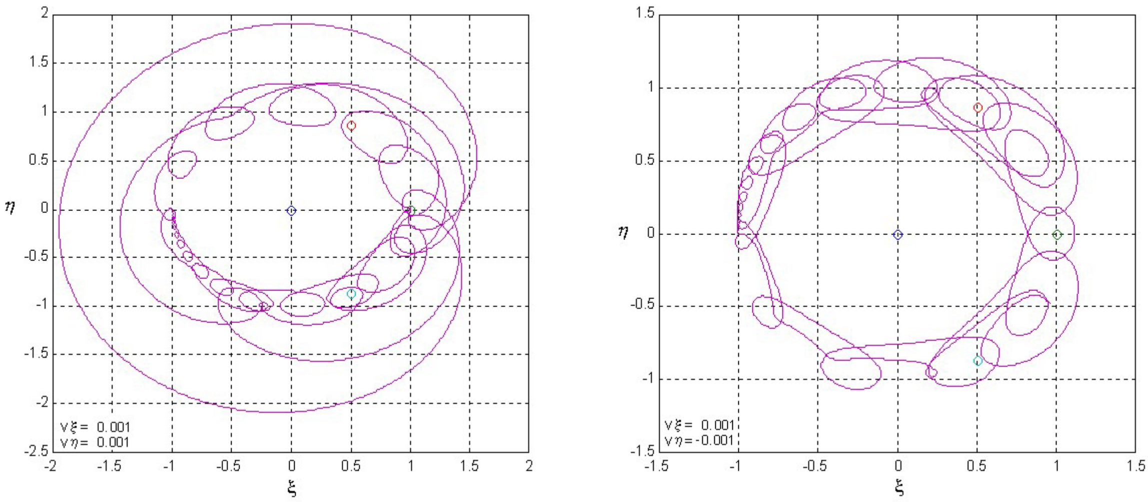

Figure 7: Asteroid's orbit about the L3 equilibrium point with the eccentricity $e = 0.1$ and $f = 0$. The starting conditions of the left's graphic are $\dot{\xi} = 0.001$, $\dot{\eta} = 0.001$ and for the one of the right are $\dot{\xi} = 0.001$, $\dot{\eta} = -0.001$.

The increase of the eccentricity to a value over 0.1 will leave the asteroid’s orbit not coorbital, as it is shown in Figure 7.

CONCLUSION

In this work is shown that in the rotating frame the eccentricity has a notable effect on the asteroid's orbit near the Lagrangian points L4, L5, and L3. The eccentricity makes the asteroid's orbit increases its libration's amplitude near L4 and L5 points, as is shown in Figures 1 to 4. Therefore, we may suppose that the increase of eccentricity makes the orbits less stable, up to a limit, wherever the particle will escape. The case more notable of the simulations has happened when the asteroid, starting near L3 has been captured under special conditions, with eccentricity of the order of Jupiter's, by the L4 and L5 Lagrangian points. Several simulations have been done and when the time is increased out of 10^5, the asteroid's orbit stops librating about L4. So we have a more durable capture at L5, even if we take the aphelion and not the perihelion as initial condition, as shows Peale (1993) suppositions. Therefore, a non zero eccentricity for Jupiter's orbit increases the stability of asteroids at L5 while decreases the stability at L4.

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